GRADE 12 AP MATHS PAPER 1 PRELIM SEPT 2019 MEMO

QUESTION 1

1.1 $|x|^2 - 4|x| - 12 = 0$

$\therefore (|x| - 6)(|x| + 2) = 0$ ✓ ✓

$\therefore |x| = 6$ ✓ or $|x| = -2$ ✓

$\therefore \underline{x = \pm 6}$ ✓ no soln ✓

Alt: let $k = |x|$ ✓ (6)

$\therefore k^2 - 4k - 12 = 0$

$\therefore (k-6)(k+2) = 0$

$\therefore k = 6$ or $k = -2$ ✓

$\therefore |x| = 6$ ✓ or $|x| = -2$ ✓

$\therefore x = \pm 6$ ✓ no soln ✓

1.2 $\dfrac{3 + \ln x^3}{\ln e + \ln x} = \dfrac{3 + 3\ln x}{1 + \ln x} = \dfrac{3(1 + \ln x)}{1 + \ln x} = 3$ ✓ ✓ ✓ (3)

1.3 (a) $60 = 20 + (90 - 20)e^{-10k}$ ✓ ✓ ✓ ✓ ✓

$\therefore 40 = 70e^{-10k}$

$\therefore \frac{4}{7} = e^{-10k}$ ✓ (8)

$\therefore \log_e\left(\frac{4}{7}\right) = -10k$ ✓

$\therefore -\frac{1}{10}\ln\left(\frac{4}{7}\right) = k$ ✓

(b) $T = 20 + 70e^{-15\left(-\frac{1}{10}\ln\left(\frac{4}{7}\right)\right)}$ ✓ ✓ (3)

$= 50°C$ ✓

QUESTION 2

2.1 $x = 2 - 3i$

$x = 2 + 3i$ ✓

Sum of roots $= 2 - 3i + 2 + 3i = 4$ ✓

Product of roots $= (2-3i)(2+3i) = 4 - 9i^2 = 13$ ✓

(6)

Quad factor: $x^2 - 4x + 13 = 0$ ✓

$\therefore b = -4$ ✓ and $c = 13$ ✓

2.2 $x^2 + bx + c = 0$

$\therefore (2-3i)^2 + b(2-3i) + (-4+19i) = 0$ ✓ ✓

$\therefore 4 - 12i + 9i^2 + 2b - 3bi - 4 + 19i = 0$ ✓

$\therefore -9 + 7i + 2b - 3bi = 0$

$\therefore b(2-3i) = 9 - 7i$ ✓

$\therefore b = \dfrac{9-7i}{2-3i}$ ✓

$\therefore b = \dfrac{9-7i}{2-3i} \cdot \dfrac{2+3i}{2+3i}$ ✓

$\therefore b = \dfrac{18 + 13i - 21i^2}{4 - 9i^2}$ ✓ ✓

(10)

$\therefore b = \dfrac{39 + 13i}{13}$ ✓

$\therefore \underline{b = 3 + i}$ ✓

QUESTION 3

$$\sum_{p=1}^{n} \frac{1}{(2p-1)(2p+1)}$$

$$= \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$ ✓

For $n=1$ $\quad T_1 = S_1 = \frac{1}{3}$ ✓

$\therefore$ True for $n=1$

Assume true for $n=k$, $k \in \mathbb{N}$ ✓

$$\therefore \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$ ✓

Prove true for $n=k+1$

$$\frac{1}{3} + \frac{1}{15} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$ ✓✓

$$= \frac{k}{2k+1} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$ ✓✓

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k(2k+3) + 1}{(2k+1)(2k+3)}$$

(12)

$$= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$$

$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$ ✓

$$= \frac{k+1}{2k+3}$$

$$= \frac{k+1}{2k+2+1} = \frac{k+1}{2(k+1)+1}$$ ✓ ✓

By the P.O.M.I, true for all $n \in \mathbb{N}$. ✓

QUESTION 4

4.1 (a) $\left.\begin{array}{l}\lim\limits_{x \to 0^-} e^x = e^0 = 1 \\ \lim\limits_{x \to 0^+} (x^2+1) = 1\end{array}\right\}$ ✓

$\therefore \lim\limits_{x \to 0} f(x) = 1$ (6)

$f(0) = (0)^2+1 = 1$

$\therefore \lim\limits_{x \to 0} f(x) = f(1)$ ✓

$\therefore f$ is continuous at $x = 0$ ✓

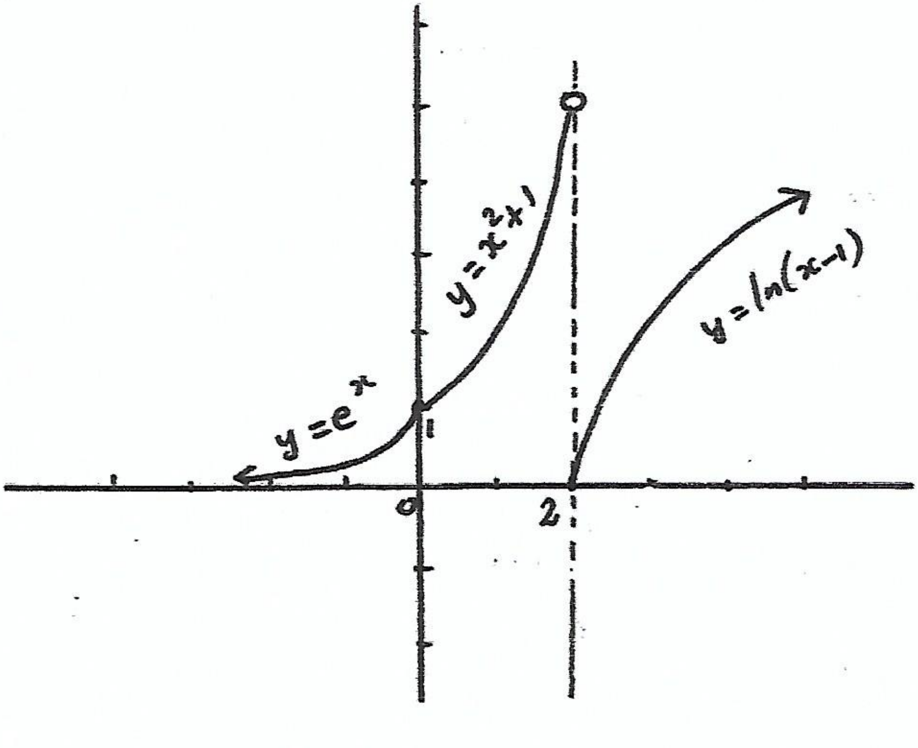
$\lim\limits_{x \to 0^-} D_x(e^x) = \lim\limits_{x \to 0^-} e^x = 1$ ✓

$\lim\limits_{x \to 0^+} (2x) = 0$ ✓

$\therefore \lim\limits_{x \to 0} f'(x)$ doesn't exist

✓

$\therefore f$ is not differentiable at $x = 0$.



5

(b) $\lim_{x \to 2^-} (x^2+1) = 5$ ✓

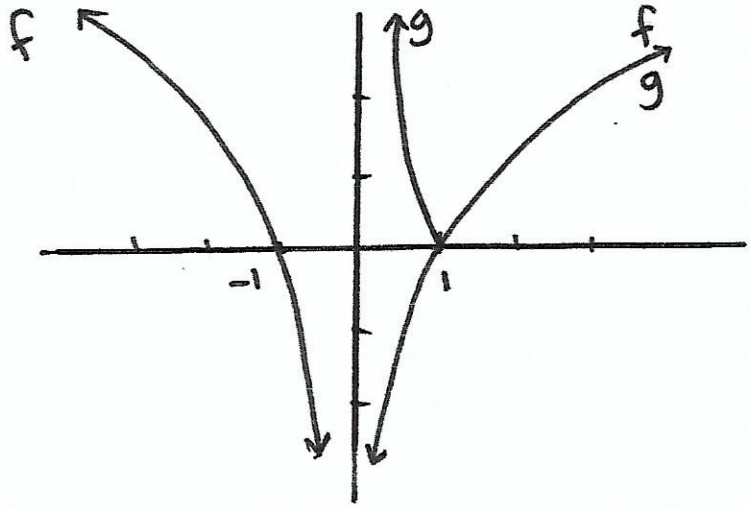
$\lim_{x \to 2^+} \ln(x-1) = 0$ ✓ (4)

$\therefore \lim_{x \to 2} f(x)$ doesn't exist.

$\therefore f$ is not continuous at $x = 2$. ✓

$\therefore f$ is not differentiable at $x = 2$. ✓

4.2 (a) $f(x) = \ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$



f ✓✓✓ (6)

g ✓✓✓

(b) $f(x) = g(x)$ (1)

$\therefore x \geq 1$ ✓

QUESTION 5

5.1 $y = e^{2x} . \ln 2x$

$$\frac{dy}{dx} = 2e^{2x} . \ln 2x + e^{2x} . \frac{1}{2x} . 2 \qquad (4)$$

$$= 2e^{2x} . \ln 2x + \frac{e^{2x}}{x}$$

5.2 (a) $f(x) = \cot x$

$$f'(x) = -\operatorname{cosec}^2 x$$

$$f''(x) = -2\operatorname{cosec} x . -\operatorname{cosec} x \cot x$$

$$= 2\operatorname{cosec}^2 x . \cot x$$

$$= 2 . \frac{1}{\sin^2 x} . \frac{\cos x}{\sin x} \qquad (4)$$

$$= \frac{2\cos x}{\sin^3 x}$$

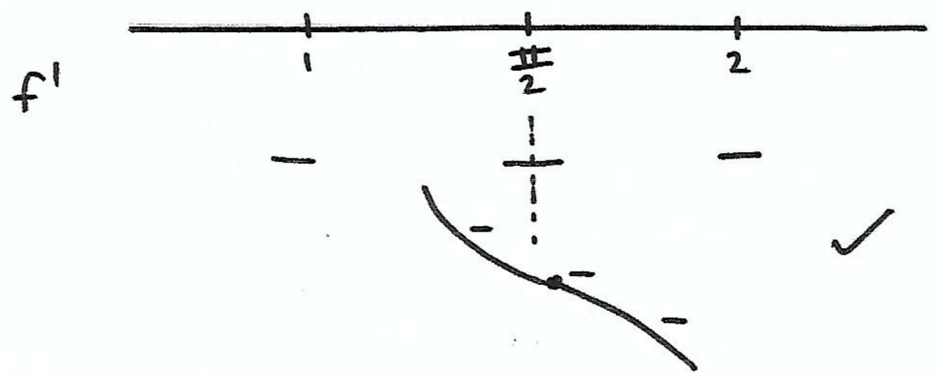
(b) $f'\left(\frac{\pi}{2}\right) = -\operatorname{cosec}^2\left(\frac{\pi}{2}\right) = -1 \neq 0$

$\therefore$ Non-stationary at $x = \frac{\pi}{2}$

$$f''\left(\frac{\pi}{2}\right) = \frac{2\cos\frac{\pi}{2}}{\sin^2\frac{\pi}{2}} = 0$$

$\therefore$ Possible point of inflection at $x = \frac{\pi}{2}$.

Use first derivative test to verify.



(4)

$\therefore$ Non-stationary point of inflec at $x = \frac{\pi}{2}$.

5.3 $g(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$

$g'(x) = 4x^3 - 12x^2 + 12x - 4$ ✓

$g''(x) = 12x^2 - 24x + 12$ ✓

$\therefore 0 = 12x^2 - 24x + 12$

$\therefore 0 = x^2 - 2x + 1$

$\therefore 0 = (x-1)^2$

$\therefore x = 1$ ✓

	Try $x = 0$	Try $x = 2$
g'	$4(0)^3 - 12(0)^2 + 12(0) - 4$	$4(2)^3 - 12(2)^2 + 12(2) - 4$
	$= -4$	$= 4$
	$-$	$+$

1

(6)

✓✓

There is a stationary point at $x = 1$.
statement is not true for g. ✓

QUESTION 6

6.1 (a) $y = \frac{3}{4}$ ✓ (1)

(b) $y = 0$ ✓ (1)

(c)
$$\begin{array}{r|l} & 3x - 5 \\ \hline x+1 & 3x^2 - 2x + 1 \\ & 3x^2 + 3x \\ \hline & -5x + 1 \\ & -5x - 5 \\ \hline & 6 \end{array}$$ ✓✓ (3)

$y = 3x - 5$ ✓

6.2 $f(x) = \dfrac{(x+3)(x-4)}{(2x-1)(x-4)}$

$= \dfrac{x+3}{2x-1}$ ✓ $x \neq 4$ (removable disc)

$\therefore$ The vertical asymptote is $x = \frac{1}{2}$. ✓ (2)

6.3 $f(x) = \dfrac{x}{\ln x}$

$f'(x) = \dfrac{(1)(\ln x) - (x).\left(\frac{1}{x}\right)}{(\ln x)^2}$ ✓✓✓

$0 = \dfrac{\ln x - 1}{(\ln x)^2}$ ✓

$\therefore 0 = \ln x - 1$ (8)

$\therefore \ln x = 1$ ✓

$\therefore \log_e x = 1$ $\therefore y = \dfrac{e}{\ln e} = e$ ✓

$\therefore x = e$ ✓

$(e;e)$ ✓

QUESTION 7

7.1 (a) $2x + 3y - xy^2 + 4 = 0$

$D_x(2x) + D_x(3y) - D_x(xy^2) + D_x(4) = 0$

$\therefore 2 + 3.\frac{dy}{dx} - [1.y^2 + x.D_x(y^2)] + 0 = 0$

$\therefore 2 + 3.\frac{dy}{dx} - [y^2 + x.2y.\frac{dy}{dx}] = 0$

$\therefore 2 + 3.\frac{dy}{dx} - y^2 - 2xy.\frac{dy}{dx} = 0$ ✓✓✓✓

$\therefore \frac{dy}{dx}(3 - 2xy) = y^2 - 2$ ✓ (5)

$\therefore \frac{dy}{dx} = \frac{y^2 - 2}{3 - 2xy}$

(b) At $(-1;-2)$, $\frac{dy}{dx} = \frac{(-2)^2 - 2}{3 - 2(-1)(-2)} = \frac{2}{-1} = -2$ ✓

$y - (-2) = -2(x - (-1))$ ✓

$\therefore y + 2 = -2(x+1)$

$\therefore y + 2 = -2x - 2$ (3)

$\therefore y = -2x - 4$ ✓

7.2 (a) $D = 3\sin 4x - [\frac{1}{2}(x+1)^2 - 6]$

$\therefore D = 3\sin 4x - \frac{1}{2}(x+1)^2 + 6$

$\therefore D' = 3.\cos 4x.4 - \frac{1}{2}.2(x+1)^1.1 + 0$

$\therefore D' = 12\cos 4x - x - 1$ ✓ (4)

$\therefore 0 = 12\cos 4x - x - 1$ ✓

$\therefore -12\cos 4x = -x - 1$

$\therefore 12\cos 4x = x + 1$ ✓

(b) $12\cos 4x = x + 1$

$\therefore 12\cos 4x - x - 1 = 0$

$\therefore F(x) = 12\cos 4x - x - 1$ ✓

$\therefore F'(x) = 12.-\sin 4x.4 - 1$

$= -48\sin 4x - 1$ ✓ (4)

$$x_{n+1} = x_n - \frac{12\cos 4x - x - 1}{-48\sin 4x - 1}$$ ✓✓

(c) $x_0 = 0{,}5$

$x_1 = 0{,}3545508708$ ✓

$x_2 = 0{,}3642421086$ ✓

$x_3 = 0{,}3642163293$ ✓ (5)

$x_4 = 0{,}3642163292$ ✓

$\therefore \underline{x \approx 0{,}36422}$ ✓

QUESTION 8

8.1 (a) $\int e^{2x+1}\,dx$

Method 1

$\int e^{2x+1}\,dx$

$= \frac{1}{2}\int 2e^{2x+1}\,dx$ ✓✓

$= \frac{1}{2}e^{2x+1} + c$ ✓

Method 2

Let $u = 2x+1$ ✓ (3)

$\therefore \frac{du}{dx} = 2$

$\therefore \frac{du}{2} = dx$

$\int e^{2x+1}\,dx$

$= \int e^{u} \cdot \frac{du}{2}$ ✓

$= \frac{1}{2}\int e^{u}\,du$

$= \frac{1}{2}e^{u} + c$

$= \frac{1}{2}e^{2x+1} + c$ ✓

(b) $\int \frac{4x}{\sqrt{x^2-1}}\,dx$

Method 1

$\int 4x(x^2-1)^{-\frac{1}{2}}\,dx$ ✓

$= 2\int 2x(x^2-1)^{-\frac{1}{2}}\,dx$ ✓✓

$= 2 \cdot \frac{(x^2-1)^{\frac{1}{2}}}{\frac{1}{2}} + c.$ ✓✓

$= 4\sqrt{x^2-1} + c$ ✓

Method 2

Let $u = x^2-1$ ✓ (6)

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$ ✓

$\int 4 . x\,dx . (x^2-1)^{-\frac{1}{2}}$

$= 4\int \frac{du}{2} \cdot u^{-\frac{1}{2}}$ ✓

$= 2\int u^{-\frac{1}{2}}\,du$

$= 2\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$ ✓✓

$= 4\sqrt{x^2-1} + c$ ✓

(c) $\int \frac{4x}{x^2-1}\,dx$

Method 1

$$\int \frac{4x}{x^2-1}$$

$$= 2\checkmark \int \frac{2x\checkmark}{x^2-1} \quad \frac{f'}{f}$$ (4)

$$= 2\ln|x^2-1| + c \quad \checkmark\checkmark$$

Method 2

Let $u = x^2-1$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x\,dx \quad \checkmark$$

$$\therefore \int \frac{4x}{x^2-1}\,dx = \int \frac{4 \cdot \frac{du}{2}}{u} =$$

$$= 2\int \frac{1}{u}\,du \quad \checkmark$$ (4)

$$= 2\ln|u| + c \quad \checkmark$$

$$= 2\ln|x^2-1| + c \quad \checkmark$$

Method 3

$$\frac{4x}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$\therefore 4x = A(x-1) + B(x+1)$

Let $x = 1$

$\therefore 4 = 2B$

$\therefore B = 2$

Let $x = -1$

$\therefore -4 = A(-1-1) + 0$

$\therefore -4 = -2A$

$\therefore A = 2$

$\therefore \dfrac{4x}{x^2-1} = \dfrac{2}{x+1} + \dfrac{2}{x-1}$ ✓✓

$\therefore \int \dfrac{4x}{x^2-1}\,dx$

$= \int \left(\dfrac{2}{x+1} + \dfrac{2}{x-1}\right) dx$

$= 2\int \dfrac{1}{x+1}\,dx + 2\int \dfrac{1}{x-1}\,dx$

$= 2\ln|x+1| + 2\ln|x-1| + c$

$= 2(\ln|x+1| + \ln|x-1|) + c$

$= 2\ln|x+1|.|x-1| + c$ (4)

$= 2\ln|(x+1)(x-1)| + c$

$= 2\ln|x^2-1| + c$ ✓

8.2 (a) $\dfrac{1}{x^3+x^2} = \dfrac{1}{x^2(x+1)}$

$\therefore \dfrac{1}{x^2(x+1)} \checkmark = \dfrac{a}{x} + \dfrac{b}{x^2} + \dfrac{c}{x+1}$

$\therefore 1 = ax(x+1) + b(x+1) + cx^2 \checkmark$

let $x = 0$

$\therefore 1 = b$

$\therefore b = 1 \checkmark$

$1 = ax(x+1) + 1(x+1) + cx^2$

let $x = -1$

$\therefore 1 = a(-1)(-1+1) + 1(-1+1) + c(-1)^2$

$\therefore 1 = c$

$\therefore c = 1 \checkmark$ (6)

$1 = ax(x+1) + (x+1) + x^2$

let $x = 1$

$\therefore 1 = a(1)(1+1) + (1+1) + (1)^2$

$\therefore 1 = 2a + 2 + 1$

$\therefore -2a = 2$

$\therefore a = -1 \checkmark$

$\therefore \dfrac{1}{x^3+x^2} = \dfrac{-1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x+1} \checkmark$

(b) $\int \frac{1}{x^3+x^2}\,dx$

$= \int \left(\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\right) dx$

$= -\int \frac{1}{x}\,dx + \int x^{-2}\,dx + \int \frac{1}{x+1}\,dx$

$= -\ln|x| + \frac{x^{-1}}{-1} + \ln|x+1| + c$ (5)

$= -\ln|x| - \frac{1}{x} + \ln|x+1| + c$

$= \ln|x+1| - \ln|x| - \frac{1}{x} + c$

$= \ln\left|\frac{x+1}{x}\right| - \frac{1}{x} + c$

8.3 $\int 2x\, e^{3x}\,dx$

$f(x) = 2x$ $\quad g'(x) = e^{3x}$

$f'(x) = 2$ $\quad g(x) = \int e^{3x}\,dx$

$= \frac{1}{3}e^{3x}$

$\int 2xe^{3x}\,dx = 2x.\frac{1}{3}e^{3x} - \int 2.\frac{1}{3}e^{3x}\,dx$

$= \frac{2}{3}xe^{3x} - \frac{2}{3}\int e^{3x}\,dx$

$= \frac{2x}{3}e^{3x} - \frac{2}{3}.\frac{1}{3}\int 3e^{3x}\,dx$

$= \frac{2x}{3}e^{3x} - \frac{2}{9}e^{3x} + c$

(6)

QUESTION 9

9.1 $\Delta x_i = \dfrac{5-3}{n} \checkmark = \dfrac{2}{n} \checkmark$ (width) (2)

9.2 Length ✓

9.3 Width $= \dfrac{2}{5} = 0{,}4$ ✓✓

$\Delta x_1 = 0{,}4$

$\therefore x_1 = 3{,}4$

$\therefore f(x_1) = \ln 3{,}4 = 1{,}223775432$ ✓

$\therefore \text{Area}_1 = f(x_1) . \Delta x_1 = 0{,}4895$

$\Delta x_2 = 0{,}4$

$\therefore x_2 = 3{,}8$

$\therefore f(x_2) = \ln 3{,}8 = 1{,}335001067$ ✓

$\therefore \text{Area}_2 = f(x_2) \Delta x_2 = 0{,}5340004267$

$\Delta x_3 = 0{,}4$

$\therefore x_3 = 4{,}2$

$\therefore f(x_3) = \ln 4{,}2 = 1{,}435084525$ ✓

$\therefore \text{Area}_3 = f(x_3) \Delta x_3 = 0{,}5740338101$

$\Delta x_4 = 0{,}4$

$\therefore x_4 = 4{,}6$

$\therefore f(x_4) = \ln 4{,}6 = 1{,}526056303$

$\therefore \text{Area}_4 = 0{,}6104225214$ ✓

$\Delta x_5 = 0{,}4$

$\therefore x_5 = 5$

$f(x_5) = \ln 5 = 1{,}609437912$

$\therefore \text{Area}_5 = 0{,}643775165$ ✓

$\therefore$ Sum of the areas of the 5 rectangles

$= 2,852$ ✓✓✓ (10)

9.4 $\int_3^5 \ln x \, dx = 2,751$ ✓✓ (2)

9.5 Over-approximation ✓✓ (area > actual area) (2)

QUESTION 10

10.1 Radius = 5cm ✓

$BC^2 = (5)^2 - (3)^2$ ✓ $OC \perp AB$

$\therefore BC = 4$ cm ✓ (4)

$\therefore AB = 8$ cm ✓

10.2 Let $A\hat{O}B = \theta$ ✓

In $\triangle OCB$: $\sin\frac{\theta}{2} = \frac{4}{5}$ ✓

$\therefore \frac{\theta}{2} = 0,927295218$ ✓

$\therefore \theta = 1,854590436$ ✓

Area segment $= \frac{1}{2}(5)^2(\theta - \sin\theta)$ ✓✓ (9)

$= 11,18238045$ ✓

Volume of water = area segment $\times 50$ ✓

$= 559,12 \text{ cm}^3$ ✓

QUESTION 11

11.1 (a) $f(\frac{\pi}{3}) = \tan\frac{\pi}{3} . \sec\frac{\pi}{3} = 2\sqrt{3}$ ✓

$A(\frac{\pi}{3}; 2\sqrt{3})$ ✓ (4)

$B(\frac{2\pi}{3}; 2\sqrt{3})$ ✓✓

(4)

(b) $\int_0^{\frac{\pi}{3}} f(x)\,dx$ ✓ $+ \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 2\sqrt{3}\,dx$ ✓✓ $+ \int_{\frac{2\pi}{3}}^{\pi} f(x)\,dx$ ✓

(c) $A = 2\int_0^{\frac{\pi}{3}} \sec x \tan x\,dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 2\sqrt{3}\,dx$

✓✓✓✓

$= 2 + 3{,}627598728$ (4)

$= 5{,}63$

11.2 $V = \pi \int_0^a (\tan x \sec x)^2\,dx$ ✓ ✓ ✓ ✓

$\therefore \frac{\pi}{3} = \pi \int_0^a \tan^2 x . \sec^2 x\,dx$ ✓

$\therefore \frac{1}{3} = \int_0^a \sec^2 x . (\tan x)^2\,dx$

$\therefore \frac{1}{3} = \left[\frac{(\tan x)^3}{3}\right]_0^a$ ✓✓

$\therefore \frac{1}{3} = \frac{(\tan a)^3}{3} - 0$ (8)

$\therefore 1 = (\tan a)^3$

$\therefore \tan a = 1$ ✓

$\therefore \underline{a = \frac{\pi}{4}}$ ✓