Herzlia 2019

AP Paper 2

1.1 $\binom{14}{5}\binom{9}{2}\binom{7}{3}\binom{4}{4} = \frac{14!}{\cancel{9!}\,5!} \times \frac{\cancel{9!}}{\cancel{7!}\,2!} \times \frac{\cancel{7!}}{\cancel{4!}\,3!} \times \frac{\cancel{4!}}{0!\,4!}$

$= \frac{14!}{5!\,2!\,3!\,4!}$

$= 2522520$

.2 $P(\text{zero undersized}) = \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}}$

$= \frac{1}{6}$ $[= 0{,}1667]$

$= 16{,}67\%$

.3 $X \sim B(20; 0{,}01)$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - \left[\binom{20}{0}(0{,}01)^0(0{,}99)^{20} + \binom{20}{1}(0{,}01)^1(0{,}99)^{19}\right]$

$= 0{,}0169$

$= 1{,}69\%$

..1 $X = 1$ when $\begin{cases} (1;1) & (1;2) & (1;3) & \ldots & (1;6) \\ & (2;1) & (3;1) & \ldots & (6;1) \end{cases}$

$\therefore P(X=1) = \frac{11}{36}$

similarly $P(X=2) = \frac{9}{36}$ etc...

$\therefore$

x	1	2	3	4	5	6
$P(X=x)$	$11/36$	$9/36$	$7/36$	$5/36$	$3/36$	$1/36$

.2 $\therefore E(X) = 1\left(\frac{11}{36}\right) + 2\left(\frac{9}{36}\right) + 3\left(\frac{7}{36}\right) + 4\left(\frac{5}{36}\right) + 5\left(\frac{3}{36}\right) + 6\left(\frac{1}{36}\right)$

$= 2{,}53$

3.1 $$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$ *law of total probability

$$\therefore \; c \int_0^5 (25t - t^3)\,dt = 1$$

$$\therefore \; c \left[\frac{25}{2} t^2 - \frac{1}{4} t^4 \right]_0^5 = 1$$

$$\therefore \; c \left(\left[\frac{25}{2}(5)^2 - \frac{1}{4}(5^2) \right] - 0 \right) = 1$$

$$\therefore \; c \left(\frac{625}{4} \right) = 1$$

$$\therefore \; c = \frac{4}{625}$$

3.2 $$P(2 \leq X \leq 4) = \frac{4}{625} \int_2^4 (25t^{\cancel{2}} - t^3)\,dt$$

accept

$$P(2 < X < 4) \qquad = \frac{4}{625} \left[\frac{25}{2} t^2 - \frac{1}{4} t^4 \right]_2^4$$

$$= 0{,}576$$

$$= 57{,}6\,\%$$

3.3 Median : $\displaystyle\int_{-\infty}^{m} f_x(x)\,dx = \frac{1}{2}$

$$\therefore \frac{4}{625}\int_0^m (25t - t^3)\,dt = \frac{1}{2}$$

$$\therefore \frac{25}{2}m^2 - \frac{1}{4}m^4 = \frac{625}{8}$$

$$\therefore 2m^4 - 100m^2 + 625 = 0$$

$$\therefore m^2 = 25 \pm \frac{25\sqrt{2}}{2}$$

$$\therefore m = 2,71 \quad \text{or} \quad m \neq 6,53$$

$$\because m \leq 5$$

4.1 $P(T \leq 10) = \displaystyle\int_0^{10} 0,2\, e^{-0,2t}\,dt$

accept $P(T < 10)$

$$= \left[-e^{-0,2t}\right]_0^{10}$$

$$= (-e^{-2}) - (-e^{0})$$

$$= 1 - e^{-2}$$

$$= 0,8647 \quad = 86,47\%$$

.2 $P(5 \leq x \leq 15) = \displaystyle\int_5^{15} 0,2\, e^{-0,2t}\,dt$

accept $P(5 < x < 15)$

$$= \left[-e^{-0,2t}\right]_5^{15}$$

$$= -e^{-3} - (-e^{-1})$$

$$= 0,3181$$

$$= 31,81\%$$

4.3 $E(x) = \int_{-\infty}^{\infty} x \, f_x(x) \, dx$

integration by parts

$\therefore E(T) = \int_0^{\infty} t \cdot 0{,}2 e^{-0{,}2t} \, dt$

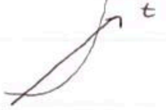
let $f = t \qquad g' = 0{,}2 e^{-0{,}2t}$

$\therefore f' = 1 \qquad g = -e^{-0{,}2t}$

$\therefore \int_0^{\infty} t \cdot 0{,}2 e^{-0{,}2t} \, dt = \left[-t e^{-0{,}2t}\right]_0^{\infty} - \int_0^{\infty} (-e^{-0{,}2t}) \, dt$

(i) intuitivly: $\lim_{t \to \infty} \frac{t}{e^{0{,}2t}}$ $\rightarrow$ $e^{0{,}2t}$ tends to ∞ WAY faster than t

$\therefore \lim_{t \to \infty} = 0$



(ii) L'Hopital's rule:

if $\frac{\infty}{\infty}$ or $\frac{0}{0}$: $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$

$\therefore \lim_{t \to \infty} \frac{t}{e^{0{,}2t}} = \lim_{t \to \infty} \frac{1}{0{,}2 e^{0{,}2t}} = \frac{1}{\infty}$

$= 0$

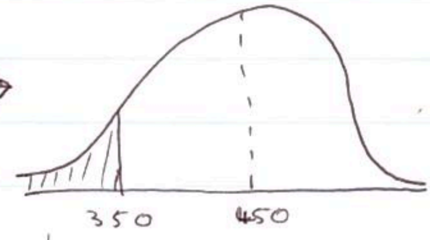
$\therefore \int_0^{\infty} t \cdot 0{,}2 e^{-0{,}2t} \, dt = 0 \cancel{-} \left[\cancel{-} 5 e^{-0{,}2t}\right]_0^{\infty}$

$= [0 - (-5)]$

$= 5$

5.1 $x \sim N(450; 120^2)$ $\left(z = \frac{\bar{x} - \mu}{\sigma}\right)$

$P(x < 350)$



$= P\left(z < \frac{350-450}{120}\right)$

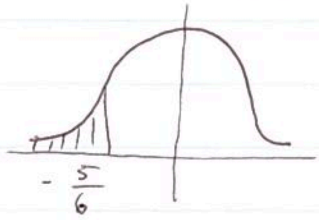
$= P\left(z < -\frac{5}{6}\right) \rightarrow -0{,}833$

$= P\left(z > \frac{5}{6}\right)$ (symmetry)

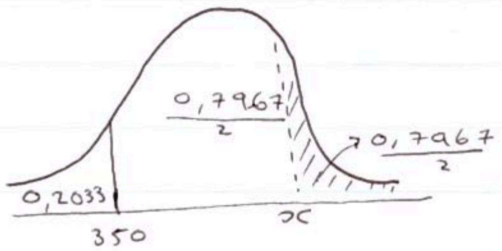
$= 0{,}5 - H\left(\frac{5}{6}\right)$

$= 0{,}5 - 2967$

$= 0{,}2033$

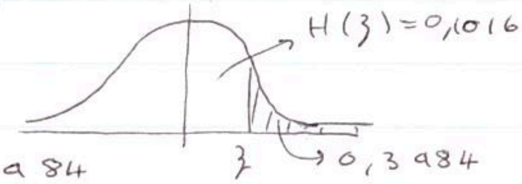


.2



$\therefore P(x > x) = \frac{0{,}7967}{2}$

$\therefore P(z > z) = 0{,}3984$



from table $H(0{,}26) \approx 0{,}5 - 0{,}3984$
closest $\approx 0{,}1016$

$\therefore 0{,}26 = \frac{x - 450}{120}$

$\therefore x = 0{,}26(120) + 450$

$= 481{,}2g$

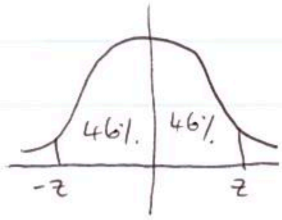
6.1 given a sample proportion p

$$\therefore S = \sqrt{p(1-p)}$$

sample standard deviation

$$p \pm z\sqrt{\frac{p(1-p)}{n}} \equiv \bar{x} \pm z\frac{\sigma}{\sqrt{n}} \approx \bar{x} \pm z\frac{s}{\sqrt{n}}$$

c.i. 92% $\therefore$



$$\therefore z = 1,75 \text{ (from table)}$$

$\therefore$ a 92 % c.i. for the true mean of the womans weight is $61,236 \pm 1,75 \dfrac{5,443}{\sqrt{100}}$

$$= (60,28 \,;\, 62,19)\text{kg}$$

6.2 $z = 2,58$ from table

$$2,58 \frac{7,651}{\sqrt{n}} \leq 1,5$$

$$\therefore n \geq \left[\frac{(2,58)(7,651)}{1,5}\right]^2$$

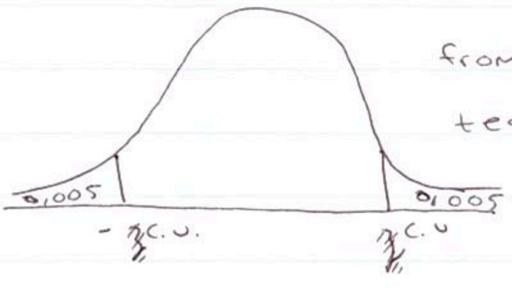
$$\therefore n \geq 173,17$$

$\therefore$ min 174 people

4.1 $H_0 : \mu = 100g$

$H_1 : \mu \neq 100g$ ($\therefore$ 2-tailed)

↳ i.e. 

4.2 

from table : critical value = 2,58

test statistic : $\dfrac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$

$= \dfrac{99,5 - 100}{\left(\frac{1,5}{\sqrt{\cancel{100}\ 40}}\right)}$

$= -2,11$

$-2,58 < -2,11 < 2,58$

$\therefore$ there is not enough evidence to reject H_0