I. Fell

GRADE 12. A.P. MATHS PRE-LIMS: 2019.

PAPER 1. MEMO

Q1.

1.1. $|x-5| + 4x = 10$

if $x \geq 5$, $\Rightarrow$ $x-5+4x = 10$ ✓a.

$5x = 15$

✓m $x = 3$. $\not\geq 5$ ✓ea.

$\therefore$ N/A.

OR, if $x \leq 5$, $\Rightarrow$ $-x+5+4x = 10$ ✓a.

$3x = 5$

$x = \frac{5}{3} \leq 5$

$\therefore x = \frac{5}{3}$ only ✓ (5).

1.2. a) $\log_x e = \frac{\log_e e}{\log_e x} = \frac{1}{\log_e x} = \frac{1}{\ln x}$

✓ change of base formula. ✓ (2)

b) $\ln x + 2\log_x e = 3$.

$\therefore \ln x + \frac{2}{\ln x} = 3$ ✓m

$\therefore (\ln x)^2 + 2 = 3\ln x$ ✓

$\therefore k^2 - 3k + 2 = 0$ ✓ quadratic

$(k-2)(k-1) = 0$ (OK if use eq. mode).

✓ ea.

$\therefore \ln x = 2$ or 1

$\therefore x = e^2$ or e^1 ✓m

$\approx 7{,}39$ or $2{,}72$ ✓a. (7).

✓a

1.3 a) $\therefore f(x) = 3\ln(x-e)$ ✓

and $x - e > 0$ ✓

$\therefore x > e$ i.e. $x > 2{,}72$. ✓ (3)

b) @ $y = 0$, $3\ln(x-e) = 0$

$\therefore \ln(x-e) = 0$ ✓m $\therefore x - e = 1$ ✓a ✓ (4)

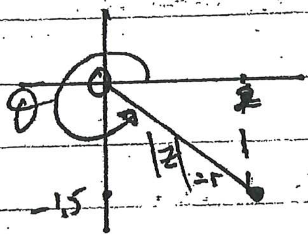
$\therefore x = 1 + e = +3{,}72$

✓a

[21]

(Q2) 2.1 $z = 2 - 1{,}5i$

a.)



quadrant ✓a (2)

b.) $|z| = \sqrt{2^2 + 1{,}5^2}$ ✓m $= 2{,}5$ ✓a (2)

c.) $\tan\theta = \frac{y}{x} = \frac{-1{,}5}{2}$ ✓m

$\therefore \theta = -0{,}64$ ✓a $(\text{OR } 2\pi - 0{,}64 = 5{,}64)$ (2)

2.2 For $n = 1$,

$LHS = (\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$

$RHS = \cos 1.\theta + i\sin 1.\theta = \cos\theta + i\sin\theta$

$\therefore LHS = RHS$ ✓m ✓a

$\therefore$ Formula true for $n = 1$.

Now, assume true for $n = k$

i.e. assume $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ ✓m ✓a

$\therefore$ For $n = k+1$; ✓m

$LHS = (\cos\theta + i\sin\theta)^{k+1}$ ✓a

$= (\cos\theta + i\sin\theta)^1 (\cos\theta + i\sin\theta)^k$ ✓m

$= (\cos\theta + i\sin\theta).(\cos k\theta + i\sin k\theta)$ ✓a

$= \cos\theta.\cos k\theta + i\sin\theta\cos k\theta + i\cos\theta\sin k\theta + i^2\sin\theta\sin k\theta$ ✓a

$= \cos\theta\cos k\theta + i(\sin\theta\cos k\theta + \cos\theta\sin k\theta) - \sin\theta\sin k\theta$ ✓m

$= \cos(\theta + k\theta) + i(\sin(\theta + k\theta))$ ✓a. identity

$= \cos(1+k)\theta + i\sin(1+k)\theta$ ✓

$= RHS$ for $n = k+1$ $\therefore$ formulae true for $n = k+1$ if true for $n = k$ ✓

$\therefore$ By principle of m. induct., true for $n = 1$ $\therefore$ true for $n = 1+1 = 2$ $\therefore$ for $2+1 = 3$ etc. $\therefore$ true for all $n \in N$. ✓

2.3. $m = (a+2i)(b+3i)$

$= ab + 2bi + 3ai + 6i^2$ ✓

$= (ab-6) + (2b+3a)i$ ✓

a.) $\therefore Re(m) = ab-6$ ✓

b.) $Im(m) = 2b+3a$ ✓ (5)

2.4. a.) $3+i$ ✓ (1)

b.) $x^2 - \text{sum}\, x + \text{prod} = 0$ ✓m

$\therefore -p = \text{sum} = (3+i)+(3-i) =$ ✓m

$= 6 \qquad \therefore p = -6$ ✓a.

$q = \text{prod} = (3+i)(3-i) = 9 - i^2 = 10 = q$ ✓ (5)

c.) $(x-(3-i))(x-(3+i))(2x-3) = 0$

$\therefore (x^2 - 6x + 10)(2x-3) = 0.$

$\therefore a = 2.(1) = 2$ ✓ and $b = 10(-3) = -30$ ✓

(4)

[35)

Q.3 $f(x) = \dfrac{x^2+3x-4}{x-p}$.

3.1. $f(x) = \dfrac{(x+4)(x-1)}{(x-p)}$ ✓ (5)

If $p = 1$ ✓ or -4, ✓ $f(x) = x+4$ or $x-1$ respectively

if $p=1$, $\therefore \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = 5$ ✓ But $f(x)$ undefined @ $x=1$ ✓

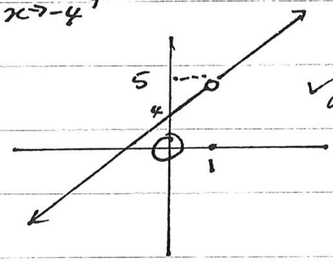
$\therefore \lim\limits_{x \to 1} f(x)$ exists but $\neq f(1)$

just explain one.

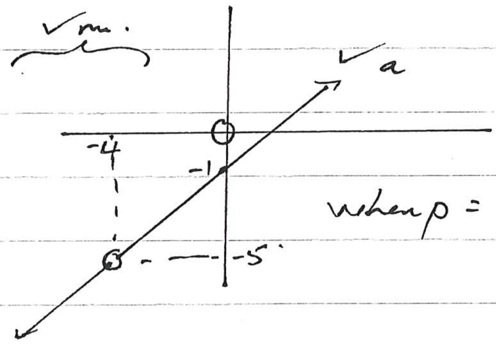
Similarly,

if $p=-4$, $\lim\limits_{x \to -4^-} f(x) = \lim\limits_{x \to -4^+} f(x) = -5 \neq f(-4)$ which is undefined.

OR show graphs: ✓a ✓m. ✓a



when $p=1$ ✓



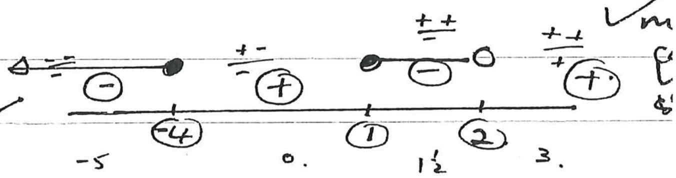
when $p=-4$

3.2. $p=2$. $\therefore f(x) = \dfrac{(x+4)(x-1)}{(x-2)}$

a.) $x=2$ ✓ vert. asymp. (1)

b.) $\therefore \dfrac{(x+4)(x-1)}{x-2}$ ✓ ≤ 0 i.e. $-$ve. or 0. ✓m

crit. v.'s $= -4 ; 1 ; 2$. ca ✓



ca ✓

$\therefore x \in (-\infty; -4]$ (or $x \leq -4$, $x \in R$.)

OR $x \in [1; 2)$ (or $1 \leq x < 2$, $x \in R$)

✓ca (6)

c.) $f(x) = \dfrac{x^2+3x-4}{x-2} = \dfrac{x(x-2)+2x+3x-4}{x-2}$ ✓m

(or long division) $= x + \dfrac{5x-4}{x-2} = x + \dfrac{5(x-2)+10-4}{x-2}$.

$= x + 5 + \dfrac{6}{x-2}$. ✓a ✓a.

(3)

3.2.

d.) ∴ oblique asymptote: $y = x+5$ ✓m ✓ca (2)

e.) Because $\lim_{x\to\infty} f(x)$ ✓ $= \lim_{x\to\infty}\left(x+5 + \frac{6}{x-2}\right)$

$= \lim_{x\to\infty}(x+5) + 0.$ (2).

& this limit does not exist ✓ ..
(as a value)

f.) $f'(x) = 0$ ✓m @ stat. pts

$f(x) = \frac{x^2+3x-4}{(x-2)} = \frac{u}{v}$ ∴ quotient rule. ✓m ✓a

$\therefore f'(x) = \frac{(x-2)(2x+3) - (x^2+3x-4)(1)}{(x-2)^2} = 0$

✓m

$\therefore 2x^2 - x - 6 - x^2 - 3x + 4 = 0.$

$\therefore x^2 - 4x - 2 = 0.$ ✓ca.

$\therefore x = 4{,}45$ OR $-0{,}45.$

and $f(4{,}45) = 11{,}90$ & $f(-0{,}45) = 2{,}10$ ✓m.

∴ st. pts $= (4{,}45;\ 11{,}9)$ & $(-0{,}45;\ 2{,}10)$. (5)

g.) $f'(x) = \frac{x^2-4x-2}{(x-2)^2} = (x^2-4x-2)(x-2)^{-2}$ ∴ prod. rule.
(or quot.)

$\therefore f''(x) = (x-2)^{-2}\cdot(2x-4) + -2(x-2)^{-3}\cdot 1\cdot(x^2-4x-2)$ ✓m.

$= \frac{2x-4}{(x-2)^2} - \frac{2(x^2-4x-2)}{(x-2)^3}$

$= \frac{(2x-4)(x-2) - 2x^2+8x+4}{(x-2)^3}$ ✓ca.

✓m $= \frac{2x^2-8x+8-2x^2+8x+4}{(x-2)^3} = \frac{12}{(x-2)^3}$

& $\frac{12}{(x-2)^3} \neq 0$ $\left(\therefore f''(x) = -ve \text{ for } x<2;\ = +ve \text{ for } x>2.\right)$ (3)

$\Downarrow$

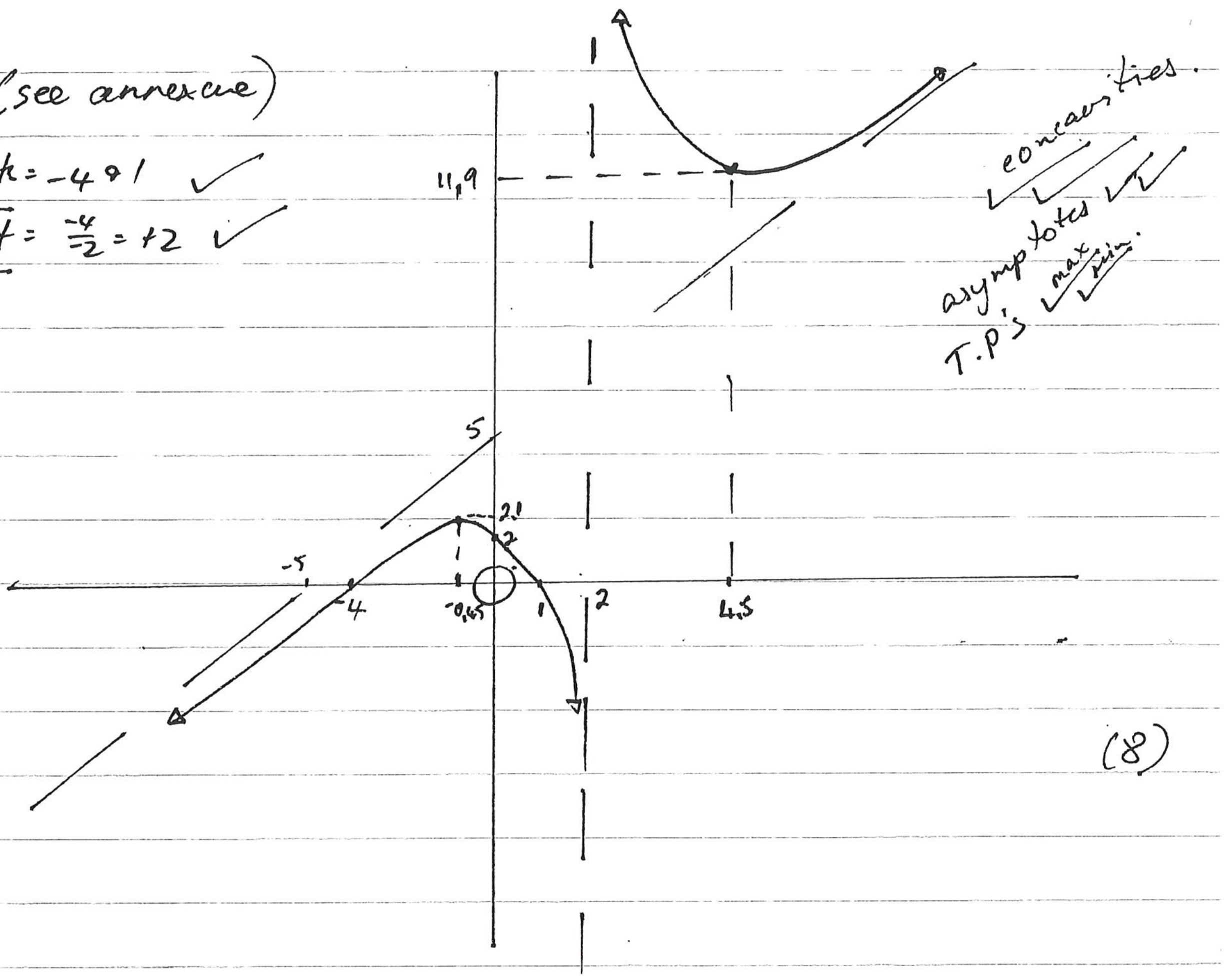
∴ no pt. of inflection.

3.2.

h.) (see annexure)

$X_{int} = -4 \; ; 1$ ✓

$Y_{int} = \frac{-4}{-2} = +2$ ✓



(8)

i.) $x+5 + \frac{6}{x-2} = f(x)$ ✓ form to one. (1)

j.) $A = \int_{-4}^{1} f(x)\,dx \approx$ 6,75 sq. units ✓a (2)

[38

Q4.

4.1 $x^2+y^2 = 9-3xy^2.$

$\therefore \frac{d}{dx}(\quad) = \frac{d}{dx}(\quad)$ ✓m

prod. rule.

$\therefore 2x ✓ + 2y.\frac{dy}{dx} ✓ = 0 - (3y^2.1 ✓ + 3.2y.\frac{dy}{dx}.x ✓)$

$\therefore 2x + 2y.\frac{dy}{dx} = -3y^2 - 6xy\frac{dy}{dx}.$

✓m

$\therefore \frac{dy}{dx}(2y+6xy) = -3y^2-2x$

$\therefore \frac{dy}{dx} = \frac{-3y^2-2x}{2y+6xy}.$ ✓ca

$= \frac{-3(3)^2-0}{2(3)-0}$ ca ✓

$= \frac{-27}{6} = \frac{-9}{2} = -4{,}5$

(9).

4.2. A is $f(g(x))$ ✓ca

B is $g(f(x))$. ✓ca

Justification:

$f(g(x)) = f(|x|) = |x|^2-3|x|+2$ ✓a

$\therefore$ domain is split at $x=0$ & $\therefore y = x^2-3x+2,\ x \geq 0$ ✓m

& $y = x^2+3x+2,\ x<0$,

which is A.

Whereas $g(f(x)) = g(x^2-3x+2) = |x^2-3x+2|$ ✓a

$= |(x-2)(x-1)|$

$\therefore$ domain is split at $+2$ & $+1$

$\left.\begin{array}{l}\therefore y = x^2-3x+2 \text{ if } x<1 \text{ or } x>2 \\ \& \ y=-x^2+3x-2 \text{ if } 1<x<2\end{array}\right\}$ ✓m

which is B.

(6).

4.2.b.) at $x = [0 ; 1]$ OR $[2 ; \infty)$ ✓m (4)

c.) $k(x) = f(g(x)) = |x|^2 - 3|x| + 2$ (graph A)

* cont. but not differentiable @ $x = 0$ ✓a

as $\lim\limits_{x \to 0^-} k'(x) = \lim\limits_{x \to 0^-} 2x + 3 = 3$

and $\lim\limits_{x \to 0^+} k'(x) = \lim\limits_{x \to 0^+} 2x - 3 = -3$ ✓m

$\therefore \lim\limits_{-} \neq \lim\limits_{+} \quad \therefore \lim\limits_{x \to 0} k'(x)$ does not exist ✓m

$\therefore$ k not diff. @ $x = 0$.

* $\lim\limits_{x \to 0^-} k(x) = \lim\limits_{x \to 0^-} x^2 + 3x + 2 = 2.$

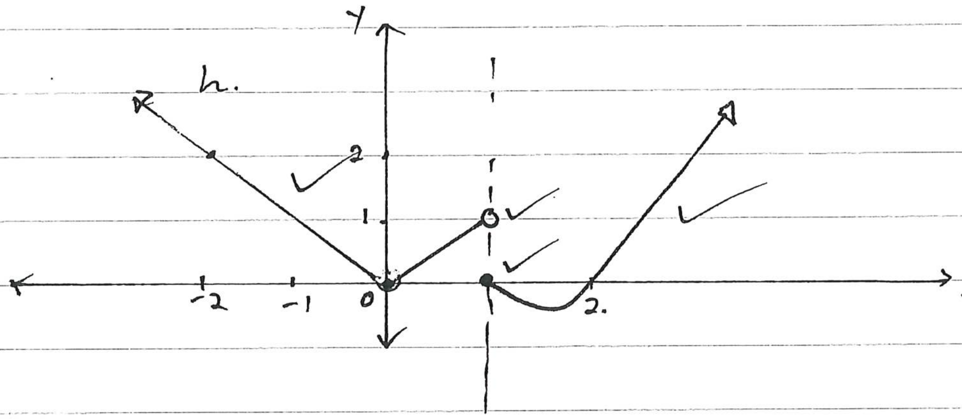
$\lim\limits_{x \to 0^+} k(x) = \lim\limits_{x \to 0^+} x^2 - 3x + 2 = 2$ ✓

$\therefore \lim\limits_{x \to 0} k(x)$ exists ✓ and $= 2$ which $= k(0)$ ✓

$\therefore$ k is continuous @ $x = 0$.

(6)

d.) (i.)



(4)

(ii.) h discontinuous @ $x = 1$. ✓

Jump discontinuity ✓ (2)

[31]

Q5

5.1. $20000 = x^3 + 2x^2 + 15x$

$f(x) = x^3 + 2x^2 + 15x - 20000$ a cont. function.

a.) $f(25) = -2750 < 0$ ∴ f below x axis ✓

$f(30) = 9250 > 0$ ∴ f above x axis. ✓

∴ $f(x) = 0$ for $x \in [25; 30]$. (2)

b.) $a_1 = 25 - \frac{f(25)}{f'(25)}$ ✓m $f'(x) = 3x^2 + 4x + 15$. ✓a

$= 25 - \frac{25^3 + 2(25)^2 + 15(25) - 20000}{3(25)^2 + 4(25) + 15}$

∴ $a_1 = 26{,}3819\ldots$ ✓ = ANS

∴ $a_2 = \text{ANS} - \frac{f(\text{ANS})}{f'(\text{ANS})} = 26{,}3141\ldots$ ✓m

∴ $a_3 = 26{,}3139\ldots$

$a_4 = 26{,}3139\ldots$ (5)

∴ $x = 26{,}31$ ✓ca (2d.pl.)

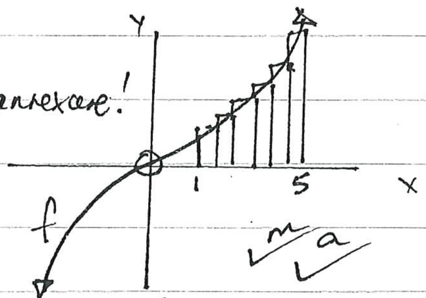
5.2. $f(x) = x^3$

a.) $A_6 = 156 + \frac{248}{6} + \frac{96}{6^2}$ ✓m $= 200$ sq. units ✓a (2)

b.) exact $A = \lim_{n \to \infty} \left(156 + \frac{248}{n} + \frac{9}{n^2}\right)$ ✓ (2)

$= 156 + 0 + 0$ $= 156$ sq. units ✓

c.) see annexure!



✓m ✓a

A_6 goes above curve of f bet. 1 & 5.

∴ $A_6 >$ exact A. (2)

d.) ∴ % error $= \frac{200 - 156}{156} \times 100\% \approx 28\%$ ✓m ✓ca (2)

5.3.

a.) $\dfrac{3}{(x-1)(2x-3)} = \dfrac{A}{x-1} + \dfrac{B}{2x-3}$ ✓m

using cover · up method,

✓m $\begin{cases} \text{sub } x=1, & \therefore A = \dfrac{3}{2(1)-3} = -3. \checkmark \\ \text{sub } x = \frac{3}{2} & \therefore B = \dfrac{3}{\frac{3}{2}-1} = 6 \checkmark \end{cases}$

$\therefore \dfrac{3}{(\ )(\ )} = \dfrac{-3}{x-1} + \dfrac{6}{2x-3}$ (4)

b.) $\therefore \displaystyle\int \frac{3}{(\ )(\ )}\,dx = \int \frac{-3}{x-1}\,dx + \int \frac{6}{2x-3}\,dx$ ✓m

$= -3\ln|x-1| + 3\displaystyle\int \frac{1}{2x-3} \cdot \underbrace{2dx}$

$= -3\ln|x-1| + 3\displaystyle\int \frac{1}{u}\,du.$ ✓ ✓

$= \underline{-3\ln|x-1| + 3\ln|2x-3| + c}$ ✓

(4).

[23]

(Q6) 6.1.

a.) $\int (6x-9)(x^2-3x)^6\,dx = 3\int (2x-3)(x^2-3x)^6\,dx.$

let $u = x^2 - 3x$ m $\therefore \dfrac{du}{dx} = 2x-3. \quad \therefore du = (2x-3)dx.$ ca

$\therefore \int \ldots\ldots = 3\int u^6\,du$ m

$= 3\left[\dfrac{u^7}{7}\right] + c$ (5).

$= \dfrac{3(x^2-3x)^7}{7} + c$

b.) $\int \sin 4x.\sin 3x\,dx$. identity.

$= \frac{1}{2}\int \cos(4x-3x)dx - \frac{1}{2}\int \cos(4x+3x)dx$

$= \frac{1}{2}\int \cos x\,dx - \frac{1}{2}\int \cos 7x\,dx.$ a.

$= \frac{1}{2}\sin x - \frac{1}{2}\dfrac{\sin 7x}{7} + c.$

$= \dfrac{\sin x}{2} - \dfrac{\sin 7x}{14} + c$ (4).

-1 if "c" not included in (a), (b) and

c.) $= \int \sin^3 x\,dx. + \int \sec^2 x\,dx = \left(\int \sin^3 x\,dx\right) + \tan x + c$

m

$= \int \sin x.\sin^2 x\,dx = \int \sin x(1-\cos^2 x)dx.$ m

$= \int \sin x - \sin x.\cos^2 x\,dx$

let
$u = \cos x$
$\therefore du = -\sin x\,du$

$= \int \sin x\,dx - \int \sin x.(\cos x)^2\,dx.$

$= -\cos x - \int u^2(-du)$

$= -\cos x + \dfrac{u^3}{3} + c$

$\therefore \int \sin^3 x + \sec^2 x\,dx = -\cos x + \dfrac{\cos^3 x}{3} + \tan x + c$ (8)

Q6) 6.1.

a.) $\int (6x-9)(x^2-3x)^6\,dx = 3\int (2x-3)(x^2-3x)^6\,dx.$ ✓

let $u = x^2 - 3x$ ✓ m $\therefore \frac{du}{dx} = 2x-3.$ $\therefore du = (2x-3)dx.$ ✓ ca

$\therefore \int \ldots = 3\int u^6\,du$ ✓ m

(5).

$= 3\left[\frac{u^7}{7}\right] + c$ ✓

$= \frac{3(x^2-3x)^7}{7} + c$

b.) $\int \sin 4x . \sin 3x\,dx$. ✓ identity.

−1 if "c" not included in (a), (b) or (c)

$= \frac{1}{2}\int \cos(4x-3x)dx - \frac{1}{2}\int \cos(4x+3x)dx$

$= \frac{1}{2}\int \cos x\,dx - \frac{1}{2}\int \cos 7x\,dx.$ ✓ a.

$= \frac{1}{2}\sin x$ ✓ $- \frac{1}{2}\frac{\sin 7x}{7}$ ✓ $+ c.$

$= \frac{\sin x}{2} - \frac{\sin 7x}{14} + c$

(4).

c.) $= \int \sin^3 x\,dx + \int \sec^2 x\,dx = \left(\int \sin^3 x\,dx\right) + \tan x + c$ ✓

$= \int \sin x . \sin^2 x\,dx$ ✓ m $= \int \sin x(1-\cos^2 x)dx.$ ✓ m

$= \int \sin x - \sin x . \cos^2 x\,dx$

$\left\{\begin{array}{l}\text{let} \\ u = \cos x \\ \therefore du = -\sin x\,du\end{array}\right\}$ ✓ ✓

$= \int \sin x\,dx - \int \sin x . (\cos x)^2\,dx.$ ✓

$= -\cos x$ ✓ $- \int u^2(-du)$

$= -\cos x + \frac{u^3}{3} + c$

$\therefore \int \sin^3 x + \sec^2 x\,dx = -\cos x + \frac{\cos^3 x}{3}$ ✓ $+ \tan x + c$ (8)

6.2.

$$A = \int_1^p \ln x \, dx = (10\ln 10) - 9.$$

$$\therefore \int_1^p 1.\ln x \, dx = (10\ln 10) - 9$$

$$\therefore \int_1^p u\,dv = (10\ln 10) - 9.$$

✓m

where $u = \ln x$ and $dv = 1dx$.

$\therefore \frac{du}{dx} = \frac{1}{x}$ i.e. $\frac{dv}{dx} = 1$

$\therefore du = \frac{1}{x}dx.$ ✓ca $\therefore v = \int 1dx = x.$ ✓ca

$$\int_1^p u\,dv = \left[uv - \int v\,du.\right]_1^p$$ ✓m (int. by parts)

$$\therefore \int \ln x.1dx = \left[(\ln x).x\right]_1^p - \int_1^p x.\frac{1}{x}dx$$

$$= p\ln p - (\ln 1).1 - \int_1^p 1.\,dx$$ ✓

$$= p\ln p - 0 - \left[x\right]_1^p$$ ✓

$$= p\ln p - \left[p - 1\right].$$

$$= p\ln p - p + 1$$

$$\therefore (p\ln p) - p + 1 = (10\ln 10) - 9.$$ ✓

$$= 10\ln 10 - 10 + 1$$

$\therefore p = 10$ ✓ (8)

6.3

$$V = \pi \int_{-2}^{a} [f(x)]^2 dx = 23{,}1922 cm^3.$$

$$\therefore \pi \int_{-2}^{a} [e^{-\frac{x}{2}}]^2 dx = 23{,}1922$$

$$\therefore \int_{-2}^{a} e^{-x} dx = \frac{23{,}1922}{\pi}$$

$$\left\{ \begin{array}{l} \text{if } u = -x \\ du = -1.dx. \end{array} \right\} \text{m}$$

$$\therefore -\int_{x=-2}^{x=a} e^{u}.du = \frac{23{,}1922}{\pi}$$

$$\therefore \left[e^{u}\right]_{x=-2}^{x=a} = -\frac{23{,}1922}{\pi}$$

$$\therefore \left[e^{-x}\right]_{-2}^{a} = \frac{-23{,}1922}{\pi} = -7{,}3823\ldots$$

$$\therefore [e^{-(a)}] - [e^{-(-2)}] = -7{,}3823\ldots$$ sub.

$$\therefore e^{-a} - e^{+2} = -7{,}3823\ldots$$

$$\therefore e^{-a} = -7{,}3823 + e^{2} = 0{,}006749\ldots$$

$$\therefore e^{-a} = 0{,}006749\ldots$$

$$\therefore -a = \ln(0{,}006749) = -4{,}998\ldots$$ ca

$$\therefore a = +4{,}998\ldots \approx 5 \text{ (2d.pl)}$$

(9).

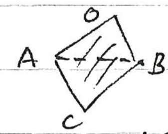
[33]

Q7. 7.1.

a.) $\theta = \dfrac{\text{arclength}}{r} = \dfrac{10}{r}$ ✓

$\therefore r = \dfrac{10}{\theta}$ ✓ (2)

b.) Shaded A = Area 2Δ's ($\Delta AOB + \Delta ACB$) $-$ Area of sector AOB ✓m



$= 2.\left(\frac{1}{2} r.r.\sin\theta\right) - \frac{1}{2} r^2.\theta$ ✓ ✓

$= r^2\sin\theta - \frac{1}{2} r^2\theta$

sub $r = \dfrac{10}{\theta}$, $\therefore A = \left(\dfrac{10}{\theta}\right)^2 \cdot \sin\theta - \dfrac{1}{2}\left(\dfrac{10}{\theta}\right)^2 \theta$ ✓m

$\therefore A = \dfrac{100\sin\theta}{\theta^2} - \dfrac{100}{2\theta^2}.\theta$ ✓ (5).

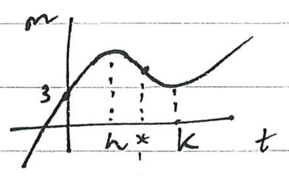
$= \dfrac{100}{\theta^2}.\sin\theta - \dfrac{50}{\theta}$.

c.) $\therefore$ if $\theta = \frac{\pi}{5}$, $A = \dfrac{100}{\left(\frac{\pi}{5}\right)^2}\sin\frac{\pi}{5} - \dfrac{50}{\frac{\pi}{5}}$ ✓

$= 69{,}31$ sq. units ✓ (2)

7.2. a.) @ $t=0$, ✓m $m = \frac{1}{28}(0+0+0+84) = 3\text{kg}$ ✓a (2)

b.)



(graph optional.)

h & k @ T.P's. ✓

$\therefore$ when $\dfrac{dm}{dt} = 0$ ✓ (5)

$\therefore \frac{1}{28}(18t^2 - 90t + 108) = 0$ ✓

$\therefore t^2 - 5t + 6 = 0$

$\therefore (t-3)(t-2) = 0$ ✓

$\therefore h = 2$ mths & $k = 3$ mths ✓

c.) ie. @ pt. of infl. ✓m

$\therefore m''(t) = 0$

$\therefore \frac{1}{28}(36t - 90) = 0$ ✓ca

$t = \frac{90}{36} = 2{,}5$ mths ✓ca (3)

[19]

GRADE 12 ADVANCED PROGRAMME MATHS PAPER 1 ASSESSMENT LEVELS

TRIALS 2019

Question	Level1	Level 2	Level 3	Level 4	Totals
1.1 1.2 a) + b) 1.3 a) + b)	6 2 4	3	7		21
2.1 – 2.4 b) 2.4. c)		31	4		35
3.		38			38
4.			31.		31
5.		23			23
6.1. a) 6.1. b. + c) 6.2 + 6.3		4	12	17.	33
7.1 7.2				8 11	19
TOTALS:	11.	99	54	36	200

110 90