****

**KEARSNEY COLLEGE TRIAL EXAMINATION**

**23 AUGUST 2019**

**ADVANCED PROGRAMME MATHEMATICS: PAPER I**

**MODULE 1: CALCULUS AND ALGEBRA**

Time: 2 hours 200 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

1. This question paper consists of 19 pages and an Information Sheet.

Please check that your question paper is complete.

2. Non-programmable and non-graphical calculators may be used, unless otherwise

indicated.

3. All necessary calculations must be clearly shown and writing should be legible.

4. Diagrams have not been drawn to scale.

5. Round off your answers to two decimal digits, unless otherwise indicated.

6. Ensure that your calculators are set to **RADIAN** mode.

**MEMORANDUM**

|  |  |  |
| --- | --- | --- |
| **QUESTION 1** | |  |
| 1.1 | (a) Express as a single logarithm. | (2) |
|  | (b) Hence, given that *x* satisfies the equation  show that *x* is a root of the quadratic equation | (3) |
| 1.2 | Solve the equation  Let | (4) |
| 1.3 | Solve for *x*: | (5) |

|  |  |  |
| --- | --- | --- |
| 1.5 | In a chemical reaction, the mass *m* grams of a chemical after *t* minutes is modelled by the equation |  |
|  | (a) Find the initial mass of the chemical.  *t* = 0: | (1) |
|  | (b) What is the mass of the chemical in the *long term*? | (2) |
|  | (c) Find the time when the mass is 30 grams. | (3) |
|  | (d) Sketch the graph of *m* against *t*. | (3) |

|  |  |  |
| --- | --- | --- |
| 1.6 | Solve for *x*:  Let | (6) |
|  |  | **[29]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 2** | |  |
| 2.1 | Solve for , given that  is a factor of  and . | (7) |

|  |  |  |
| --- | --- | --- |
| 2.2 | For how many values of *n* is  a real number if ?    Create a pattern: *n* = 0  *n* = 1  *n* = 2  …  *n* = 155    ∴ | (6) |
| 2.3 | Solve for x, where *x* is a real number: | (6) |
|  |  | **[19]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 3** | |  |
| Use mathematical induction to prove that  is divisible by 3, when .  Step 1: *n* = 1:  i.e. divisible by 3, so true for *n* = 1  Step 2: assume true for  Step 3: prove true for  or  ∴ true for *n* = 1; *n* = *k* and *n* = *k* + 1, ∴ true for all values of *n*. | |  |
|  |  | **[10]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 4** | |  |
| 4.1 | Given that , show that . | (5) |
| 4.2 | The function  is defined as follows:    Determine the value(s) of *a* and *b* if is differentiable at .  If differentiable, then continuous:    differentiable: | (10) |

|  |  |  |
| --- | --- | --- |
| 4.3 | The function  is defined by  for .  Show that  and state the domain of this function.    Range of  Domain of  or    based on sketch: | (6) |
|  |  | **[21]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 5** | |  |
| Fig. 1 shows a greenhouse which is built against a wall.    **Fig. 1**  **Fig. 2**  ,  ,  ,  , | | |
| The greenhouse is a prism of length 5,5m. The curve AC is an arc of a circle with centre B and radius 2,1m, as shown in Fig. 2. The sector angle ABC is 1,8 radians and ABD is a straight line. The curved surface of the greenhouse is covered in polythene (light flexible synthetic resin). | | |
| 5.1 | Find the length of the arc AC, and hence find the area of polythene required for the curved surface of the greenhouse. | (3) |
| 5.2 | Show that the angle  is 1,34 radians, rounded to two decimal places. | (2) |

|  |  |  |
| --- | --- | --- |
| 5.3 | Calculate the length BD, and hence the area of BCD.    area of BCD = | (5) |
| 5.4 | Calculate the volume of the greenhouse. | (5) |
|  |  | **[15]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 6** | |  |
| The curve *C* has equation  The Point *P* on *C* has an *x*-coordinate of 1. | | |
| 6.1 | Show that the value of  at *P* is 3. | (5) |
| 6.2 | Find the equation of the normal to *C* at *P*.  *x* = 1 →  *m* of normal: | (4) |
| 6.3 | The normal meets the *x*-axis at the point . Find the value of *k*. | (2) |
|  |  | **[11]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 7** | |  |
| A curve is described by the equation . | | |
| 7.1 | Find the coordinates of the two points on the curve where .  → | (3) |
| 7.2 | Find the gradient of the curve at each of these points, using implicit differentiation.    At | (8) |
|  |  | **[11]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 8** | |  |
| Consider the function . | | |
| 8.1 | Determine the intercepts with both axes. | (6) |
| 8.2 | Determine the equations of any asymptotes.    *oblique asymptote* since deg (num) ­− 1 = deg (denom)    so oblique asymptote is: | (8) |
| 8.3 | Determine the coordinates of any stationary points. | (8) |
| 8.4 | Use the fact that  to determine the nature of the stationary points you found in Question 8.3. | (4) |
|  |  | **[26]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 9** | |  |
| 9.1 | By writing  as , show that . | (5) |
| 9.2 | Determine , given  *Simplify your final answer* | (4) |
| 9.3 | The function  is defined for the domain  The function is alongside.  Find the range of *f*.      Range: | (6) |
|  |  | **[15]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 10** | |  |
| , where *x* is in radians. | | |
| 10.1 | Show that  has a root between  and    Change in sign, ∴has a root between  and | (2) |
| 10.2 | Using  as a first approximation, apply the Newton-Raphson method ONCE to find a second approximation for *x*, giving your answer to 5 decimal places. | (6) |
|  |  | **[8]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 11** | |  |
| Given: | | |
| 11.1 | Find the values of the constants *A*, *B* and *C*. | (5) |
| 11.2 | Hence, find | (6) |
|  |  | **[11]** |

|  |  |  |
| --- | --- | --- |
| **QUESTION 12** | |  |
| 12.1 | Determine the integrals for each of the following: |  |
|  | (a) | (5) |
|  | (b)  Note:    or using → | (5) |
|  | (c)  using: | (5) |
| 12.2 | The curve with equation  is shown in Figure 1.  Find the exact area of the shaded region.  use a calculator | (3) |

|  |  |  |
| --- | --- | --- |
| 12.3 | Use integration by parts to find  of the form:  where: | (6) |
|  |  | **[24]** |