

**Advanced Programme Mathematics Paper 2**

**Grade 12**

 **Preliminary Examination**

 **2019**

**Duration: 1 hour Examiner: R. Obermeyer**

**Marks: 100 Moderator: P. Pitout**

**Date: 20 August 2019 External Moderator: I. L. Atteridge**

**INSTRUCTIONS:**

* **See overleaf for Instructions.**
* **This paper consists of 9 pages (including cover and an information sheet) and an information sheet.**

**NAME: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

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| **Question** | **Level Tested** | **Suggested Time Allocation** | **Possible mark** | **Mark Obtained** |
| 1 | 1 – 4  |  13 mins | **22** |  |
| 2 | 1 – 4  | 16 mins | **26** |  |
| 3 | 1 – 4  |  6 mins | **10** |  |
| 4 | 1 – 4  |  19 mins | **32** |  |
| 5 | 1 – 4  | 6 mins | **10** |  |
|  |  | **TOTAL:** | **100** |  |
|  |  | **PERCENTAGE:** |  |

**Teacher’s Signature:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Controller’s Signature:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Moderator’s Signature:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 **Instructions**

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 9 pages including an answer sheet and an Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round your answers off to **one** decimal digit where necessary unless otherwise stated.
7. It is in your own interest to write legibly and to present your work neatly.
8. Please hand in this question paper.
9. Answer all questions **underneath** each other.
10. Start each new question on a **new** page.



**QUESTION 1**

**PENTA** is a regular pentagon, formed by rotating point $P(26,4; 19,1)$ about the origin.

The coordinates of two other vertices are also given: $A(26,4; -19,1)$ and $N(-32,6; 0)$.



1. Explain why $P\hat{O}E = 72°$. (2)
2. Use a matrix calculation to determine the coordinates of **E**, correct to the nearest integer. (6)
3. **PENTA** is the image of another regular pentagon, after an enlargement through the origin. If the area of **PENTA** is ***k*** times the area of the original pentagon, give the matrix in terms of ***k***, that maps the original figure onto **PENTA** (2)
4. Line segment PA is to be reflected about a line with equation $y=mx$.

 The images of these respective points are then $P^{1}(31; -10)$ and $A^{'}(19,2;26,3)$.

Find the angle of inclination of the line of reflection, correct to the nearest degree. (12)

  **[22]**

**QUESTION 2**

1. Given matrix $P=\left(\begin{matrix}-1&1&5\\2&4&1\\-2&2&3\end{matrix}\right)$

 Find the matrix ***Q*** so that $PQ=\left(\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right)$ (10)

1. Solve the following equations simultaneously, using Gaussian reduction.

 Be sure to show the relevant working in the process of obtaining the solutions.

 $x+y-2z=6$

 $-2x+z=5$

 $x-y-3z=-15$

 (10)

1. Consider four planes ***A, B,*** and ***C*:**

 **A:** $x-2y+z=9$

 **B:** $4x-y+2z=20$

 **C:** $2x+3y=-6$

Prove that planes ***A, B,*** and ***C*** do not intersect at a unique point. (6)

 **[26]**

**QUESTION 3**

**Wheel graphs** consist of a central vertex, which is surrounded by a ring of peripheral vertices. Each peripheral vertex is directly connected to the central vertex, as well as to only two adjacent peripheral vertices. Four wheel graphs have been sketched below:



1. For a wheel graph $W\_{n}$, where $n$ is the number of vertices, state in terms of $n$:
2. The number of internal regions (2)
3. The number of edges in the graph (2)
4. The minimum number of edges that need to be added to create an Eulerian circuit for odd values of $n$. (2)
5. Sketch the graph $W\_{3}$, clearly showing the correct number of vertices, edges and internal regions. (4)

 **[10]**

**QUESTION 4**

24

20

24

31

23

29

25

22

15

Hout Bay

City C

Airport A

Milnerton

V & A

Sea Point

Blue Route

Tygerberg

18

18

30

12

The map above represents a section of the Cape Town metropolitan area, with the weights of the edges representing the time in minutes expected for journeys along the edges.

The owner of a business which first opened at Blue Route, has expanded to each of the centres on the network, whilst maintaining his Operations Office at Blue Route (marked accordingly)

1. Use Kruskal’s algorithm to determine the shortest (and most cost effective) route that Eskom can take in order to up-grade the lines so as to connect each of the centres to each other. State the minimum distance of this spanning tree. (8)
2. Use Dijkstra’s algorithm to determine the quickest route the Sales’ Manager can take in order get from his home in Hout Bay to Tygerberg. State the length of this shortest route. (8)
3. The Sales Manager must travel each route to identify advertising boards.

Use the Chinese Postman algorithm to determine the **quickest route** the Sales’ Manager can take in order to cover each route starting at, and returning back to, the Blue Route Office. (8)

1. Determine a lower bound, based on Prim’s algorithm and initially leaving out Milnerton for the Sales Manager to visit each centre once, starting and ending at Blue Route.

Record the order in which you choose the edges. (8)

**[32]**

**QUESTION 5**

Three graphs are given in the form of five-by-five adjacency matrices.



1. Draw a graph of matrix **K.** Clearly indicate the weight of each edge in the graph. (6)
2. State which matrix represents a graph that is connected, but not simple. Give a reason for your answer. (4)

**[10]**

**Answer Sheet**

**Name: ……………………………………**

24

20

24

31

23

29

25

22

15

Hout Bay

City C

Airport A

Milnerton

V & A

Sea Point

Blue Route

Tygerberg

18

18

30

12

24

20

24

31

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Hout Bay

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Tygerberg

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12