Question 1 [9 marks]

a) Given that $x = 1 - \sqrt{5}\ and\ x = 1 + \sqrt{5}$ are roots of the equation

$x^4 - 6x^2 - 12x - 8 = 0$

Determine all Complex roots. (5)

b) If $\frac{a+bi}{i} = 3 + 5i\ ;\ a, b \in R$ evaluate $a^2 - b^2$ (4)

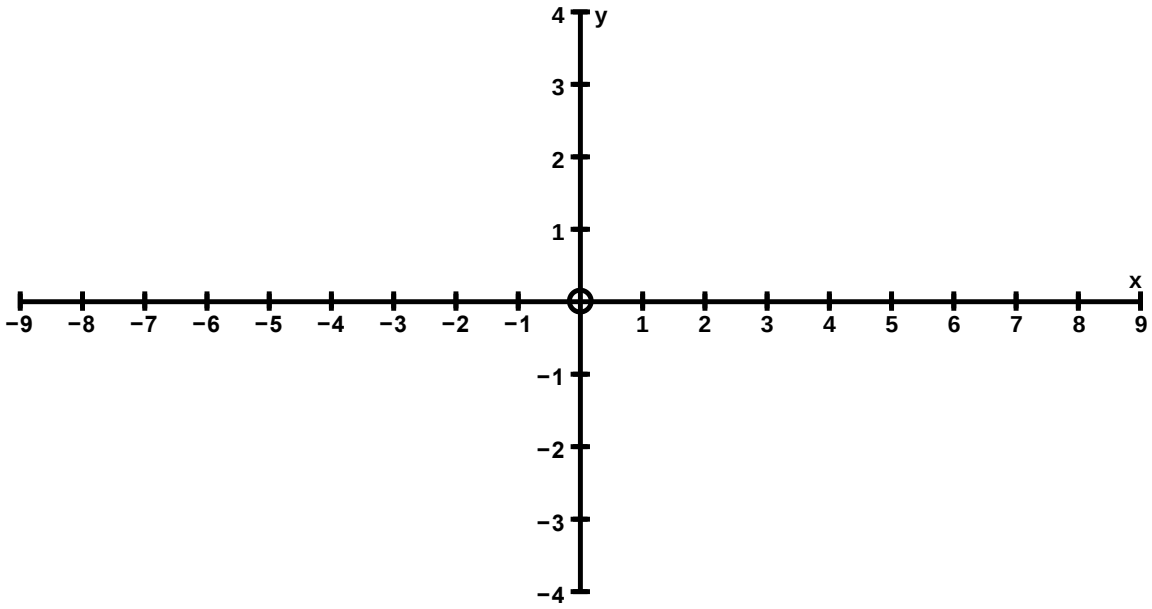
Question 2 [14 marks]

Solve for x

a) $|x|^2 - |x| - 12 = 0$ (4)

b) $|x - 2| > x^2$ (5)

c) Sketch the graph of $y = \big||x + 4| - 2\big|$ (use the axes below) (5)



Question 3 [13 marks]

a) Solve for x (show all working) $\ln(e^{2x} - 12) = x$ (5)

b) If $f(x) = \ln(x-1) + 2$ and $g(x) = e^{x-2} + 1$ determine $g(f(x))$ and state its Domain and Range. (8)

Question 4 [14 marks]

Use Mathematical Induction to prove

$$logx + 2\,logx + 3\,logx + \ \ldots\ + n\,logx = \frac{n}{2}\,logx^{n+1}$$

Where n ∈ N ; x > 0 (14)

Question 5 [6 marks]

Given $f(x) = \begin{cases} ax + b + 1 & if\ x < -1 \\ ab & if\ x = -1 \\ x - 3b & if\ x > -1 \end{cases}$

Determine the values of a and b such that f is continuous for all values of x (6)

Question 6 [10 marks]

a) A cubic function $y = ax^3 + bx^2 + cx$ has a gradient of - 3 at the origin, and has a point of inflection at (2 ; -22).

Determine the values of a ; b and c (10)

Question 7 [6 marks]

a) Consider the equation $x^2 = 3$

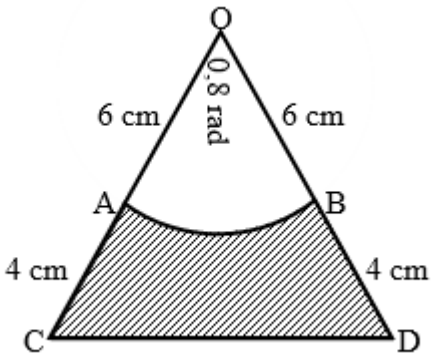
Use Newton–Rhapson method to the calculate the value of $\sqrt{3}$ (5)

Start with $a_1 = 2$ as your first approximation and show your working to calculate a_2 , then work until correct to 5 decimal places

b) Explain what will happen if you chose $a_1 = -1$ (1)

Question 8 [16 marks]

a)



In the diagram, OCD is an isosceles triangle with OC = OD and $C\hat{O}D = 0,8$ radians. The points A and B, on OC and OD respectively, are joined by an arc of a circle with centre O and radius 6 cm. Find:

i) The area of the shaded region. (4)

ii) The perimeter of the shaded region. (4)

b) Evaluate the following limits

i) (4)

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

ii) (4)

$$\lim_{x\to 0} \frac{\sin x}{\sin 3x}$$

Question 9 [15 marks]

Differentiate with respect to x (do not leave answers with negative exponents)

a) $g(x) = \sqrt{4x^2 + 1}$ (5)

b) $y = \frac{\sec 2x}{x^2}$ (6)

c) Use implicit differentiation to find $\frac{dy}{dx}$ if $\sin y = \cos x$ (4)

Question 10 [10 marks]

Given $x^2 - xy + y^2 = 1$

a) show that $\frac{dy}{dx} = \frac{y-2x}{2y-x}$ (7)

b) find the equation of the **normal** to $x^2 - xy + y^2 = 1$ at the point (1 ; 0) (3)

a normal is a line perpendicular to the tangent through the point of contact

Question 11 [3 marks]

If $y = ln\frac{x+1}{3x-4}$ show that $\frac{dy}{dx} = \frac{1}{x+1} - \frac{3}{3x-4}$ (3)

Question 12 [26 marks]

$$f(x) = \frac{x^2}{x+1}$$

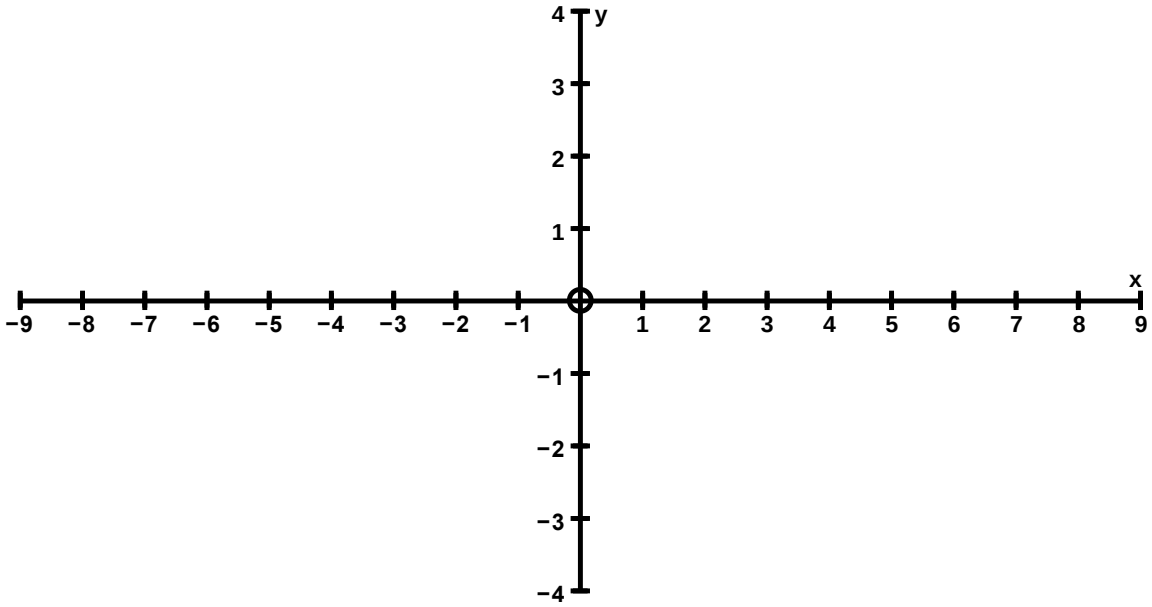
a) Find the intercepts with the axes (2)

b) Find $f'(x)$ and simplify (6)

c) Determine the turning points of f and classify them as local maxima or minima (6)

d) Determine all asymptotes for the function (6)

e) Sketch the graph of $y = f(x)$ on the axes below (6)



Question 13 [39 marks]

a) Determine the value(s) of k such that (6)

$$\int_{-1}^{k} (4 - 3x^2)dx = 3$$

b) Determine the following integrals.

i) $\int sin3x\ cos3x\ dx$ (5)

ii) $\int \frac{1}{1-4x}\ dx$ (4)

iii) $\int \frac{\ln x}{x}\ dx$ (8)

c) Determine A and B if (8)

$$\frac{3}{x^2 - 3x} = \frac{A}{x} + \frac{B}{x - 3}$$

and hence determine

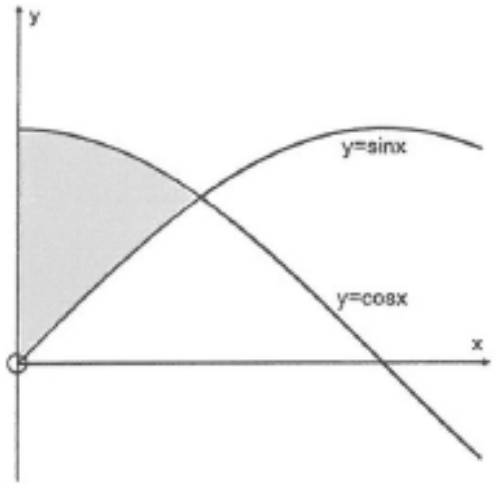
$$\int \frac{3}{x^2 - 3x} dx$$

d) Use integration by parts to determine $\int x\, e^x\, dx$ (8)

Question 14 [6 marks]

The diagram shows the sketch graphs of $y = \sin x \ \ and \ \ y = \cos x$

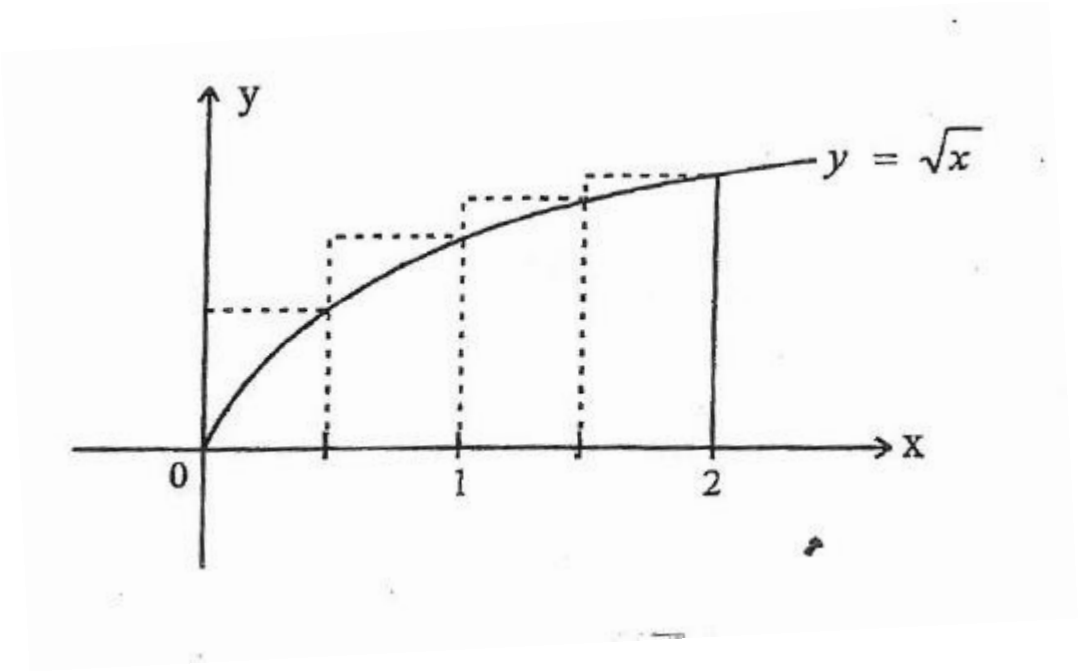
Without use of a calculator, calculate the shaded area. (6)



Question 15 [13 marks]

A parabolic cylinder is obtained by revolving the curve $y = \sqrt{x}$ about the x-axis

on the interval $0 \leq x \leq 2$ (x is measured in metres)



Give your answers in terms of π and show working

a) The volume is estimated by partitioning the interval [0 ; 2] into 4 equal sub-intervals with equal width, and the height on the right of the sub-interval as shown

Calculate this estimated volume (7)

b) How could this estimate be improved? (1)

c) Calculate the exact volume (5)

TOTAL = 200