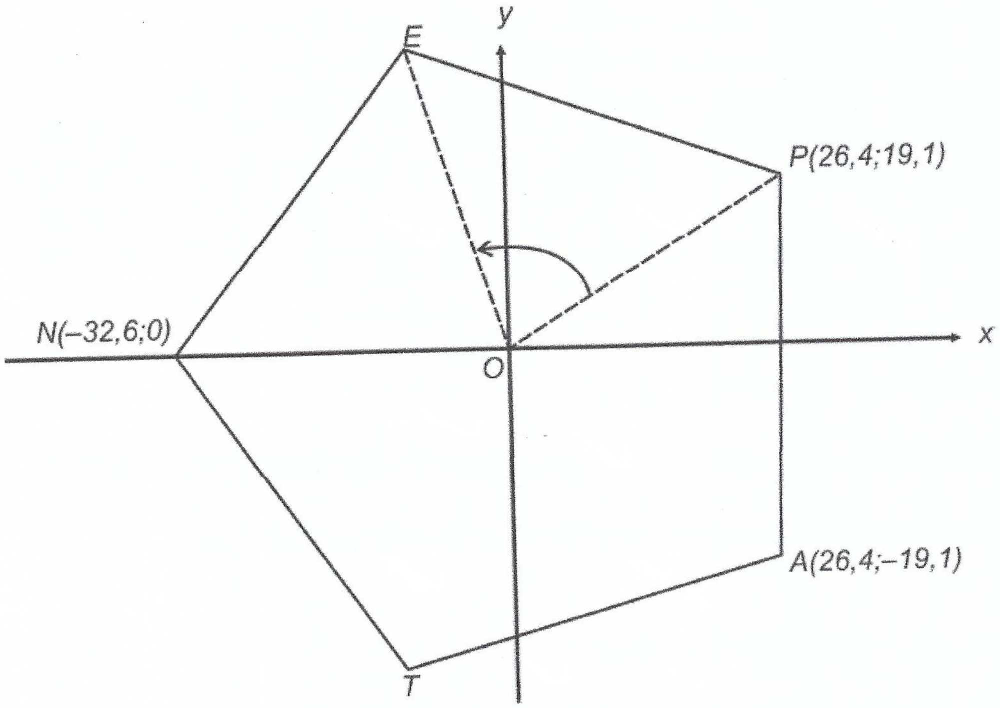
Gr 12 AP Maths Prelim 2019

SECTION B GRAPHS and MATRICES **(100 marks)**

QUESTION 1

PENTA is a regular pentagon, formed by rotating point **P(26,4 ; 19,1)** about the origin. The coordinates of two other vertices are also given: **A(26,4 ; −19,1)** and **N(−32,6 ; 0)**.



a) Explain why $\mathbf{P\hat{O}E = 72^\circ}$

Pentagon: $360^\circ \div 5 = 72^\circ$ (2)

b) Use a matrix calculation to determine the coordinates of **E**, correct to the nearest integer

Anti-clockwise rotation: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$\therefore \begin{pmatrix} \cos 72^\circ & -\sin 72^\circ \\ \sin 72^\circ & \cos 72^\circ \end{pmatrix}\begin{pmatrix} 26,4 \\ 19,1 \end{pmatrix} = \begin{pmatrix} -10 \\ 31 \end{pmatrix}$$

E: $(-10;31)$ (6)

.3 c PENTA is the image of another regular pentagon, after an enlargement through the origin. If the area of **PENTA** is k times the area of the original pentagon, give the matrix in terms of k, that maps the original figure onto **PENTA**

Enlargement matrix: $\begin{pmatrix} \sqrt{k} & 0 \\ 0 & \sqrt{k} \end{pmatrix}$ ✓✓ (2)

.4 d Line segment PA is to be reflected about a line with equation $y = mx$. The images of these respective points are then **P'(31 ; −10)** and **A'(19,2 ; 26,3)**. Find the angle of inclination of the line of reflection, Correct to the nearest degree.

$$\therefore \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}\begin{pmatrix} 26,4 & 26,4 \\ 19,1 & -19,1 \end{pmatrix} = \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix}$$ ✓✓✓

Multiplying LHS: (any two of ...)

$26,4\cos 2\theta + 19,1\sin 2\theta = 31$ *i)* ✓ $26,4\cos 2\theta - 19,1\sin 2\theta = 19,2$ *ii)* ✓

$-19,1\cos 2\theta + 26,4\sin 2\theta = -10$ *iii)* ✓ $19,1\cos 2\theta + 26,4\sin 2\theta = 26,3$ *iv)* ✓

$i) + ii):$ $52,8\cos 2\theta + 0 = 50,2$ ✓

$\cos 2\theta = \dfrac{50,2}{52,8} \Rightarrow 2\theta = 18,055^\circ$ ✓✓

$\Rightarrow \theta = 9^\circ$ ✓✓

(12)
[22]

3

QUESTION 2

a) Given matrix $P = \begin{pmatrix} -1 & 1 & 5 \\ 2 & 4 & 1 \\ -2 & 2 & 3 \end{pmatrix}$

Find the matrix Q so that $PQ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

any method.

$$\text{det P} = -1(12-2) - 1(6+2) + 5(4+8) = 42 \checkmark\checkmark$$

$$P = \begin{pmatrix} -1 & 1 & 5 \\ 2 & 4 & 1 \\ -2 & 2 & 3 \end{pmatrix} \rightarrow \text{co-factor matrix:} \begin{pmatrix} 10 & 8 & 12 \\ -7 & 7 & 0 \\ -19 & -11 & -6 \end{pmatrix} \checkmark\checkmark$$

$$\begin{pmatrix} 10 & 8 & 12 \\ -7 & 7 & 0 \\ -19 & -11 & -6 \end{pmatrix} \rightarrow \text{apply signs matrix:} \begin{pmatrix} 10 & -8 & 12 \\ 7 & 7 & 0 \\ -19 & 11 & -6 \end{pmatrix} \checkmark\checkmark$$

$$\begin{pmatrix} 10 & -8 & 12 \\ 7 & 7 & 0 \\ -19 & 11 & -6 \end{pmatrix} \rightarrow \text{transpose matrix:} \begin{pmatrix} 10 & 7 & -19 \\ -8 & 7 & 11 \\ 12 & 0 & -6 \end{pmatrix} \checkmark\checkmark$$

$$\therefore Q: \frac{1}{42}\begin{pmatrix} 10 & 7 & -19 \\ -8 & 7 & 11 \\ 12 & 0 & -6 \end{pmatrix} \checkmark\checkmark$$

(10)

b) Solve the following equations simultaneously, using Gaussian reduction.
Be sure to show the relevant working in the process of obtaining the solutions.

$-2x+z=5 \qquad x-y-3z=-15 \qquad x+y-2z=6$

$$\begin{pmatrix} -2 & 0 & 1 & 5 \\ 1 & -1 & -3 & -15 \\ 1 & 1 & -2 & 6 \end{pmatrix}$$

$$\begin{array}{c} R_1 \\ R_1+2R_2 \\ R_1+2R_3 \end{array} \begin{pmatrix} -2 & 0 & 1 & 5 \\ 0 & -2 & -5 & -25 \\ 0 & 2 & -3 & 17 \end{pmatrix}$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_2+R_3 \end{array} \begin{pmatrix} -2 & 0 & 1 & 5 \\ 0 & -2 & -5 & -25 \\ 0 & 0 & -8 & -8 \end{pmatrix}$$

R_3: $-8z=-8 \Rightarrow z=1$

R_2: $-2y-5=-25 \Rightarrow y=10$

R_2: $-2x+1=5 \Rightarrow x=-2$

(10)

c) Consider four planes A, B, C

$A:\quad x-2y+z=9 \qquad B:\quad 4x-y+2z=20$
$C:\quad 2x+3y=-6$

Prove that planes $A, B,$ and C do not intersect at a unique point.

Matrix of coefficients $= M = \begin{pmatrix} 1 & -2 & 1 \\ 4 & -1 & 2 \\ 2 & 3 & 0 \end{pmatrix}$

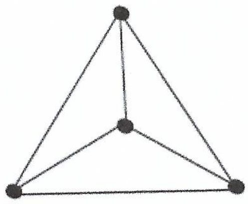
Det $M = 1(0-6)+2(0-4)+1(12+2)=0$

$\therefore$ **planes A, B and C do not intersect at a point**

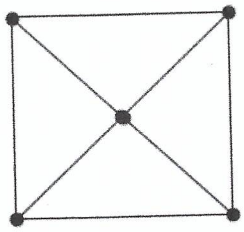
(6)

QUESTION 3

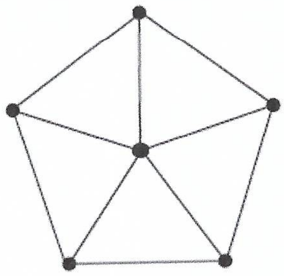
Wheel graphs consist of a central vertex, which is surrounded by a ring of peripheral vertices. Each peripheral vertex is directly connected to the central vertex, as well as to only two adjacent peripheral vertices. Four wheel graphs have been sketched below:



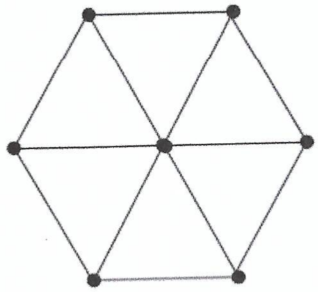
W_4: 4 vertices
6 edges
3 internal regions



W_5: 5 vertices
8 edges
4 internal regions



W_6: 6 vertices
10 edges
5 internal regions



W_7: 7 vertices
12 edges
6 internal regions

a) For a wheel graph W_n, where n is the number of vertices, state in terms of n:

1) The number of internal regions

$\exists\ n-1$ internal regions (2)

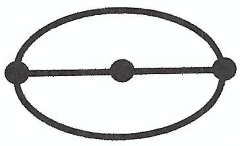
2) The number of edges in the graph

$\exists\ 2n-2$ edges (2)

3) The minimum number of edges that need to be added to create an Eulerian circuit for odd values of n.

Add $\frac{n-1}{2}$ edges (2)

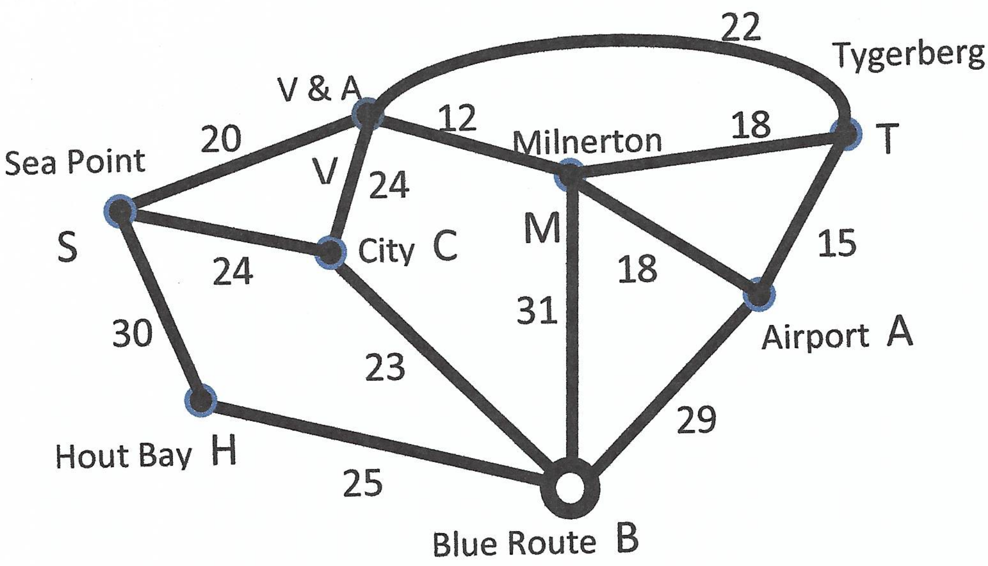
b) Sketch the graph W_3, clearly showing the correct number of vertices, edges and internal regions.



(4)

[10]

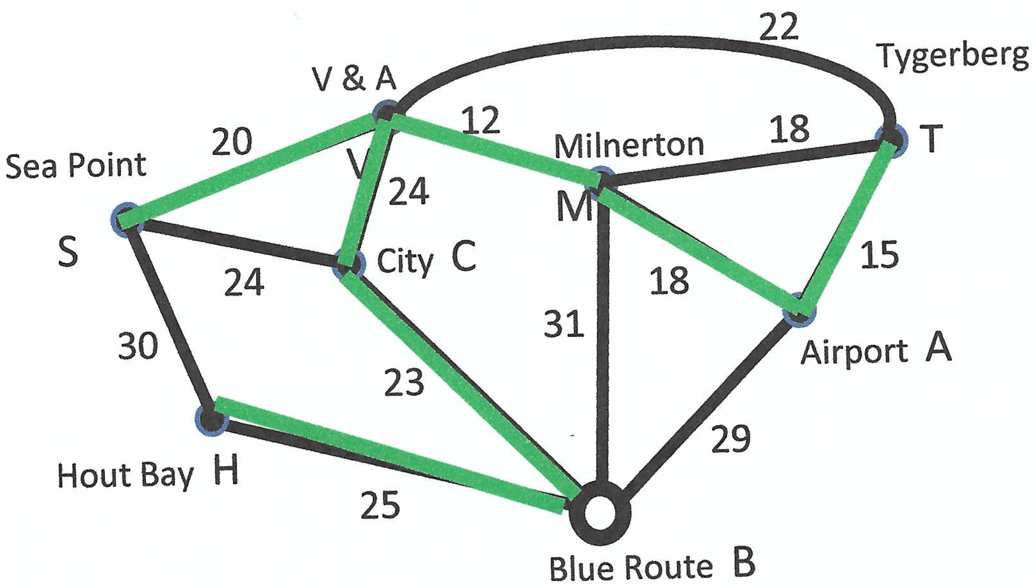
QUESTION 4



The map above represents a section of the Cape Town metropolitan area, with the weights of the edges representing the time in minutes expected for journeys along the edges.

The owner of a business which first opened at Blue Route, has expanded to each of the centres on the network, whilst maintaining his Operations Office at Blue Route (marked accordingly)

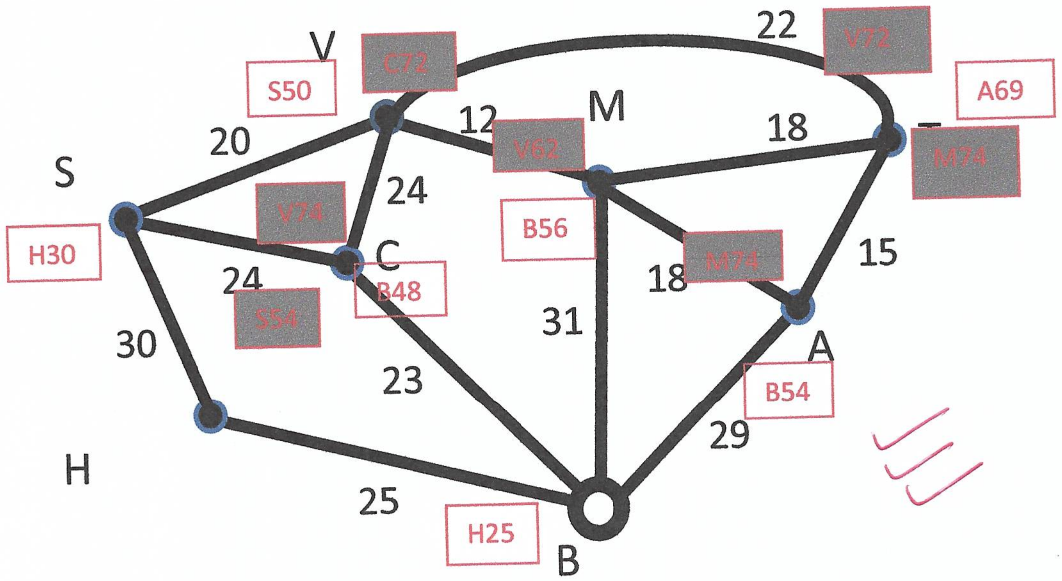
a) Use Kruskal's algorithm to determine the shortest (and most cost effective) route that Eskom can take in order to up-grade the lines so as to connect each of the centres to each other. State the minimum distance of this spanning tree.



(8)

$VM:12$ ✓ $AT:15$ ✓ $AM:18$ ✓ $VS:20$ ✓ $BC:23$ ✓ $VC:24$ ✓ $BH:25$ ✓ $Total:137$ ✓

b) Use Dijkstra's algorithm to determine the quickest route the Sales' Manager can take in order get from his home in Hout Bay to Tygerberg. State the length of this shortest route. (8)



Start at H:-

Visited	Visit(RouteCost)	Yes/No
H	S(H 30) B(H 25)	
(H)S	V(HS 50) C(HS 54)	No
(HS)B	C(HB 48) M(HB 56) A(HB 54)	Yes (delete above)
(HSB)V	T(HSV 72) M(HSV 62) C(HSV 74)	No No
(HSBV)C	V(HBC 72)	No
(HSBVC)M	T(HBM 74) A(HBM 74)	No No
(HSBVCM)	A(HBA 72)	No
(HSBVCMA)	T(HBA 69)	

Quickest route: $H \rightarrow B \rightarrow A \rightarrow T = 69$

(8)

3.c) The Sales Manager must travel each route to identify advertising boards. Use the Chinese Postman algorithm to determine the **quickest route** the Sales' Manager can take in order to cover each route starting at, and returning home to, the Blue Route Office.

We need an Eulerian Circuit, i.e. all vertices of even degree

But: A, C, S, T all have odd degree

So, find shortest distance of pairings A, C, S, T

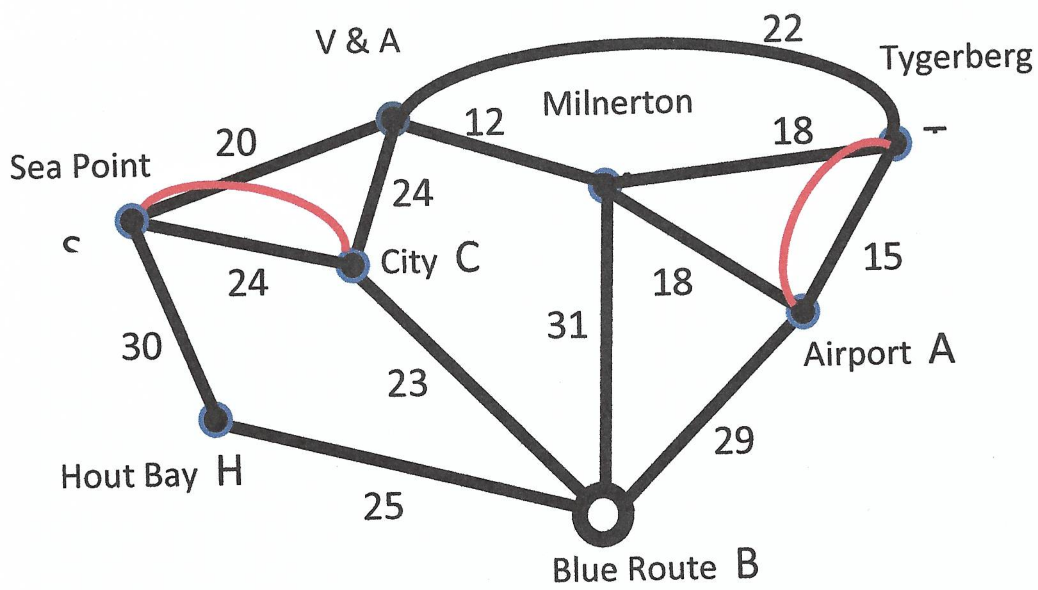
	A	C	S	T
A	-	52	50	15
C	52	-	24	46
S	50	24	-	42
T	15	46	42	-

AC & ST $= 52 + 42 = 94$ ✓

AS & CT $= 50 + 46 = 96$ ✓

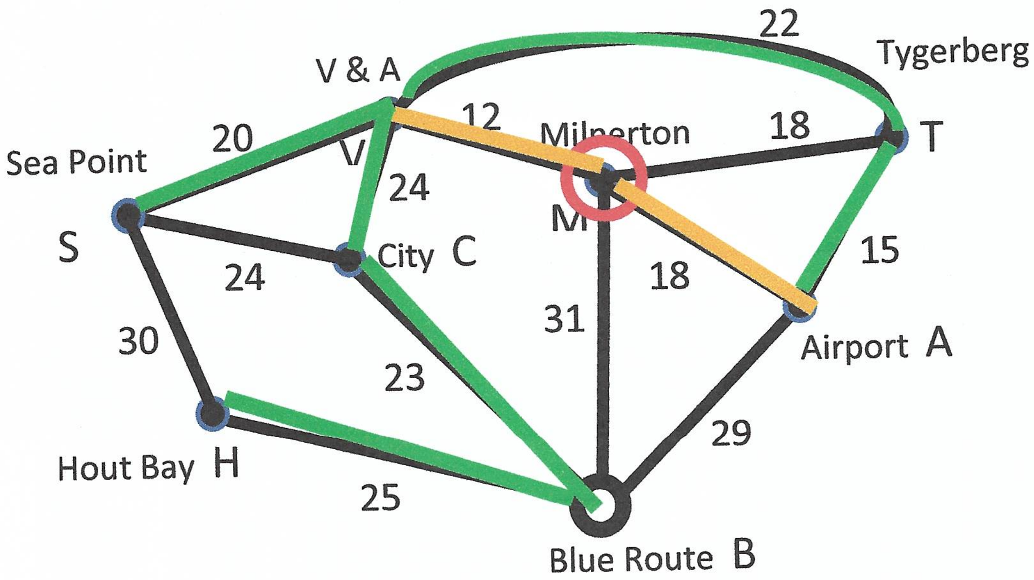
AT & CS $= 15 + 24 = 39$ **use this pair** ✓

$\therefore$ **Add chains: AT & CS to the graph** $\rightarrow$ **Eulerian Circuit** ✓



One option:– $B \rightarrow H \rightarrow S \rightarrow C \rightarrow S \rightarrow V \rightarrow C \rightarrow B \rightarrow M \rightarrow V \rightarrow T$ $\rightarrow M \rightarrow A \rightarrow T \rightarrow A \rightarrow B = 330$ ✓✓✓✓ (8)

d) Determine a lower bound, based on Prim's algorithm and initially leaving out Milnerton for the Sales Manager to visit each centre once, starting and ending at Blue Route. Record the order in which you choose the edges.



Remove M

BC : 23   *CV* : 24   *VS* : 20   *VT* : 22   *TA* : 15   *BH* : 25   *Total* : 129

Reconnect M with 2 shortest edges

VM : 12   &   *AM* : 18 or *TM* : 18

Total : 159

(8)
[32]

10

QUESTION 5

Three graphs are given in the form of five-by-five adjacency matrices.

Matrix G

	G_1	G_2	G_3	G_4	G_5
G_1			5		
G_2				3	8
G_3	5				
G_4		3			6
G_5		8		6	

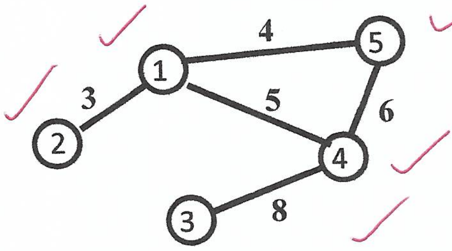
Matrix H

	H_1	H_2	H_3	H_4	H_5
H_1					5
H_2			5		
H_3		8			8
H_4					6
H_5	5		8	6	

Matrix K

	K_1	K_2	K_3	K_4	K_5
K_1		3		5	4
K_2	3				
K_3				8	
K_4	5		8		6
K_5	4			6	

a) Draw a graph of matrix **K.** Clearly indicate the weight of each edge in the graph.



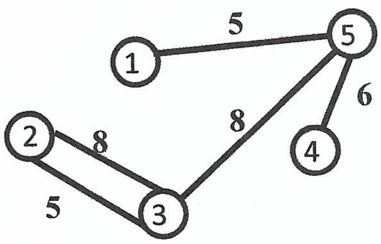
(6)

b) State which matrix represents a graph that is connected, but not simple. Give a reason for your answer.

***H* :** **either: matrix not symmetrical,**

or: double edge between 2 & 3

or: sketch



(4)

[10]