MEMO

AP Maths P2 : Statistics
Prelim Exam 2019

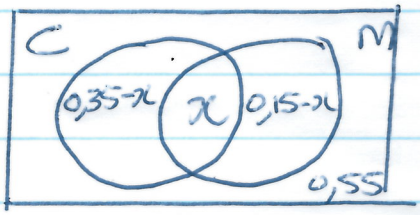
Question 1

1.1.

a) $0,35 - x + x + 0,15 - x = 0,45$ ✓ ✓

$x = \frac{1}{20}$ ✓

or $x = 0,05$



$\therefore P(C \cap M) = \frac{1}{20}$ (3)

b) $P(C|M) = \frac{P(C \cap M)}{P(M)}$ ✓ use "given" formula

$= \frac{\frac{1}{20}}{0,15}$ ✓

$= \frac{1}{3}$ ✓ ✓ (4)

1.2. $P(X = x) = \frac{\mu^x e^{-\mu}}{x!}$ $x = 0; 1; 2; \ldots$

a) $P(X = 2) = 2P(X = 1)$

$\therefore \frac{\mu^2 \cdot e^{-\mu}}{2!}$ ✓ $= 2\left(\frac{\mu \cdot e^{-\mu}}{1!}\right)$ ✓

$\therefore \frac{\mu^2 e^{-\mu}}{2}$ ✓ $= 2\mu e^{-\mu}$ ✓

$\therefore \mu = 4$ ✓ (5)

b) $P(x \geq 2)$ ✓ $= 1$ ✓ $- P(X = 1 \text{ or } X = 0)$ ✓

$= 1 - \left(\frac{4 \cdot e^{-4}}{1!}\right.$ ✓ $\left. + \frac{4^0 e^{-4}}{0!}\right)$ ✓

$= 0,9084$ ✓ (6)

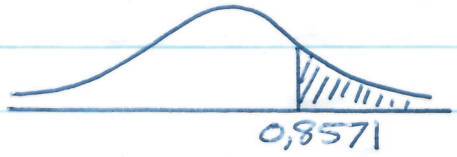
Question 2

2.1. $\bar{x} = 640$; $\sigma = 70$

$P(x \geq 700)$ ✓

$= P\left(z \geq \frac{700 - 640}{70}\right)$ ✓ ✓ ✓

$= P(z \geq 0,8571)$ ✓



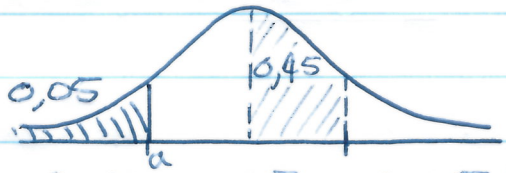
$= 0,5 - 0,3051$ ✓

$= 0,1949$ ✓

(6)

2.2. Probability $\leq 0,05$

$\therefore P(z \leq a) = 0,05$ ✓ ✓



$0,5 - 0,05 = 0,45$ ✓

$a = -1,645$

$\therefore -1,645$ ✓ $= \frac{X - 640}{70}$ ✓

$\therefore$

$\therefore X = 524,85$ weeks ✓

(6)

Question 3

3.1. $P(\text{defective}) = p = 0,1$

$\therefore 1 - p = 0,9$ ✓

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - ({}^{15}C_0 p^0 q^{15} + {}^{15}C_1 p^1 q^{14} + {}^{15}C_2 p^2 q^{13})$

$= 0,1841$ ✓

(14)

3.2. $P(\text{reject}) = 0,1841$

$P(\text{accept}) = 0,8159$

$E(X) = (0,8159 \times 38,00) + (0,1841 \times 5)$

$= R31,93$ ✓

$\therefore$ Expected profit $= 31,93 - 20,00$ ✓

$= R11,93$

(6)

Question 4

$$P(X = 2) = \frac{\binom{20}{5}\binom{50-20}{5-2}}{\binom{50}{5}}$$ ✓ use Hg distrib.

$= 0,3641$

(8)

Question 5

$n = 200$

$p = P(\text{even no}) = \frac{1}{2}$ ✓ $\quad q = \frac{1}{2}$ ✓

$\mu = np$ ✓

$= 200 \times \frac{1}{2}$

$= 100$ ✓

$\sigma = \sqrt{npq}$ ✓

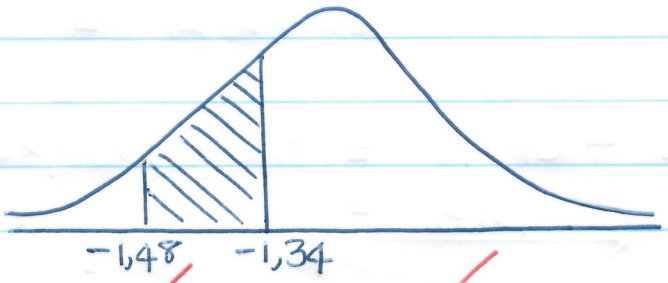
$= \sqrt{200(\frac{1}{2})(\frac{1}{2})}$

$= \sqrt{50}$ ✓

$\therefore P(89,5 \leq x \leq 90,5)$ ✓ ✓ binomial ↔ normal approx

$= P\left(\frac{89,5-100}{\sqrt{50}} \leq z \leq \frac{90,5-100}{\sqrt{50}}\right)$

$= P(-1,48 \leq z \leq -1,34)$ ✓ ✓



$= 0,4306 - 0,4099$ ✓ ✓

$= 0,0207$ ✓

(13)

Question 6

$H_0: \mu = 100$ ✓

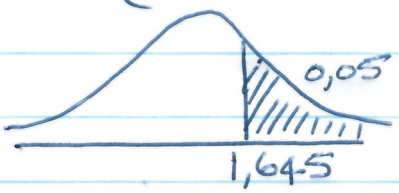
$H_1: \mu > 100$ ✓

one-tailed: $z = 1,645$ ✓ (critical value)

$z = \frac{112 - 100}{\frac{15}{\sqrt{30}}}$ ✓ ✓ ✓

$z = 4,3818$ ✓

(in the 5% area)



$\therefore$ We reject H_0 at 5% signif level ✓

(9)

Question 7

7.1. $\bar{x} = 12,5$

$\sigma = \sqrt{4,5}$

a) For 94% $z = \pm 1,88$

$\therefore \pm 1,88 = \dfrac{12,5 - \mu}{\sqrt{\frac{4,5}{6}}}$

$\mu = (10,8719 \; ; \; 14,1281)$ (6)

b) There is a 94% probability that the true population mean lies between the two above lengths (2)

7.2. a) $\hat{p} = \dfrac{25}{400} = \dfrac{1}{16} = 0,0625$

$16 \times 400 = 6400$ in population (3)

b) For 95% CI $z = \pm 1,96$

$p = \frac{1}{16} \pm 1,96 \sqrt{\dfrac{(\frac{1}{16})(\frac{15}{16})}{400}}$

$= (0,1375 \; ; \; 0,0388)$ (pop prob of success [tagged bird] in CI)

$\therefore$ # of birds $= (2909,09 \; ; \; 10\,309,28)$

ie 95% confident there are between 2909 & 10 309 birds in the population (9)