

Herzlia 2019

AP Paper 2

$$\begin{aligned} 1.1 \quad \binom{14}{5} \binom{9}{2} \binom{7}{3} \binom{4}{4} &= \frac{14!}{9!5!} \times \frac{9!}{7!2!} \times \frac{7!}{4!3!} \times \frac{4!}{0!4!} \\ &= \frac{14!}{5!2!3!4!} \\ &= 2522520 \end{aligned}$$

$$\begin{aligned} 2 \quad P(\text{zero undersized}) &= \frac{\binom{3}{0} \binom{7}{4}}{\binom{10}{4}} \\ &= \frac{1}{6} \quad [= 0,1667] \\ &= 16,67\% \end{aligned}$$

$$3 \quad x \sim B(20; 0,01)$$

$$\begin{aligned} P(x \geq 2) &= 1 - P(x < 2) \\ &= 1 - [\binom{20}{0} (0,01)^0 (0,99)^{20} + \binom{20}{1} (0,01)^1 (0,99)^{19}] \\ &= 0,0169 \\ &= 1,69\% \end{aligned}$$

$$1 \quad x = 1 \quad \text{when} \begin{cases} (1; 1) & (1; 2) & (1; 3) & \dots & (1; 6) \\ & (2; 1) & (3; 1) & \dots & (6; 1) \end{cases}$$

$$\therefore P(x=1) = \frac{11}{36}$$

$$\text{similarly } P(x=2) = \frac{9}{36} \text{ etc...}$$

$$\therefore$$

x	1	2	3	4	5	6
$P(x=x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

$$\begin{aligned} 2 \quad \therefore E(x) &= 1 \left(\frac{11}{36} \right) + 2 \left(\frac{9}{36} \right) + 3 \left(\frac{7}{36} \right) + 4 \left(\frac{5}{36} \right) + 5 \left(\frac{3}{36} \right) + 6 \left(\frac{1}{36} \right) \\ &= 2,53 \end{aligned}$$

3.1 $\int_{-\infty}^{\infty} f_X(x) dx = 1$ ^{law of total probability}

$$\therefore c \int_0^5 (25t - t^3) dt = 1$$

$$\therefore c \left[\frac{25}{2} t^2 - \frac{1}{4} t^4 \right]_0^5 = 1$$

$$\therefore c \left(\left[\frac{25}{2} (5)^2 - \frac{1}{4} (5^4) \right] - 0 \right) = 1$$

$$\therefore c \left(\frac{625}{4} \right) = 1$$

$$\therefore c = \frac{4}{625}$$

3.2 $P(2 \leq X \leq 4) = \frac{4}{625} \int_2^4 (25t^2 - t^3) dt$

↓
accept

$$P(2 < X < 4) = \frac{4}{625} \left[\frac{25}{2} t^2 - \frac{1}{4} t^4 \right]_2^4$$

$$= 0,576$$

$$= 57,6\%$$

3.3 Median : $\int_{-\infty}^m f_x(x) dx = \frac{1}{2}$

$$\therefore \frac{4}{625} \int_0^m (25t - t^3) dt = \frac{1}{2}$$

$$\therefore \frac{25}{2} m^2 - \frac{1}{4} m^4 = \frac{625}{8}$$

$$\therefore 2m^4 - 100m^2 + 625 = 0$$

$$\therefore m^2 = 25 \pm \frac{25\sqrt{2}}{2}$$

$$\therefore m = 2,71 \quad \text{or} \quad m = 6,53$$

$$\therefore m \leq 5$$

1.1 $P(T \leq 10) = \int_0^{10} 0,2 e^{-0,2t} dt$

↓
accept
 $P(T < 10)$

$$= [-e^{-0,2t}]_0^{10}$$

$$= (-e^{-2}) - (-e^0)$$

$$= 1 - e^{-2}$$

$$= 0,8647 = 86,47\%$$

1.2 $P(5 \leq x \leq 15) = \int_5^{15} 0,2 e^{-0,2t} dt$

↓
accept
 $P(5 < x < 15)$

$$= [-e^{-0,2t}]_5^{15}$$

$$= -e^{-3} - (-e^{-1})$$

$$= 0,3181$$

$$= 31,81\%$$

$$4.3 \quad E(x) = \int_{-\infty}^{\infty} x f_x(x) dx$$

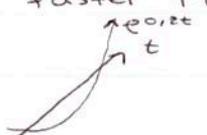
integration by parts

$$\therefore E(T) = \int_0^{\infty} t \cdot 0,2 e^{-0,2t} dt$$

$$\begin{aligned} \text{let } f &= t & g' &= 0,2 e^{-0,2t} \\ \therefore f' &= 1 & g &= -e^{-0,2t} \end{aligned}$$

$$\therefore \int_0^{\infty} t \cdot 0,2 e^{-0,2t} dt = [-t e^{-0,2t}]_0^{\infty} - \int_0^{\infty} (-e^{-0,2t}) dt$$

(i) intuitively: $\lim_{t \rightarrow \infty} \frac{t}{e^{0,2t}}$ } $e^{0,2t}$ tends to ∞ WAY faster than t
 $\therefore \lim_{t \rightarrow \infty} = 0$



(ii) L'Hopital's rule:

$$\text{if } \frac{\infty}{\infty} \text{ or } \frac{0}{0} : \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$$

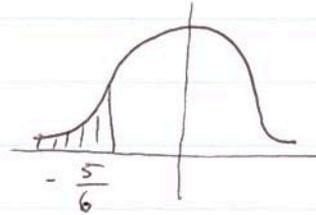
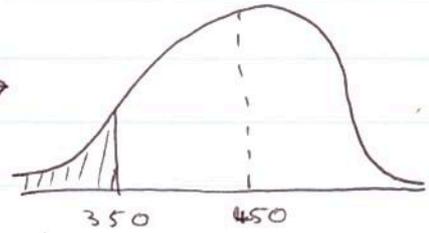
$$\therefore \lim_{t \rightarrow \infty} \frac{t}{e^{0,2t}} = \lim_{t \rightarrow \infty} \frac{1}{0,2 e^{0,2t}} = \frac{1}{\infty} = 0$$

$$\begin{aligned} \therefore \int_0^{\infty} t \cdot 0,2 e^{-0,2t} dt &= 0 \stackrel{1/4}{=} [-5 e^{-0,2t}]_0^{\infty} \\ &= [0 - (-5)] \\ &= 5 \end{aligned}$$

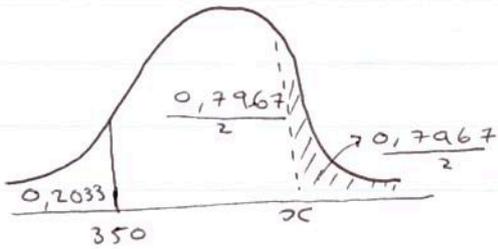
5.1 $X \sim N(450, 120^2)$ $\left(z = \frac{\bar{x} - \mu}{\sigma}\right)$

$P(X < 350)$

$$\begin{aligned}
 &= P\left(z < \frac{350 - 450}{120}\right) \\
 &= P\left(z < -\frac{5}{6}\right) \rightarrow -0,833 \\
 &= P\left(z > \frac{5}{6}\right) \text{ (symmetry)} \\
 &= 0,5 - H\left(\frac{5}{6}\right) \\
 &= 0,5 - 2967 \\
 &= 0,2033
 \end{aligned}$$

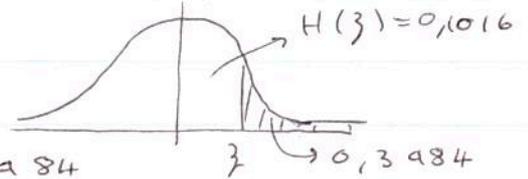


2



$\therefore P(X > x) = \frac{0,7967}{2}$

$\therefore P(Z > z) = 0,3984$



from table $H(0,26) \approx 0,5 - 0,3984$
 \downarrow
 closest $\approx 0,1016$

$\therefore 0,26 = \frac{x - 450}{120}$

$\therefore x = 0,26(120) + 450$
 $= 481,2g$

6.1

given a sample proportion p

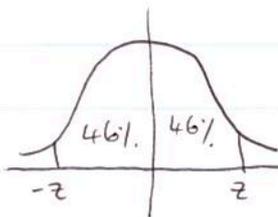
$$\therefore S = \sqrt{p(1-p)}$$

sample standard deviation

$$p \pm z \sqrt{\frac{p(1-p)}{n}} \equiv \bar{x} \pm z \frac{\sigma}{\sqrt{n}} \approx \bar{x} \pm z \frac{s}{\sqrt{n}}$$

c.i. 92%.

∴



$$\therefore z = 1,75 \text{ (from table)}$$

∴ a 92% c.i. for the true mean of the woman's weight is $61,236 \pm 1,75 \frac{5,443}{\sqrt{100}}$

$$= (60,28 ; 62,19) \text{ kg}$$

6.2

~~z~~ = 2,58 from table

$$2,58 \frac{7,651}{\sqrt{n}} \leq 1,5$$

$$\therefore n \geq \left[\frac{(2,58)(7,651)}{1,5} \right]^2$$

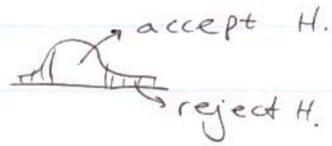
$$\therefore n \geq 173,17$$

∴ min 174 people

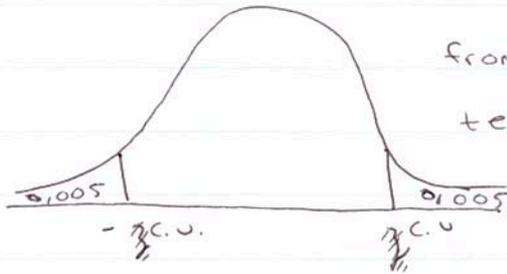
t.1 $H_0 : \mu = 100g$

$H_1 : \mu \neq 100g$ (\therefore 2-tailed)

i.e.



t.2



from table: critical value = 2,58

test statistic: $\frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

$$= \frac{99,5 - 100}{\left(\frac{1,5}{\sqrt{40}}\right)}$$

$$= -2,11$$

$$-2,58 < -2,11 < 2,58$$

\therefore there is not enough evidence to reject H_0

Year	Q1	Q2	Q3	Q4	Total
2018	100	150	200	250	700
2019	120	180	220	280	800
2020	150	200	250	300	900
2021	180	220	280	350	1030
2022	200	250	300	400	1150
2023	220	280	350	450	1300
2024	250	300	400	500	1450
2025	280	350	450	550	1630
2026	300	380	500	600	1780
2027	320	400	550	650	1920
2028	350	450	600	700	2100
2029	380	500	650	750	2280
2030	400	550	700	800	2450
2031	420	600	750	850	2620
2032	450	650	800	900	2800
2033	480	700	850	950	2980
2034	500	750	900	1000	3150
2035	520	800	950	1050	3320
2036	550	850	1000	1100	3500
2037	580	900	1050	1150	3680
2038	600	950	1100	1200	3850
2039	620	1000	1150	1250	4020
2040	650	1050	1200	1300	4200
2041	680	1100	1250	1350	4380
2042	700	1150	1300	1400	4550
2043	720	1200	1350	1450	4720
2044	750	1250	1400	1500	4900
2045	780	1300	1450	1550	5080
2046	800	1350	1500	1600	5250
2047	820	1400	1550	1650	5420
2048	850	1450	1600	1700	5600
2049	880	1500	1650	1750	5780
2050	900	1550	1700	1800	5950
2051	920	1600	1750	1850	6120
2052	950	1650	1800	1900	6300
2053	980	1700	1850	1950	6480
2054	1000	1750	1900	2000	6650
2055	1020	1800	1950	2050	6820
2056	1050	1850	2000	2100	7000
2057	1080	1900	2050	2150	7180
2058	1100	1950	2100	2200	7350
2059	1120	2000	2150	2250	7520
2060	1150	2050	2200	2300	7700
2061	1180	2100	2250	2350	7880
2062	1200	2150	2300	2400	8050
2063	1220	2200	2350	2450	8220
2064	1250	2250	2400	2500	8400
2065	1280	2300	2450	2550	8580
2066	1300	2350	2500	2600	8750
2067	1320	2400	2550	2650	8920
2068	1350	2450	2600	2700	9100
2069	1380	2500	2650	2750	9280
2070	1400	2550	2700	2800	9450
2071	1420	2600	2750	2850	9620
2072	1450	2650	2800	2900	9800
2073	1480	2700	2850	2950	9980
2074	1500	2750	2900	3000	10150
2075	1520	2800	2950	3050	10320
2076	1550	2850	3000	3100	10500
2077	1580	2900	3050	3150	10680
2078	1600	2950	3100	3200	10850
2079	1620	3000	3150	3250	11020
2080	1650	3050	3200	3300	11200
2081	1680	3100	3250	3350	11380
2082	1700	3150	3300	3400	11550
2083	1720	3200	3350	3450	11720
2084	1750	3250	3400	3500	11900
2085	1780	3300	3450	3550	12080
2086	1800	3350	3500	3600	12250
2087	1820	3400	3550	3650	12420
2088	1850	3450	3600	3700	12600
2089	1880	3500	3650	3750	12780
2090	1900	3550	3700	3800	12950
2091	1920	3600	3750	3850	13120
2092	1950	3650	3800	3900	13300
2093	1980	3700	3850	3950	13480
2094	2000	3750	3900	4000	13650
2095	2020	3800	3950	4050	13820
2096	2050	3850	4000	4100	14000
2097	2080	3900	4050	4150	14180
2098	2100	3950	4100	4200	14350
2099	2120	4000	4150	4250	14520
2100	2150	4050	4200	4300	14700