HERSCHEL GIRLS HIGH SCHOOL

ADVANCED PROGRAMME MATHEMATICS- GRADE 12

Paper 1-Calculus and Algebra

DATE: August 2019   **TIME: 2 HOURS**
MARKS: 200

INSTRUCTIONS TO CANDIDATES:

1.   This paper consists of 9 questions and a formula sheet.

2.   Answer ALL questions.

3.   All the necessary working details must be shown with your answer.

4.   Clearly number all your questions correctly.

5.   Non-programmable calculators may be used unless the question states otherwise.

6.   Round off your answers to TWO decimal digits correctly unless the question states otherwise.

7.   Write answers with positive exponents.

8.   If applicable, calculations should be done using radians and answers should be given in radians.

Question 1[40]

1.1.1 Draw the graphs of $f(x) = 2e^{-x}$ and $g(x) = e^{2x} - 3$ on the same set of axes. Clearly indicate any asymptotes as well as intercepts with the axes.

(6)

1.1.2 Calculate the co-ordinates of the point of intersection of $f(x)$ and $g(x)$. [Give answer correct to 2 decimal places]

(7)

1.1.3 Determine the equation of the tangent to $g(x)$ at $x = 0$.

(5)

1.1.4 Determine the equation of $y = g^{-1}(x)$ and write it in the form $y = \cdots$

(3)

1.1.5 On a separate set of axes, draw the graph of $y = |g^{-1}(x)|$. Clearly indicate any asymptotes as well as intercepts with the axes.

(4)

1.2 The spread of information through a population is modelled by the logistic equation: $\qquad P = \frac{1}{1+Ce^{-kt}}$

where P represents the proportion of stockbrokers that are aware of the information after time t, measured in hours, and C and k are constants.

At the start, 10% of stockbrokers have heard about the impending financial collapse of a company.

Two hours later, 25% of stockbrokers have heard about it.

Determine C and k and hence calculate how long it will take before 75% of stockbrokers have heard the news.

(8)

1.3 Two complex numbers are defined as $w = -15 + qi$ and $z = 2 + 2i$. Determine p and q if $(p + qi).z = w^*$

(7)

Question 2[12]
Prove, using the principle of Mathematical Induction,

that $\sum_{r=1}^{n}(5r - 3) = \frac{n}{2}(5n - 1)$ all $n \in N$.

(12)

Question 3[23]

3.1 Functions $f(x)$ and $g(x)$ are defined by: $\quad f(x) = x^2 - 5x$
$$g(x) = |x|$$

Determine the values of k if $(fog)(k) = 6$ $\hspace{2cm}$ (11)

3.2 Sketch a graph of a function $y = f(x)$ with the following characteristics:
$$f(-2) = 0 \; ; f(0) = 0 \; ; f(1) = 3 \; ;$$
$$\lim_{x \to 1^-} f(x) = 1 \text{ and } \lim_{x \to 1^+} f(x) = -1$$

Identify the kind of discontinuity that occurs at $x = 1$. [No justification required] $\hspace{2cm}$ (6)

3.3 Given: $\quad f(x) = \begin{cases} 2e^x + 1 \; if \; x > 0 \\ |x + 1| + 3 \; if \; -2 \le x \le 0 \\ -x^2 + 8 \; if \; x < -2 \end{cases}$

$f(x)$ is continuous at $x = -2$. Determine whether or not $f(x)$ is differentiable at $x = -2$. Explain fully. $\hspace{2cm}$ (6)

Question 4[15]

4.1 The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle $AOB = 2,4$ radians.
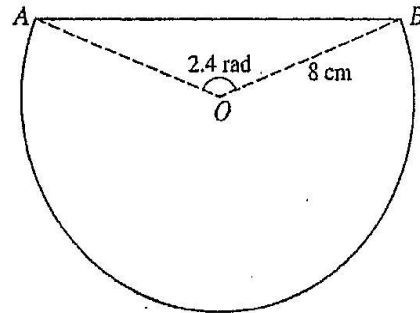
Calculate:

4.1.1 the length of AB. $\hspace{1cm}$ (2)

4.1.2 the perimeter of the plate. $\hspace{1cm}$ (4)

4.1.3 the area of the plate. $\hspace{1cm}$ (4)

4.2.1 Simplify $\frac{2\tan x}{\sin 2x}$ and write the answer as a single trig ratio. (3)

4.2.2 Hence determine $\int \frac{2\tan x}{\sin 2x} dx$ (2)

Question 5[22]

Given: $y = \frac{x^2+4x}{2x-1}$

5.1 Determine the equations of any asymptotes. (5)

5.2 Determine the co-ordinates of any stationary points. (9)

5.3 Draw a sketch graph of $y = \frac{x^2+4x}{2x-1}$. On the graph clearly indicate any intercepts with the axes, asymptotes as well as stationary points. (8)

Question 6[15]

6.1 Sketch the graphs of $y = e^{-x}$ and $y = \sqrt{x}$ on the same set of axes. (3)

6.2 Taking a suitable integer as a first approximation, use Newton's Method to determine the x value of the point of intersection of the 2 graphs. [Give answer correct to 6 decimal places] (12)

Question 7[27]

7.1 Determine the equation of the tangent to $y = x.\sqrt{x^2-3}$ at the point $(2; 2)$ (7)

7.2 Determine the gradient of the normal to $3x^2 - 2xy + y^2 = 9$ at the point where $y = -3$ and $x < 0$. (12)

7.3 Given: $f(x) = \frac{\cos x}{2-\sin x}$; $x\varepsilon[0; \pi]$

Determine the x values of the stationary points. (8)

Question 8[24]

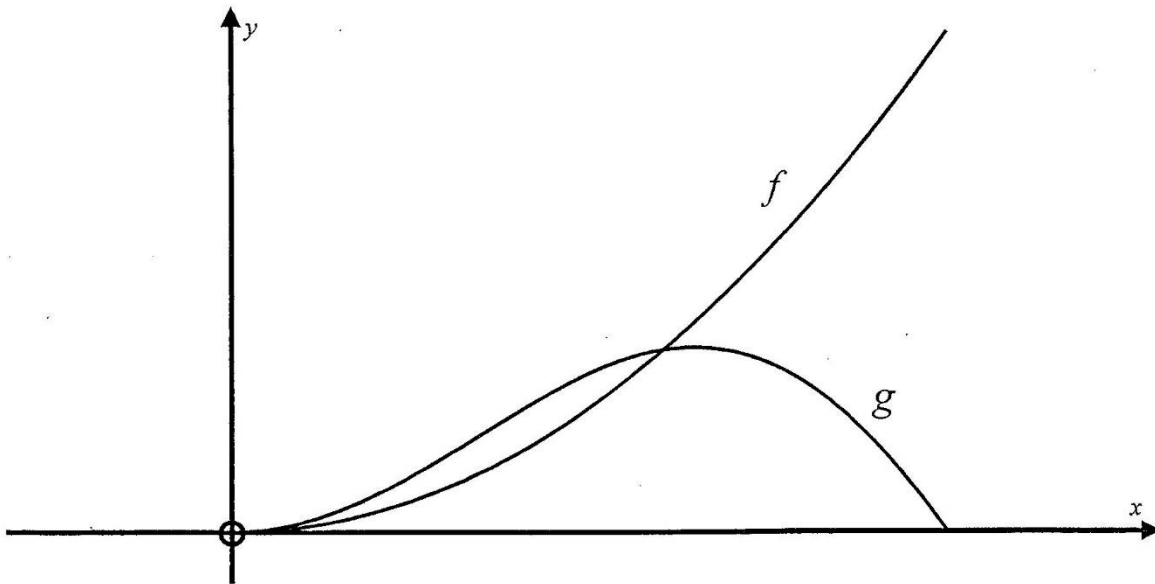8.1.1 Resolve into partial fractions: $\dfrac{-4x^2+8x+4}{(x-1)(2x^2-3x+5)}$ (8)

8.1.2 Hence determine $\int \dfrac{-4x^2+8x+4}{(x-1)(2x^2-3x+5)}dx$ (6)

8.2   Solve for k if $k > 0$ and $\int_1^k \dfrac{12x}{\sqrt{3x^2+1}}dx = 20$ (10)

Question 9[22]

9.1.1 Integrate $\int 2x.\sin x \, dx$ (7)

9.1.2 The sketch below represents the graphs of $f(x) = x^2$ and
        $g(x) = 2x.\sin x$. The graphs intersect at $x = 0$ and $x = 1,9$.

Determine the area between the 2 graphs for the interval $x\varepsilon[0\ ;1,9]$ (4)

9.2.1 Integrate: $\int(2x+3)^3 dx$ (5)

9.2.2 Determine the area between the graph $y = (2x+3)^3$ and the x axis for the interval $x\varepsilon[-3\,;1]$ (6)

Total: 200