Grade 12: Advanced Programme Mathematics Preliml

Paper 1: Calculus & Algebra

2 Hour

200 Marks

September, 2019

Examiner: M Klein

Name: _____

Teacher: _____

1. This paper consists of 6 pages and an information sheet.

2. Show ALL calculations, answers only will NOT be awarded full marks.

3. Approved non-programmable calculators are permissible unless stated otherwise. Ensure your calculator is set to RADIANS

4. Round off answers to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total
Mark Achieved													
Possible Mark													
Marker													

Question 1 [32 Marks]

(a) Determine a cubic equation with roots -3 and $(2 - i)$. [4]

(b) Solve the equation $ln(e^{2x} - 6) = x$ [7]

(c) Solve for x if $log_{\frac{1}{10}}(x - 20) + log(2x) = 1$ [6]

(d) Solve for $x \in R$ if $\dfrac{x+4}{|x+1|} < x$. [9]

(e) Solve for x and y, if $(x + 2i)(4 - i) = 14 + iy$. [6]

Question 2 [12 Marks]

Use mathematical induction to prove that

$$8^n - 7n + 6$$

is divisible by 7 for all $n \in N$. [12]

Question 3 [11 Marks]

Determine the value of a and b if $f(x)$ is differentiable at $x = 2$. [11]

$$f(x) = \begin{cases} ax - b - 1 & \text{if } x < 2 \\ bx^2 - ax + 5 & \text{if } x \geq 2 \end{cases}$$
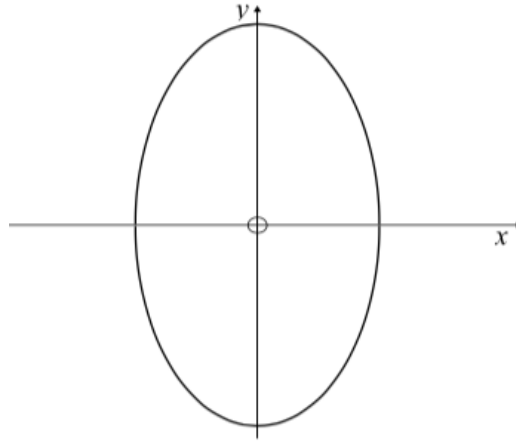
Question 4 [17 Marks]

Differentiate the following:

(a) $\cos^3(2x + 1) \times \tan(x^2 + 2)^3$ [9]

(b) $\dfrac{\operatorname{cosec} x^2}{\cot^2 x}$ [8]

Question 5 [8 Marks]

Determine an expression for the nth derivative of $f(x) = \dfrac{1}{3x + 1}$ [8]

Question 6   [15 Marks]
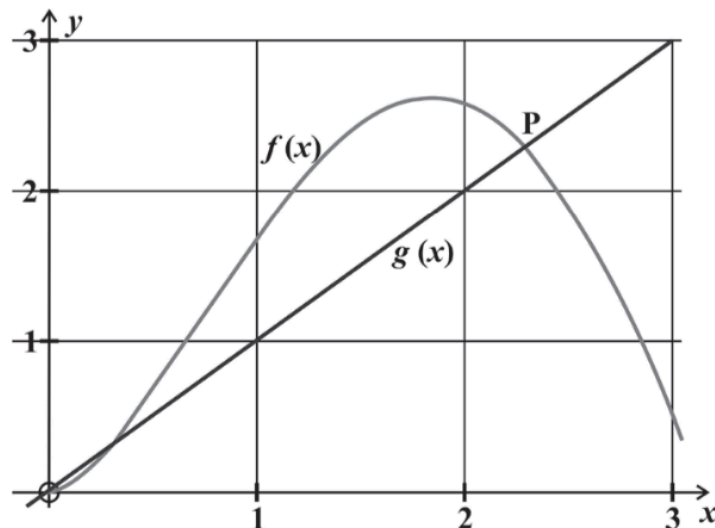
The ellipse $4x^2 + y^2 = 72$ is given below.

Determine the equations of the tangents to the ellipse which are perpendicular to the line $x + 2y + 3 = 0$ [15]

Question 7   [10 Marks]

Below is a sketch of the graphs $f(x) = 2x\sin x$ and $g(x) = x$. The graphs intersect in three places on the domain $x \in [0; 3]$. P is the point of intersection of f and g indicated on the graph.

(a) Determine $f'(x)$ [4]

(b) Use the Newton-Raphson method to determine the x-coordinate of P correct to 4 decimal places.   [6]

Question 8 [29 Marks]

$$f(x) = \frac{x^2 - x - 5}{x - 3}.$$

(a) Determine the coordinates of any stationary points of the function f. [12]

(b) Use calculations to determine whether they are local maxima or minima. [6]

(c) Determine the equations of the asymptotes of f. [3]

(d) Sketch a graph of the function f, clearly indicating all stationary points, intersections with the axes and asymptotes. [8]

Question 9 [25 Marks]

(a) $\int (2x - 1)\sqrt{3x^2 - 3x + 3}dx$ [9]

(b) $\int \sin 5x \cos 3x dx$ [5]

(c)　(i) Prove the identity $\operatorname{cosec}^4 x = \operatorname{cosec}^2 x \cot^2 x + \operatorname{cosec}^2 x$ [3]

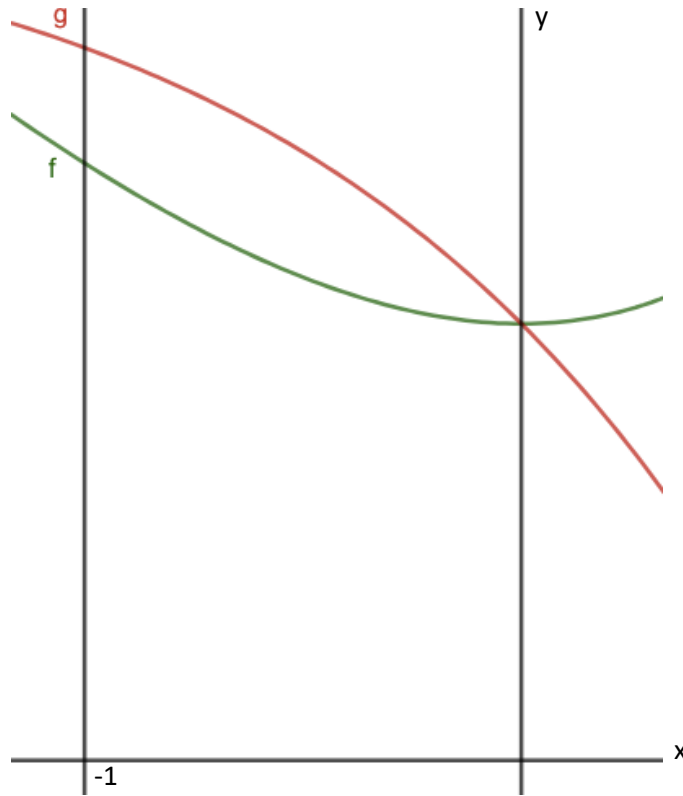　　(ii) Hence, or otherwise, determine $\int \operatorname{cosec}^4 x dx$ [8]

Question 10 [11 Marks]

$$f(x) = \frac{x^3}{x^2 - 4}$$

(a) Dissolve $f(x)$ into partial fractions. [7]

(b) Determine $\int f(x)dx$ [4]

Question 11   [15 Marks]

$f(x) = e^x - x$ and $g(x) = -e^x + 2$

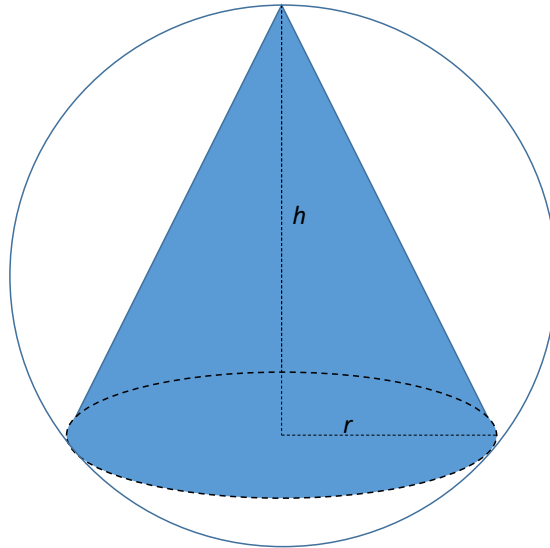(a) Determine the area between $f(x)$ and $g(x)$ for $x \in [-1; 0]$. Give your answer in terms of e.   [8]

(b) Determine the volume of the solid of revolution if $g(x)$ is rotated about the $x-$axis for $x \in [-1; 0]$. Give your answer correct to 2 decimal places   [7]

Question 12   [15 Marks]

A cone of height h and radius r has volume

$$V = \frac{1}{3}\pi r^2 h$$

(a) If the cone fits exactly inside a sphere of radius 3, show that the volume

$$V = \frac{\pi}{3}h^2(6-h)$$

.   [5]

(b) Hence find the maximum volume.   [10]