Question 1

a) $(x-[1-\sqrt{5}])(x-([1+\sqrt{5}]) = 0$ ...

$[(x-1)^2-5]$ } is a factor

or x^2-2x-4 }

$(x^2-2x-4)(x^2+2x+2)=0$

$x = 1 \pm \sqrt{5}$ or $x = -1 \pm i$

(5)

b) $a+bi = i(3+5i)$

$a+bi = 3i-5$ ✓ $a=-5$ $b=3$

$(a+bi)^2 = (3i-5)^2$ ✓m $a^2-b^2=16$

$a^2-b^2+2abi = -9+25-30i$

(4)

$\therefore a^2-b^2 = 16$ ✓

Q2

a) $x^2-x-12=0$ ✓

$x=4$ or $x\neq -3$

or $x^2+x-12=0$ ✓

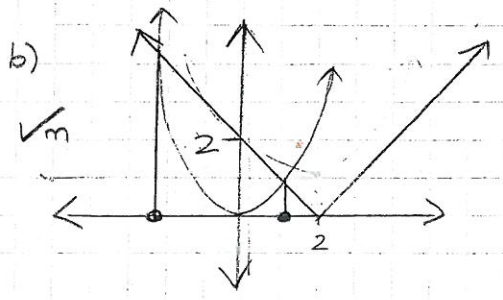
$x=-4$ or $x\neq 3$

$(|x|-4)(|x|+3)=0$

$|x|=4$ $|x|\neq -3$ ✓

(4) $x=\pm 4$

b)



✓m

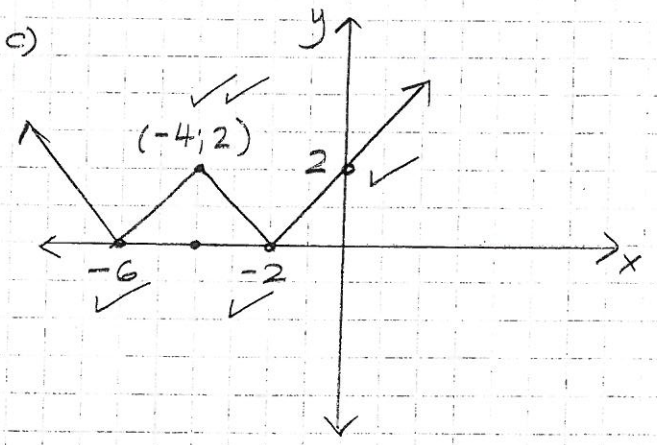
$\begin{cases} x^2 = -x+2 \\ x^2+x-2=0 \end{cases}$ ✓

$x=-2$ or $x=1$ ✓

$-2 < x < 1$ ✓ ✓

(5)

c)



(5)

Q3

a) $\ln(e^{2x}-12) = x$

$e^{2x}-12 = e^x$ ✓

$e^{2x}-e^x-12=0$

$e^x = 4$ or $e^x = -3$ N.V

$\therefore x = \ln 4$

$= 1,4$

(5)

b) $g(f(x)) = e^{[\ln(x-1)+2]-2}+1$

$= e^{\ln(x-1)}+1$

$= x-1+1$

$= x$

dom: $x > 1$

Ran: $y > 1$

(8)

Q4.

If $n=1$ ✓

LHS $= \log x$ ✓

RHS $= \frac{1}{2}\log x^{1+1}$ ✓

$= \frac{2}{2}\log x$ ✓

$= \log x$ ✓

5

$\therefore$ true for $n=1$

assume true for $n=k$ ✓

$\therefore \log x + 2\log x + \dots + k\log x = \frac{k}{2}\log x^{k+1}$

1

If $n=k+1$ ✓

LHS $= \log x + 2\log x + \dots + k\log x + (k+1)\log x$ ✓

$= \frac{k}{2}\log x^{(k+1)} + (k+1)\log x$ ✓

$= \frac{k}{2}(k+1)\log x + (k+1)\log x$ ✓

$= (k+1).\left[\frac{k}{2}\log x + \log x\right]$ ✓

$= (k+1)\left(\frac{k}{2}+1\right)\log x$ ✓

$= (k+1)\left(\frac{k+2}{2}\right)\log x$

RHS $= \frac{k+1}{2}\log x^{(k+1+1)}$ ✓

$= \frac{(k+1)(k+2)}{2}\log x$ ✓

8

$\therefore$ true for $n=k+1$ an $\therefore$ for $n \in \mathbb{N}$

✓ sub $x=-1$

Q5

$a(-1)+b+1 = ab$ ✓ and $ab = -1-3b$ ✓

$ab + a = b + 1$

✓ m simult

$a(b+1) = b+1$ ✓

$\Rightarrow -1-3b = b$

$a = 1$ ✓

$-\frac{1}{4} = b$ ✓

6

Q6

$y'(x) = 3ax^2 + 2bx + c$ ✓

$y''(x) = 6ax + 2b$ ✓

$y'(0) = -3$ ✓

$c = -3$ ✓

$y''(2) = 0$ ✓

$12a + 2b = 0$

$6a + b = 0$ ✓

and $y(2) = -22$ ✓

$8a + 4b + 2(2) = -22$

$2a + b = -4$ ✓

$a = 1$ ✓ $\quad b = -6$ ✓

10

Q7

a) $f(x) = x^2 - 3 = 0$ ✓

$f'(x) = 2x$ ✓

If $a_1 = 2$

$a_2 = 2 - \frac{(2^2-3)}{2(2)} = 1,75$ ✓ ✓

$a_3 = 1,732\ldots$

$\therefore x = \sqrt{3} = 1,73203$ ✓ (5)

b) You will find $x = -\sqrt{3}$ (the negative root) ✓ (1)

Q8

a) i) $\text{Area}_1 \triangle OCD = \frac{1}{2} \times 10 \times 10 \times \sin 0,8$ ✓ Area Rule

$\text{Area}_2 \text{ sector } OAB = \frac{1}{2} \cdot 6^2 \times 0,8$ ✓ ca ✓ formula

shaded $A = A_1 - A_2$

$= 21,5 \text{ cm}^2$ ✓ ca (4)

ii) $CD^2 = 10^2 + 10^2 - 2 \cdot 10 \cdot 10 \cdot \cos 0,8$ ✓ Cos Rule

$CD = 7,78\ldots$ ✓

$AB = 6 \times 0,8$ ✓

Perimeter $= 4 + 4 + CD + AB$

$= 29,6 \text{ cm}$ ✓ ca (4)

b) i) $\lim_{x \to \pi/4} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$ ✓

$= \lim_{x \to \pi/4} \cos x + \sin x$ ✓

$= \cos \pi/4 + \sin \pi/4$ ✓ sub

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$ (4)

$= \sqrt{2}$ ✓

ii) $= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{2x}{\sin 3x} \cdot \frac{1}{3}$ ✓ ✓

$= \frac{1}{3}(1)\left(\frac{1}{1}\right)$ ✓

$= \frac{1}{3}$ (4)

Q9. a) $g(x) = (4x^2+1)^{\frac{1}{2}}$ ✓

$g'(x) = \frac{1}{2}(4x^2+1)^{-\frac{1}{2}} \cdot 8x$ ✓ ✓

$= \frac{4x}{\sqrt{4x^2+1}}$ ✓ (5)

b) $y' = \frac{\sec 2x \cdot \tan 2x \cdot 2 \cdot x^2 - \sec 2x \cdot 2x}{(x^2)^2}$ ✓ ✓ ✓ ✓ ✓ ✓ quotient Rule

$= \frac{2x^2 \sec 2x \cdot \tan 2x - 2x \sec 2x}{x^4}$ (6)

c) $\cos y \cdot \frac{dy}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$ (4)

Q10 product Rule

a) $2x - [1 \cdot y + x \cdot \frac{dy}{dx}] + 2y \cdot \frac{dy}{dx} = 0$

$2x - y - x \cdot \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx}[2y - x] = y - 2x$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}$ (7)

b) $m_{LT} = \frac{dy}{dx} = \frac{0-2}{0-1} = 2$

$m_N = -\frac{1}{2}$

$\therefore y - 0 = -\frac{1}{2}(x-1)$ } sub in

$2y + x - 1 = 0$ (3)

Q11, log law

$y = \ln(x+1) - \ln(3x-4)$

$\frac{dy}{dx} = \frac{1}{x+1} - \frac{3}{3x-4}$ (3)

Q12

a) $(0;0)$ (2)

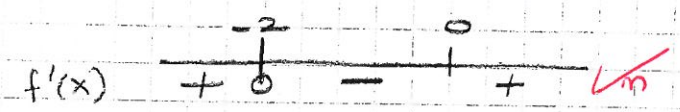
b) $f'(x) = \frac{2x(x+1) - x^2(1)}{(x+1)^2}$ quotient Rule

$= \frac{x^2+2x}{(x+1)^2}$ (6)

c) $f'(x) = 0$

$\frac{x(x+2)}{(x+1)^2} = 0$

$x = 0$ or $x = -2$



$\therefore (-2;-4)$ local max

$(0;0)$ local min (6)

d) vertical: $x = -1$

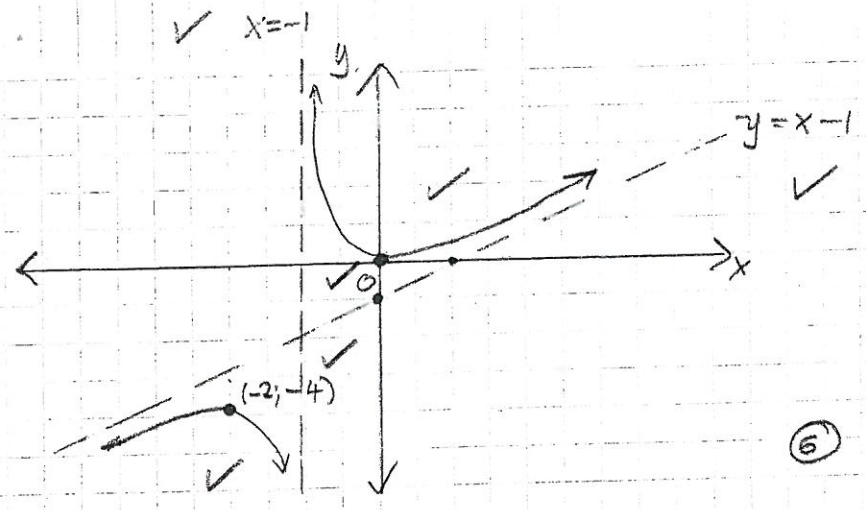
oblique: $y = \frac{x^2+x}{x+1} - \frac{x}{x+1}$

partial fractions

$= x - \frac{x+1}{x+1} + \frac{1}{x+1}$

$y = x - 1 + \frac{1}{x+1}$

$\therefore y = x - 1$ (6)



(6)

Q13

a) $\left[4x - \frac{3x^3}{3}\right]_{-1}^{k}$

$(4k - k^3) - (4(-1) - (-1)^3) = 3$

$4k - k^3 = 0$

$k(4-k^2) = 0$

$k = 0$ or $k = \pm\sqrt{4} = \pm 2$

but $k > -1$

b) i) $= \int \frac{1}{2}\left[\sin(3x+3x) - \sin(3x-3x)\right] dx$

$= \frac{1}{2}\int \sin 6x \, dx$

$= \frac{1}{2} \times \frac{1}{6} \cdot (-\cos 6x) + C$

$= -\frac{1}{12}\cos 6x + C$ (5)

ii) $= -\frac{1}{4} \ln|1-4x| + C$ (4)

iii) let $u = \ln x$ $\quad du = \frac{1}{x}dx$

$\int \frac{\ln x}{x} dx$

$= \int u \, du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$ (8)

c) $\frac{3}{x^2-3x} = \frac{-1}{x} + \frac{1}{x-3}$

$A = -1$ $\quad B = 1$

$\int \frac{3}{x^2-3x} dx$

$= \int \frac{-1}{x} dx + \int \frac{1}{x-3} dx$

$= -\ln|x| + \ln|x-3| + C$ (8)

d) let $u = x$ $\quad dv = e^x dx$

$du = 1\,dx$ $\quad v = e^x$

$\int x \cdot e^x dx = uv - \int v \, du$

$= x \cdot e^x - \int e^x dx$

$= x \cdot e^x - e^x + C$ (8)

Q14

$\sin x = \cos x$

$x = \pi/4$ ✓

$\therefore \int_0^{\pi/4} (\cos x - \sin x)\,dx$ ✓

$= \left[-\sin x + \cos x\right]_0^{\pi/4}$ ✓ ✓

$= \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - (\sin 0 + \cos 0)$ ✓sub

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1$ (6)

$= \underline{\sqrt{2} - 1}$ ✓

Q15

a) ✓

$V = \frac{\pi}{2}\left[\left(\sqrt{\frac{1}{2}}\right)^2 + \left(\sqrt{1}\right)^2 + \left(\sqrt{\frac{3}{2}}\right)^2 + \left(\sqrt{2}\right)^2\right]$ ✓ ✓ ✓ ✓

$= \frac{\pi}{2} \cdot 5$

$= 2{,}5\pi$ ✓ (7)

b) let $n \to \infty$ (or $\Delta x \to 0$) ✓ (1)

c) $V = \pi \int_0^2 (\sqrt{x})^2\,dx$ ✓ formula ✓ limits

$= \pi \int_0^2 (x)\,dx$ ✓ $= \pi \left.\frac{x^2}{2}\right|_0^2$

$= 2\pi$ ✓ (5)