FINANCE MEMO

Question 1

a) R 36 000 ✓✓ (2)

b) Jacob saves R 75 000 by 1st November 2023. ✓✓
∴ It takes Jacob 3 years and 10 months to save half the amount needed. ✓✓ (4)

c) Total savings @ 1 Dec 2023 = R 76 000 ✓
Total savings @ 1 Jan 2024 = R 104 000 ✓

Monthly installment @ 1 Jan 2024 = R 2500 ✓

∴ Lump sum = 104 000 - 76 000 - 2500
= R 25 500 → ✓ (4)

Question 2

a) $F = P(1-i)^n$ ✓

$= 90\,000\,(1-0.07)^5$ ✓

$= \underline{R\ 62\ 611.95324} \rightarrow$ ✓ (3)

b) $F = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ ✓

$= \dfrac{100\left[\left(1+\frac{0.11}{12}\right)^{60} - 1\right]}{\frac{0.11}{12}}$ ✓✓ (4)

$= \underline{R\ \ 7951.807969} \rightarrow$ ✓

c) Cost of upgrade $= 90\,000\,(1.045)^5$ ✓✓

$= R112\ 156.3744$ ✓ (5)

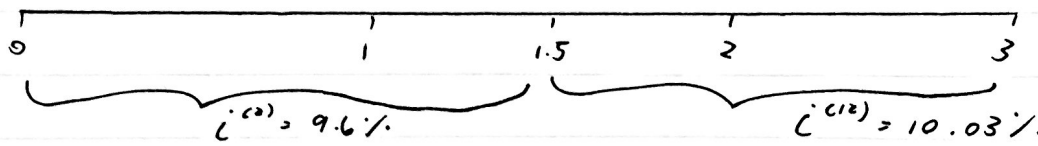
∴ Daniel needs R112 156.37 to upgrade.

If Daniel sells his equipment at R62 611.95 in 5 years time and saves R 7951.81 over the 5 years, he will contribute R 70 563.76 to his upgrade.

∴ No, Daniel will not have enough money. ✓

Question 3

a)

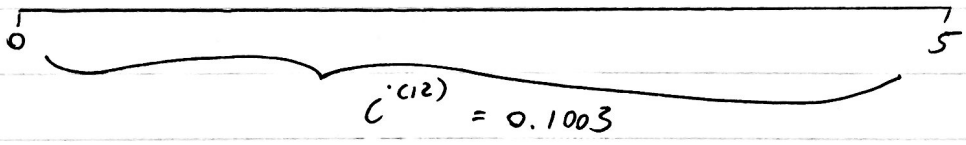


$$\text{Loan} = 51180\left(1+\frac{0.096}{2}\right)^{3}\left(1+\frac{0.1003}{12}\right)^{18} \quad \checkmark\checkmark$$
$$+ 53483.1\left(1+\frac{0.096}{2}\right)^{1}\left(1+\frac{0.1003}{12}\right)^{18} \quad \checkmark\checkmark$$
$$+ 55889.84\left(1+\frac{0.1003}{12}\right)^{12} \quad \checkmark$$
$$= 195\,301.2294$$
$$= \underline{R\ 195\ 301.23} \quad \checkmark \qquad (6)$$

b) $195\,301.23 - 51180 - 53\,483.1 - 55\,889.84$ ✓ (2)

$= \underline{R\ 34\ 748.29}$ ✓

c)



$$189\,835.93 = x\left[\frac{1-\left(1+\frac{i^{(4)}}{4}\right)^{-20}}{\frac{i^{(4)}}{4}}\right] \quad \checkmark$$

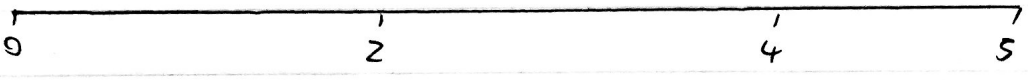
$$\left(1+\frac{i^{(4)}}{4}\right)^{4} = \left(1+\frac{0.1003}{12}\right)^{12} \quad \therefore i^{(4)} = 0.1011406\ldots\ (A) \quad \checkmark\checkmark$$

$$\therefore\ 189\,835.93 = x\left[\frac{1-\left(1+\frac{(A)}{4}\right)^{-20}}{\frac{(A)}{4}}\right] \quad \checkmark\checkmark$$

$$\therefore\ x = 12210.2569 \quad \checkmark\checkmark \qquad (7)$$

$$\therefore\ \underline{x = R\ 12210.26}$$

d)



i) $OB@2 = 12210.26\left[\frac{1-\left(1+\frac{(A)}{4}\right)^{-12}}{\frac{(A)}{4}}\right]$ ✓✓

$= 125\,033.9846$

$= \underline{125\,033.98} \rightarrow$ ✓ (3)

ii) $125\,033.98 = 15000\left[\frac{1-\left(1+\frac{(A)}{4}\right)^{-n}}{\frac{(A)}{4}}\right]$ ✓

$0.78923\ldots = \left(1+\frac{(A)}{4}\right)^{-n}$

$\therefore \log(0.78923\ldots) = -n\log\left(1+\frac{(A)}{4}\right)$ ✓✓

$\therefore n = 9.47882\ldots$

Rachel will have to make 10 installments. ✓ (4)

iii) $125033.9846\left(1+\frac{(A)}{4}\right)^{10} = 160499.9216$ ✓✓

$\therefore$ Final installment $= 160499.9216 - 15000\left[\frac{\left(1+\frac{(A)}{4}\right)^{9}-1}{\frac{(A)}{4}}\right]\left(1+\frac{(A)}{4}\right)$ ✓✓

$= \underline{R\ 7229.12} \rightarrow$ ✓✓ (6)

Modelling Memo

Question 4

a) Annual growth rate = birth rate - death rate death rate = $\frac{1}{\text{life expectancy}}$

Birth rate = % female × survival rate × size of litter × litters per cycle

$= 0.5 \times 0.4 \times 7 \times 1$ ✓✓

$= 1.4$

$\therefore$ Annual growth rate $= 1.4 - \frac{1}{6}$ ✓✓

$= 1.233$ ✓

(5)

b) $P_{n+1} = P_n(1+r)$ $P_0 = 500$ ✓

$= P_n(1+1.233)$

$= P_n(2.233)$ ✓

(2)

c) $P_6 = 61\,987.24615$ ✓

$\therefore \underline{P_6 = 61\,987}$ ✓

(2)

Question 5

a) $\frac{\Delta P}{P} = r\left(1 - \frac{P}{K}\right)$

$= -\frac{rP}{K} + r$ ✓

$\therefore \underline{r = 0.15} \rightarrow$ ✓ (2)

b) $-\frac{r}{K} = -0.000\overline{1875}$

$\therefore \frac{-0.15}{K} = -0.000\overline{1875}$ ✓ ✓

$\therefore \underline{K = 800} \rightarrow$ ✓ (3)

c) $0.055 = -0.000\overline{1875}P + 0.15$ ✓

$\therefore P = 506.66\ldots$

$\therefore \underline{P = 507} \rightarrow$ ✓ ✓ ✓ (4)

d) $P_{n+1} = P_n + r P_n \left(1 - \frac{P_n}{K}\right)$

$\therefore P_{n+1} = P_n + 0.15 P_n \left(1 - \frac{P_n}{800}\right)$ ✓

$P_0 = 102$ ✓ (3)

$P_1 = 115.34925$

$P_2 = 130.1568\ldots$

$P_3 = 146.50399\ldots$

$\approx \underline{147} \rightarrow$ ✓

Question 6

a) 17 000 ✓ (1)

b) $c = \frac{\text{cycle}}{\text{lifespan}}$ (3)

$= \frac{1}{5 \times 12}$

$= \frac{1}{60} = 0,01667$

c) $b \cdot W_n \cdot E_n$ = no. of elk killed by wolves per cycle

$b\ (15\,500)(30) = \frac{48 \times 30}{12}$ ✓ ✓ ✓

$\therefore b = 0.000258 \rightarrow$ ✓✓ (5)

d) $f \cdot b \cdot W_n \cdot E_n$ = the reproductive growth of wolves per cycle

$f\ (0.000258)(15\,500)(30) = \frac{(30)(0.4)(3)}{12}$ ✓✓✓

$\therefore f = 0.025006 \rightarrow$ ✓ (5)

e) Graph 2 ✓✓ (2)

f) Elk = 2500 ✓✓

Wolves = 130 ✓✓ (4)

Question 7

a) 15 moves ✓✓✓ (3)

b)

n	T_n
0	1
~~1~~ 1	3
~~2~~ 2	7
~~3~~ 3	15

$T_{n+1} = a T_n + b T_{n-1}$

$\therefore T_3 = a T_2 + b T_1 \Rightarrow 7 = 3a + b$ ✓

$T_4 = a T_3 + b T_2 \Rightarrow 15 = 7a + 3b$ ✓

Solve simultaneously:

$21 = 9a + 3b$

$-15 = -7a - 3b$

$6 = 2a$ ✓

$\therefore a = 3$ and $b = -2$ ✓

$T_0 = 1$ ✓

$T_1 = 3$

(6)

$\therefore T_{n+1} = 3T_n - 2T_{n-1}$ ✓