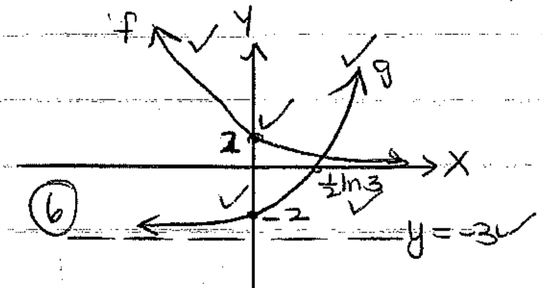
GR12 AP MATHS P1 SEP 2019 MEMO

QUESTION 1

1.1.1.



(6)

1.1.2. $2e^{-x} = e^{2x}-3$

$\frac{2}{e^{x}} = e^{2x}-3$

let $e^{x}=k$

$\frac{2}{k} = k^{2}-3$

$0 = k^{3}-3k-2$

$0 = (k-2)(k+1)^{2}$

$k=2$ OR $k \neq -1$

$e^{x}=2$

$x=\ln 2$ (7)

(-1) if not 2 dec pl

$x=0,69$ $(0,69;1)$

1.1.3. $g(x) = e^{2x}-3$

$g'(x) = e^{2x}(2)$

if leave out $m=1$ $y=x-2$ $\left(\frac{3}{5}\right)$

$x=0$ $m=2$

$y=2x+c$

$(0;-2)$ $-2=c$ (5) $y=2x-2$

1.1.4 $x=e^{2y}-3$

$x+3=e^{2y}$

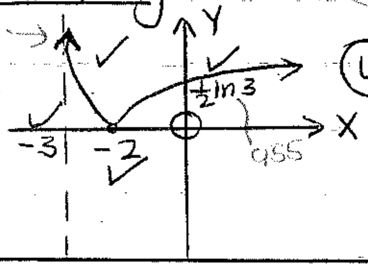
$\ln(x+3)=2y$ (3)

$\frac{1}{2}\ln(x+3)=y$

if $g^{-1}(x)=e^{2y}-3$ ✗ $x=e^{2y}-3$ $\frac{\ln(x+3)}{2}=y$ $\left(\frac{1}{3}\right)$

1.1.5

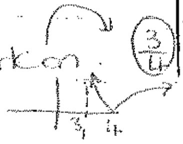
must tend towards asymp



(4)

(-1) if not ab value

if $x-3=e^{2y}$ ✗ $\frac{1}{2}\ln(x-3)=y$ $\left(\frac{2}{3}\right)$ → mark on



$\left(\frac{3}{4}\right)$

1.2. $0,1 = \frac{1}{1+Ce^{-k(0)}}$

$0,1(1+C) = 1$

$1+C = 10$

$C=9$

$0,25 = \frac{1}{1+9e^{-k(2)}}$

$1+9e^{-2k} = 4$

$9e^{-2k} = 3$

$e^{-2k} = \frac{1}{3}$

$-2k = \ln\frac{1}{3}$

$k = 0,5493\ldots$ (A)

If use 10, 25 and 75 → $k=-0,08$ $c=-0,9$ $\left(\frac{3}{8}\right)$

$0,75 = \frac{1}{1+9e^{-(A)t}}$

$1+9e^{-At} = \frac{4}{3}$

$e^{-At} = \frac{1}{27}$ (8)

$-At = \ln\frac{1}{27}$

If $t=5,992$ $\left(\frac{7}{8}\right)$

$t = 6\text{ hrs}$

1.3. $(p+qi)(2+2i) = -15-qi$

$2p+2pi+2qi+2qi^{2} = -15-qi$

$2p+2pi+2qi-2q = -15-qi$

$2p-2q = -15$ ①

$2p+2q = -q$

$2p+3q = 0$ ②

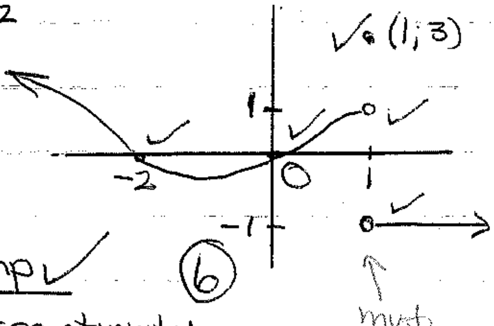
$p=-\frac{9}{2}$ $q=3$

(7)

if ✗ $-15+qi$ $p=\frac{5}{2}$ $q=-5$ $\left(\frac{5}{7}\right)$

If $2qi^{2}$ gives $2q$ ✗ $p=-\frac{45}{2}$ $q=15$ $\left(\frac{4}{7}\right)$

1.3.2



Jump discontinuity (6)

must be a function (-1) if not

If don't show $2qi^{2}$ $\left(\frac{5}{7}\right)$ if show $2qi^{2}$ and make it $2q$

must not have other x- or y int

0,5 no y int

if wrong equations (6/10)

3.3

$f(x) = -x^2+8$ | $f(x) = -x+2$

$f'(x) = -2x$ | $f'(x) = -1$

$\lim_{x\to -2^-} (-2x)$ | $\lim_{x\to -2^+} (-1)$

$= 4$ | $= -1$

if $y = x+4$, $y' = 1$ (5/6)

$\lim_{x\to -2^-} f'(x) \neq \lim_{x\to -2^+} f'(x)$

$\therefore$ not differentiable (6)

QUESTION 2

$2+7+12+\ldots+(5n-3) = \frac{n}{2}(5n-1)$

must have + signs otherwise (-2)

① $n=1$ LHS 2

RHS $\frac{1}{2}[5(1)-1]$

$= \frac{1}{2}(4)$

$= 2$ LHS=RHS

$\therefore$ true for $n=1$

② Assume true for $n=k$

$2+7+12+\ldots+(5k-3) = \frac{k}{2}(5k-1)$

③ $n=k+1$

$2+7+12+\ldots+(5k-3)+[5(k+1)-3] =$

$\frac{k}{2}(5k-1) + 5k+2$

$= \frac{k(5k-1)+10k+4}{2}$

$= \frac{5k^2-k+10k+4}{2}$

$= \frac{5k^2+9k+4}{2}$

$= \frac{(5k+4)(k+1)}{2}$

if don't prove $n=k+1$ then don't give for last step.

$= \frac{(k+1)[5(k+1)-1]}{2}$

$\therefore$ true for $n=k+1$ (12)

Proved true for $n=1$ and $n=1+1$ etc $\therefore$ true for all $n\in N$

if $(f\circ g)(k) = f(g(k))$

$= f(|k|)$ $k\in R$

$= |k|^2 - 5|k|$

QUESTION 3

3.1. $(f\circ g)(x) = f(g(x))$

$= f(|x|)$ $x\in R$

$= |x|^2 - 5|x|$ $x\in R$

$(f\circ g)(k) = |k|^2 - 5|k|$

$6 = |k|^2 - 5|k|$

$k \geq 0$: $k^2-5k-6=0$

$(k-6)(k+1)=0$

$k=6$ or $k\neq -1$

$k<0$: $k^2+5k-6=0$

$(k+6)(k-1)=0$

(11) $k=-6$ or $k\neq 1$

if $-k^2+5k-6=0$

$k^2-5k+6=0$

$(k-2)(k-3)=0$

$k\neq 2$ or $k\neq 3$

max 9/11

if not 2 cases

if $6 = (|k|)^2 - 5|k|$

$6 = k^2-5k$

$0 = k^2-5k-6$

$0 = (k-6)(k+1)$

$k=6$ or $k\neq -1$

QUESTION 4

4.1.1. $AB^2 = (8)^2 + (8)^2 - 2(8)^2 \cos 2{,}4$ ✓

$AB = 14{,}91\text{cm}$ ✓ (2) *

4.1.2. $s(AB) = (2\pi - 2{,}4)(8)$ ✓ (3,88)

$= 31{,}07\text{cm}$ ✓

$P = 31{,}07 + 14{,}91$

$= 45{,}98\text{cm}$ ✓ (4)

4.1.3 $\text{Area} = (8)^2\pi - \left(\tfrac{1}{2}(2{,}4)(8)^2 - \tfrac{1}{2}(8)(8)\sin 2{,}4\right)$ ✓ (76,8)

$= (8)^2\pi - (76{,}8 - 21{,}61\ldots)$

$= 145{,}88\text{ cm}^2$ (4)

4.2.1 $\dfrac{2\sin x}{\cos x} \times \dfrac{1}{2\sin x\cos x}$ ✓

$= \dfrac{1}{\cos^2 x}$ ✓ (3)

$= \sec^2 x$

4.2.2 $\int \sec^2 x\,dx$ ✓ ← (-1) if not 1st line

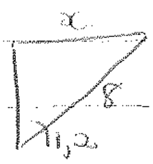
$= \tan x + c$ (2)

* $\sin 1{,}2 = \dfrac{x}{8}$ ✓

$7{,}45\ldots = x$

$AB = 2(7{,}45\ldots)$

$= 14{,}91\text{cm}$ (2)



QUESTION 5

5.1 Vert: $x = +\tfrac{1}{2}$ ✓

Oblique: $x^2 + 4x = (2x-1)(\tfrac{1}{2}x + \tfrac{9}{4}) + \tfrac{9}{4}$ ✓✓

$\dfrac{x^2 + x}{2x-1} = \tfrac{1}{2}x + \tfrac{9}{4} - \dfrac{9}{(2x-1)4}$ (-1) if error here

$y = \tfrac{1}{2}x + \tfrac{9}{4}$ ✓ (5)

5.2 $y' = \dfrac{(2x+4)(2x-1) - (x^2+4x)(2)}{(2x-1)^2}$ ✓✓✓

SP $y' = 0$

$0 = 4x^2 + 6x - 4 - 2x^2 - 8x$ ✓

$0 = 2x^2 - 2x - 4$

$0 = x^2 - x - 2$ ✓ careless mistake max (7)

$0 = (x-2)(x+1)$

$x = 2$ ✓ OR $x = -1$ ✓ $y = 1$

$y = 4$ $(-1; 1)$ ✓

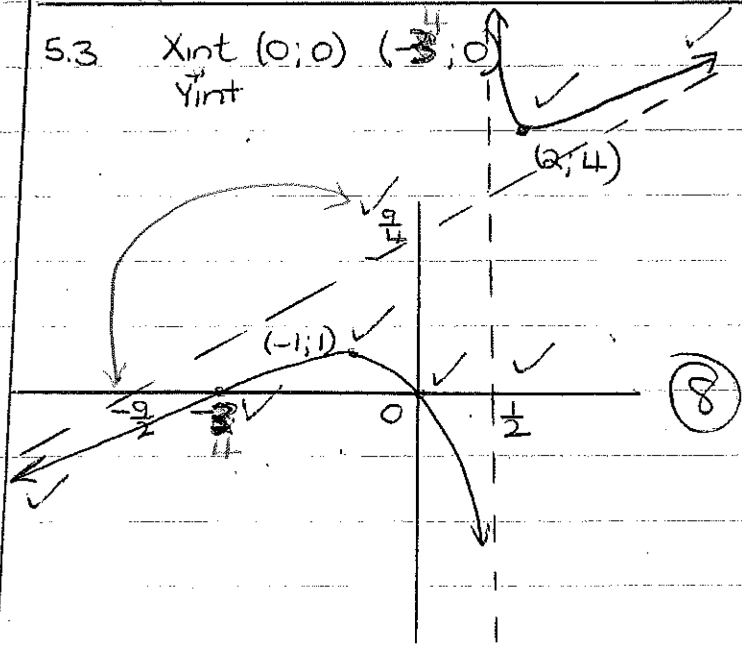
$(2; 4)$ ✓ (9)

(-1) if not co-ords

-1		½	2	
+	−		−	+

5.3 Xint $(0;0)$ $(-\tfrac{3}{4}; 0)$

Yint

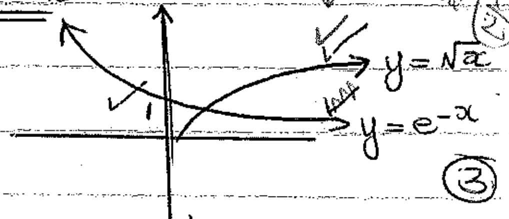


(8)

3.

QUESTION 6

6.1



(3)

6.2. $e^{-x} = x^{\frac{1}{2}}$

$e^{-x} - x^{\frac{1}{2}} = 0$

$f(x) = e^{-x} - x^{\frac{1}{2}}$

$f(x_n) = e^{-x_n} - x_n^{\frac{1}{2}}$

$f'(x_n) = e^{-x_n}(-1) - \frac{1}{2}x_n^{-\frac{1}{2}}$

$= -e^{-x_n} - \frac{1}{2x_n^{\frac{1}{2}}}$

If mistake here max 5/12

x_1 value

$x_{n+1} = x_n - \frac{e^{-x_n} - x_n^{\frac{1}{2}}}{\left(-e^{-x_n} - \frac{1}{2x_n^{\frac{1}{2}}}\right)}$

$x_1 = 1$

$x_2 = 0{,}2716493$

$x_3 = 0{,}4116023$

$x_4 = 0{,}4261836$

$x_5 = 0{,}4263027$

$x_6 = 0{,}4263027$

$x = 0{,}426303$

(12)

QUESTION 7

7.1. $y = x(x^2-3)^{\frac{1}{2}}$

$y' = (x^2-3)^{\frac{1}{2}} + x \cdot \frac{1}{2}(x^2-3)^{-\frac{1}{2}}(2x)$

$= (x^2-3)^{\frac{1}{2}} + \frac{x^2}{(x^2-3)^{\frac{1}{2}}}$

$x=2$ $\quad m = 1 + \frac{4}{1}$

$= 5$

$y = 5x + c$ (7)

$(2;2)$ $\quad 2 = 10 + c$

$-8 = c$ $\qquad y = 5x - 8$

If mistake in question simplified max 5/7

If $y' = (1)\frac{1}{2}(x^2-3)^{-\frac{1}{2}}(2x)$

If $y' = x\frac{1}{2}(x^2-3)^{-\frac{1}{2}}(2x)$ $\quad y = 4x - 6$ 4/7

If 7 $2x\frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{y-3x}{x+y}$ $\quad m_T = -\frac{3}{5}$ $\quad m_N = \frac{5}{3}$ $\quad$ 10/12

7.2. $6x - 2y - 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-2x+2y) = 2y - 6x$

$\frac{dy}{dx} = \frac{y-3x}{-x+y}$

$y = -3$ $\qquad (-2;-3)$ $\qquad m_T = \frac{-3+6}{2-3}$

$3x^2 + 6x + 9 = 9$ $\qquad = -3$

$3x(x+2) = 0$ $\qquad$ (12) $\qquad m_N = \frac{1}{3}$

$x \neq 0$ or $x = -2$

7.3 $f'(x) = \frac{-\sin x(2-\sin x) - \cos x(-\cos x)}{(2-\sin x)^2}$

SP $f'(x) = 0$

$0 = -2\sin x + \sin^2 x + \cos^2 x$

$0 = -2\sin x + 1$

$\frac{1}{2} = \sin x$

$x = \frac{\pi}{6}$ or $x = \pi - \frac{\pi}{6}$

(8) $\qquad = \frac{5\pi}{6}$

QUESTION 8

8.1.1. $\frac{-4x^2+8x+4}{(x-1)(2x^2-3x+5)} = \frac{A}{x-1} + \frac{Bx+C}{2x^2-3x+5}$

$-4x^2+8x+4 = A(2x^2-3x+5) + (Bx+C)(x-1)$

$x=1$ $\qquad 8 = 4A$

$2 = A$

$-4x^2+8x+4 = 2Ax^2 - 3Ax + 5A + Bx^2 - Bx + Cx - C$

$-4 = 4 + B$

$-8 = B$

$4 = 10 - C$

$C = 6$

$\frac{-4x^2+8x+4}{(x-1)(2x^2-3x+5)} = \frac{2}{x-1} - \frac{8x-6}{2x^2-3x+5}$ (8)

8.1.2 $\int \frac{2}{x-1}\,dx - \int \frac{8x-6}{2x^2-3x+5}\,dx$

$u = 2x^2-3x+5$

$du = (4x-3)\,dx$

(6)

$\int \frac{2}{x-1}\,dx - 2\int \frac{1}{u}\,du$

$= 2\ln|x-1| - 2\ln|u| + C$

$= 2\ln|x-1| - 2\ln|2x^2-3x+5| + C$

8.2.1. $y = \ln(\sin x)$

$y' = \frac{1}{\sin x} \cdot \cos x$

$= \frac{\cos x}{\sin x}$ (2)

8.2.2. $\int \cos x \cdot \ln(\sin x)\,dx$

$f(x) = \ln(\sin x)$ $\quad g'(x) = \cos x$

$f'(x) = \frac{\cos x}{\sin x}$ $\quad g(x) = \sin x + C$

$\int \cos x \cdot \ln(\sin x)\,dx = \ln(\sin x)\cdot \sin x - \int \frac{\cos x}{\sin x}\cdot \sin x\,dx$

(6)

$= \ln(\sin x)\cdot\sin x - \sin x + C$

8. 2. $\int_1^K 12x(3x^2+1)^{-\frac{1}{2}}\,dx = 20$

$u = 3x^2+1$ $\quad x = 1 \quad u = 4$

$du = 6x\,dx$ $\quad x = K \quad u = 3K^2+1$

$\int_4^{3K^2+1} 2u^{-\frac{1}{2}}\,du = 20$

$\left.\frac{2u^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^{3K^2+1} = 20$

If don't replace and K (5/8)

$4(3K^2+1)^{\frac{1}{2}} - 4(4)^{\frac{1}{2}} = 20$

$(3K^2+1)^{\frac{1}{2}} = 7$

$3K^2+1 = 49$

$3K^2 = 48$

If careless mistake (8/10) as long as K>1

$K^2 = 16$

$K = 4 \ (K>0)$ (10)

QUESTION 9

If leave out (−1)

9.1.1 $\int 2x\sin x\,dx$

$f(x) = 2x$ $\quad g'(x) = \sin x$

$f'(x) = 2$ $\quad g(x) = -\cos x + C$

$\int 2x\sin x\,dx = -2x\cos x + 2\int \cos x\,dx$

$= -2x\cos x + 2\sin x + C$ (7)

9.2 $A = \int_0^{1,9} (x^2 + 2x\sin x)\,dx$

$= \left.\frac{x^3}{3} - 2x\cos x + 2\sin x\right]_0^{1,9}$

$= 0,83$ units^2 (4)

9.2.1. $\frac{1}{2}\int 2(2x+3)^3\,dx$

$u = 2x+3$

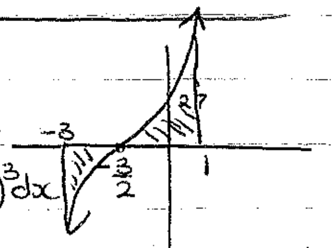
$du = 2\,dx$

$\frac{1}{2}\int u^3\,du$

$= \frac{1}{2}\left(\frac{u^4}{4}\right) + C$

(−1) if no C (5) $= \frac{(2x+3)^4}{8} + C$

9.2.2. $A = -\int_{-3}^{-\frac{3}{2}} (2x+3)^3\,dx + \int_{-\frac{3}{2}}^{1} (2x+3)^3\,dx$



if $\int (2x+3)^3\,dx$ (3/6)

$= -\left[\frac{(2x+3)^4}{8}\right]_{-3}^{-\frac{3}{2}} + \left[\frac{(2x+3)^4}{8}\right]_{-\frac{3}{2}}^{1}$

$= \frac{81}{8} + \frac{625}{8} = \frac{353}{4}$ units^2 (6)

88,25