AP Prelim August 2019 ALGEBRA

1(a) $x|x+1|+4=x$

$|x+1| = \frac{x-4}{x}$ ✓

$\therefore x+1 = \frac{x-4}{x}$ if $x+1 \geq 0$ ✓ OR $x+1 = \frac{-(x-4)}{x}$ ✓ if $x+1<0$ ✓

$x<-1$

$x^2+x = x-4$

$x^2=-4$

No real soln. ✓

$x^2+x=-x+4$

$x^2+2x-4=0$ ✓

$x = \frac{-2\pm\sqrt{20}}{2}$

$= -1\pm\sqrt{5}$ ✓ $\therefore x = 1-\sqrt{5}$ ✓ (8)

(b) $\ln(x-4) = \ln x - 4$

$x-4>0$ & $x>0$

$\therefore x>4$ ✓

$\ln(x-4)-\ln x = -4$ ✓

$\ln\left(\frac{x-4}{x}\right) = -4$ ✓

$\frac{x-4}{x} = e^{-4}$ ✓

$x = 4{,}07$ ✓ (5)

(c) $\ln(\cos x) = -2$ ✓

$\cos x = e^{-2}$ ✓

$x = \pm 1{,}44 + 2\pi k$ ✓; $k\in\mathbb{Z}$

$\therefore x = 1{,}44$ ✓ or $x=3{,}84$ ✓ (5)

(2) $x=-i$ is one root

$\therefore x=i$ is another root ✓

$(x+i)(x-i) = x^2-i^2$ ✓

$= x^2+1$ ✓

$\therefore (x^2+1)(x-6) = x^3+ax^2+bx-6$ ✓

$x^3-6x^2+x-6 = x^3+ax^2+bx-6$

$\therefore a=-6$ ✓

$b=1$ ✓ (6)

(3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Prove: $1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$ ✓✓

Prove statement is true for $n=1$

$\therefore$ LHS $=1^2=1$ RHS $= \frac{1(1+1)(2+1)}{6} = 1$ ✓

$\therefore$ LHS = RHS. i.e. statement true for $n=1$ ✓

Assume statement is true for $n=k$

i.e. $1^2+2^2+3^2+\ldots+k^2 = \frac{k(k+1)(2k+1)}{6}$ ✓✓

Prove statement is true for $n=k+1$

LHS $= 1^2+2^2+3^2+\ldots+k^2+(k+1)^2$ ✓

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ ✓

$= (k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$ ✓

$= \frac{(k+1)(2k^2+7k+6)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$ ✓

RHS $= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ ✓

$= \frac{(k+1)(k+2)(2k+3)}{6}$ ✓

$\therefore$ LHS = RHS i.e statement is true for $n=k+1$ ✓

Conclusion

4a (i) $|x-2| < 3$

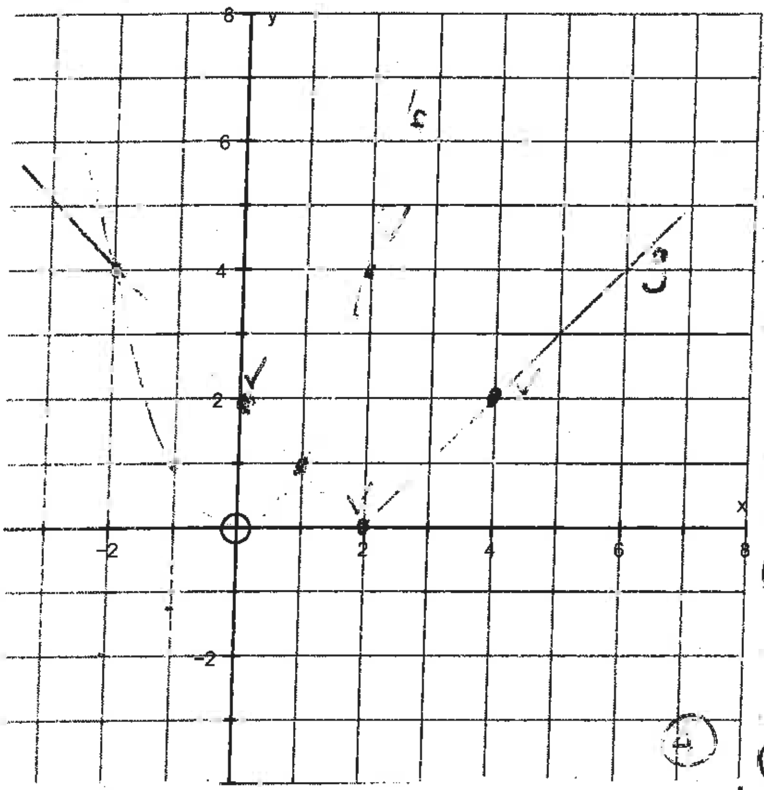
$-3 < x-2 < 3$ ✓

$-1 < x < 5$ ✓

(ii) $|x-2| = -3$

No real solution. ✓

(b)



(c) $|x-2| > x^2$ if $x > 0$

$x \in (0; 1)$

(5) $f(x) = \dfrac{2x^2-7x+4}{x-3}$

Vertical asymptote: $x = 3$ ✓

$$\begin{array}{r|l} & 2x-1 \\ x-3 & 2x^2-7x+4 \\ & 2x^2-6x \\ & -x+4 \\ & -x+3 \\ & 1 \end{array}$$

$\therefore f(x) = (2x-1) + \frac{1}{(x-3)}$

$f(x) - (2x-1) = \dfrac{1}{x-3}$ ✓

$\lim\limits_{x\to\pm\infty} [f(x)-(2x-1)] = \lim\limits_{x\to\pm\infty} \left(\frac{1}{x-3}\right)$

$= 0$ ✓

(b) $f'(x) = \dfrac{(4x-7)(x-3) - 1(2x^2-7x+4)}{(x-3)^2}$

At stat pts, $f'(x) = 0$

$\therefore 4x^2 - 19x + 21 - 2x^2 + 7x - 4 = 0$

$2x^2 - 12x + 17 = 0$

$x^2 - 6x + 9 - 9 + \frac{17}{2} = 0$

$(x-3)^2 = \frac{1}{2}$

$x = 3 \pm \sqrt{\frac{1}{2}}$

$= 3{,}71$ or $2{,}29$

(c) $f(0) = -\frac{4}{3}$

y-int $(0; -4/3)$

for x-int $2x^2 - 7x + 4 = 0$

$x = \dfrac{7 \pm \sqrt{49-32}}{4}$

$x =$ or $x =$

(d) $f'(0) = \frac{17}{9} > 0$

$\therefore$ function increasing at $x = 0$

(6) $A\hat{C}B = \frac{\pi}{2}$ ($\angle$ subt. by diam)

Perim shaded area $=$

$\frac{1}{2}$(circumf) $+ AC + CB$

$= \pi a + 2a(\sin\theta + \cos\theta)$

$\boxed{\frac{AC}{2a} = \sin\theta \quad \frac{BC}{2a} = \cos\theta}$

(b) Area $\odot = \pi r^2 = \pi a^2$

Area $\Delta ACB = \frac{1}{2} AC.BC$

$= \frac{1}{2}.2a\sin\theta.2a\cos\theta$

$= 2a^2\sin\theta\cos\theta$

$\therefore$ Shaded area: $\pi a^2 - 2a^2\sin\theta\cos\theta$

$= a^2(\pi - 2\sin\theta\cos\theta)$

$= a^2(\pi - \sin 2\theta)$

(c) $\theta \in (0; \frac{\pi}{2})$

AP MATHS PAPER 1 AUG 2019 CALCULUS MEMO

7a) $\Delta x = \frac{3-1}{6} = \frac{1}{3}$

$x_1 = 1 + \frac{1}{3} = \frac{4}{3} \quad \therefore y_1 = 3$

$x_2 = \frac{4}{3} + \frac{1}{3} = \frac{5}{3} \quad \therefore y_2 = \frac{12}{5}$

$x_3 = \frac{6}{3} = 2 \quad \therefore y_3 = 2$

$x_4 = \frac{7}{3} \quad ; \quad y_4 = \frac{12}{7}$

$x_5 = \frac{8}{3} \quad ; \quad y_5 = \frac{3}{2}$

$x_6 = 6 \quad ; \quad y_6 = \frac{4}{3}$

(10) $\text{Area} = \frac{1}{3} \sum_{1}^{6} f(x_k)$

$= \frac{1}{3}\left(3 + \frac{12}{5} + 2 + \frac{12}{7} + \frac{3}{2} + \frac{4}{3}\right)$

$= 3{,}99 \text{ u}^2$

b) $A = \int_1^3 \frac{4}{x}\,dx$

$= 4\ln x \Big|_1^3$

(4) $= 4{,}39\text{ u}^2$

c) • Too few rectangles
• All rectangles lie under the graph

(2)

8a) LHS $= \dfrac{1}{\left(\frac{\sin x}{\cos x}\right)\sqrt{\sin^2 x}}$

$= \dfrac{1}{\left(\frac{\sin^2 x}{\cos x}\right)}$

(6)

$= \dfrac{\cos x}{\sin^2 x}$

RHS $= \dfrac{1}{\sin x} \cdot \dfrac{\cos x}{\sin x}$

$=$ LHS

b) $\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \operatorname{cosec} x \cot x$

(5) $= -\operatorname{cosec} x \Big|_{\frac{\pi}{6}}^{\frac{2\pi}{3}}$

$= \frac{6 - 2\sqrt{3}}{3}$

9a) $\dfrac{x+1}{x(x^2+x-6)} = \dfrac{x+1}{x(x+3)(x-2)}$

(8) $= \dfrac{A}{x} + \dfrac{B}{x+3} + \dfrac{C}{x-2}$

$\therefore (x+1) = A(x+3)(x-2) + Bx(x-2) + C(x)(x+3)$

when $x=2$: $3 = 2C(5)$

$\therefore C = \frac{3}{10}$

when $x=-3$: $-2 = -3(B-5)$

$\therefore B = \frac{2}{15}$

when $x=0$: $1 = A(-6)$

$A = -\frac{1}{6}$

$\therefore \dfrac{x+1}{x(x+3)(x-2)} = \dfrac{-1}{6x} + \dfrac{2}{15(x+3)} + \dfrac{3}{10(x-2)}$

b) $\int \dfrac{x+1}{x(x+3)(x-2)}\,dx$

(4) $= -\frac{1}{6}\ln|x| + \frac{2}{15}\ln|x+3| + \frac{3}{10}\ln|x-2| + c$

10a) $y = \left(x^2 + x^{\frac{3}{2}}\right)^{\frac{1}{3}}$

(5) $y' = \frac{1}{3}\left(x^2 + x^{\frac{3}{2}}\right)^{-\frac{2}{3}}\left(2x + \frac{3}{2}x^{\frac{1}{2}}\right)$

b i) $y' = -e^x \sin\left(e^x + \frac{\pi}{4}\right)$

For min $y' = 0$

$\therefore \sin\left(e^x + \frac{\pi}{4}\right) = 0$

(6) $\therefore e^x + \frac{\pi}{4} = 0 + k\pi \quad k \in \mathbb{Z}$

$\therefore e^x = -\frac{\pi}{4} + k\pi$

$\therefore$ least pos value $\Rightarrow e^x = \frac{3\pi}{4}$

$\therefore x = \ln\left(\frac{3\pi}{4}\right)$

ii) $y'' = -e^{2x}\cos\left(e^x + \frac{\pi}{4}\right) - e^{2x}\sin\left(e^x + \frac{\pi}{4}\right)$

(6) $y' - e^{2x}y = -e^x\sin\left(e^x + \frac{\pi}{4}\right) - e^{2x}\cos\left(e^x + \frac{\pi}{4}\right)$

$=$ RHS

10c) $e^y = \sin(x+y)$

$y'e^y = \cos(x+y)\{1+y'\}$

$y'e^y - y'\cos(x+y) = \cos(x+y)$

(8) $y'(e^y - \cos(x+y)) = \cos(x+y)$

$y' = \dfrac{\cos(x+y)}{e^y - \cos(x+y)}$

11a) $\ln\left(\frac{x^4}{x-1}\right)$

$y = \ln x^4 - \ln(x-1)$

(4) $y = 4\ln x - \ln(x-1)$

b) $x > 0$ and $x-1>0$

(6) $\therefore x > 1$

c) $y' = \frac{4}{x} - \frac{1}{x-1} = 0$ at stat pt

$\therefore \dfrac{4(x-1)-x}{x(x-1)} = 0$

$\therefore 3x-4=0$

$x = \frac{4}{3}$

(11) $y = 2{,}25$

$\therefore$ stat pt $\left(\frac{4}{3}; 2{,}25\right)$

d) $1 < x < \frac{4}{3}$

(2) $x \in \left(1; \frac{4}{3}\right)$

12. $V = \pi\int_0^2 y^2\,dx$

$= \pi\int_0^2 xe^x\,dx$

Let $f(x) = x$ $\therefore f'(x) = 1$

$g'(x) = e^x$ $\therefore g(x) = e^x$

(10) $\therefore V = \pi\left\{xe^x\Big|_0^2 - \int_0^2 e^x\,dx\right\}$

$= \pi\left\{xe^x\Big|_0^2 - e^x\Big|_0^2\right\}$

$= \pi\left(2e^2 - e^2 + 1\right)$

$= \pi\left(e^2+1\right)$ u^2

13a) $(F\circ G)x = F(x+2)$

$= 3(x+2)+2$

(9) $= 3x+8$

b) $G(x) = x^2$

c) $F(x) = \frac{1}{x}$

14a) $k'(x) = 2 - \sec x\tan x$

$= 2 - \dfrac{\tan x}{\cos x}$

(7) $x - \dfrac{k(x)}{k'(x)} = x - \dfrac{\left(2x - \frac{1}{\cos x}\right)}{\left(2 - \frac{\tan x}{\cos x}\right)}$

$= \dfrac{\cancel{2x} - \frac{x\tan x}{\cos x} - \cancel{2x} + \frac{1}{\cos x}}{\frac{2\cos x - \tan x}{\cos x}} \times \dfrac{\cos x}{\cos x}$

$= \dfrac{1 - x\tan x}{2\cos x - \tan x}$

b) Let $x_1 = \frac{1}{2}$

(5) $x_2 = 0{,}601266$

$x_3 = 0{,}609953$

$x_4 = 0{,}61003 = x_5$

$\therefore x \approx 0{,}61$

15a) Area $A = \int_0^b h(x)\,dx - \int_0^b f(x)\,dx$

(3)

b) Area $B = \int_0^b g(x)\,dx - \int_0^d (f(x) - g(x))\,dx$

(4)

AP MATHS PRELIM AUG 2019 PAPER 2 STATISTICS

MEMO

1(a) $^8P_4 = 1680$ (4)

b) $\binom{17}{7} \times \binom{10}{2} \times \binom{8}{3} \times \binom{5}{5}$ (6)

$= 49\,008\,960$

2.a) $\dfrac{4 \times \binom{13}{5} \times \binom{39}{0}}{\binom{52}{5}}$

$= \dfrac{33}{16660}$ (5)

$= 0{,}00198$

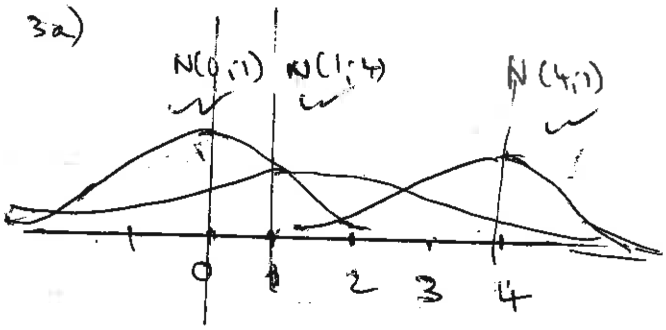
b) P(≥ 2 in same month)

= 1 − P(all 6 different)

$= 1 - \frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12}$

$= 0{,}78$ (8)

3a)



b.i) $X \sim N(500;30^2)$

$P(X \le 473 \text{ or } X \ge 482)$

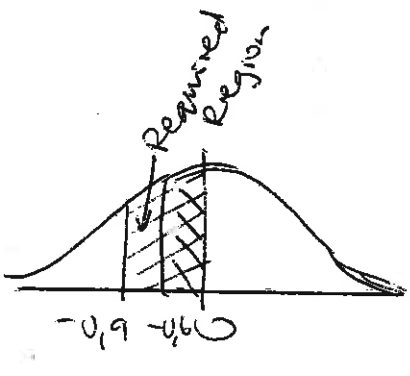
$= 1 - P(473 < x < 482)$

$= 1 - P\left(\frac{473-500}{30} < z < \frac{482-500}{30}\right)$

$= 1 - P(-0{,}9 \le z < -0{,}6)$

$= 1 - (0{,}3159 - 0{,}2257)$

$= 0{,}9098$ (8)



b) P(3 pkts between 473 g and 482g)

$= (0{,}9098)^3$

$= 0{,}7531$ (4)

3(c i) $p = 0{,}8$ (gun works) $\}$ 2

$q = 0{,}2$

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$ ✓

$= 1 - \binom{10}{0}(0{,}2)^{10} - \binom{10}{1}(0{,}8)^1(0{,}2)^9$ ✓✓

$= 0{,}99 \quad (\approx 1)$ ✓ (6)

ii) $n = 500 \quad (> 30)$ ✓

$np = 500 \times 0{,}7 = 350 \quad (> 5)$ ✓

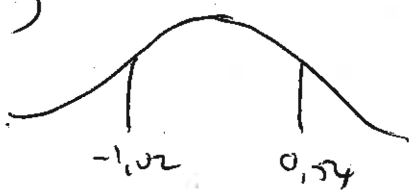
$s^2 = npq = 500 \times 0{,}7 \times 0{,}3 = 105$ ✓

$P(340 < x < 355)$

$= P(339{,}5 < x < 355{,}5)$ ✓✓

$= P\left(\frac{339{,}5 - 350}{\sqrt{105}} < Z < \frac{355{,}5 - 350}{\sqrt{105}}\right)$ ✓

$= P(-1{,}02 < Z < 0{,}54)$ ✓



$= 0{,}3461 + 0{,}2054$

$= 0{,}5515$ ✓ (8)

4(a) $\bar{x} = 28{,}15$ ✓ ; $s = 4{,}28$ ✓

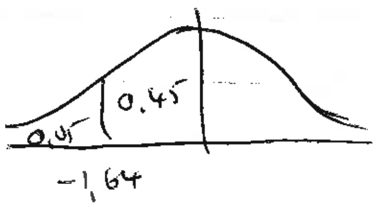
$CI = \left[28{,}15 - 1{,}96 \times \frac{4{,}28}{\sqrt{20}} ; 28{,}15 + 1{,}96 \times \frac{4{,}28}{\sqrt{20}}\right]$ ✓ ✓

$= [26{,}27 ; 30{,}03]$ ✓✓ (6)

b) $H_0 : \mu = 29$ ✓) (1 tailed

$H_1 : \mu < 29$ ✓

$z_c = -1{,}64$ ✓✓



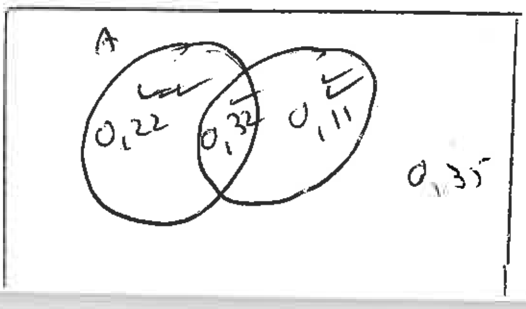
Test statistic $= \frac{28{,}15 - 29}{\frac{4{,}28}{\sqrt{20}}}$ ✓ ✓ (8)

$= -0{,}88$

∴ Not enough evidence to reject H_0 in favour of H_1

∴ accept that call response time is not less ✓✓

5a



b) $P(A) = 0{,}54 \quad P(B) = 0{,}43$

c) $P(A|B') = \frac{P(A \cap B')}{P(B')}$ ✓ $= \frac{0{,}22}{0{,}57}$ ✓✓

$= 0{,}3859$ ✓✓

6a) $P(4 \text{ girls invited}) = \dfrac{\binom{6}{4}\binom{9}{3}}{\binom{15}{7}}$ (6)

$= 0{,}1958$

b) i) $\sum P(X=x) = k + 0{,}6k + (0{,}6)^2 k + (0{,}6)^3 k + \dots$

$\therefore 1 = \dfrac{k}{1-0{,}6}$

$\therefore 0{,}4 = k$ (6)

ii) $P(X \geq 3) = 1 - \{P(X=0) + P(X=1) + P(X=2)\}$

$= 1 - 0{,}4 - (0{,}4)(0{,}6) - (0{,}4)(0{,}6)^2$

$= 0{,}216$ (6)