Our Lady of Fatima
Dominican Convent School

ADVANCED PROGRAMME MATHEMATICS
TRIAL EXAMINATION

PAPER 2 – STATISTICS

Grade 12 August 2019

Time: 1 hour Marks: 100

EXAMINER: Mrs. D. Fell **MODERATOR: Mrs. E. O'Connell**

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This examination paper consists of 5 pages including this coversheet.
2. Please refer to the information sheet supplied.
3. Round off to **4 decimal places** unless otherwise stated.
4. Calculators may be used.
5. Number your answers exactly as the questions are numbered.
6. Write only in black or blue ink.
7. All necessary working as per mark allocation must be clearly shown.
8. Please **write legibly** and present your work as neatly as possible.
9. Read the questions very carefully, focus and do your best!

QUESTION 1

1.1 If $P(A \cap B) = 0{,}29$ and $P(A' \cap B) = 0{,}35$, find the probability that the event A occurred, given that event B occurred. (4)

1.2 A certain international swimming pool has 10 lanes.

a.) How many different ways can 10 swimmers be arranged on the starting blocks? (2)

b.) If 3 of the swimmers **cannot** swim in lanes next to each other, in how many different ways can the 10 swimmers be arranged on the starting blocks? (4)

1.3 Consider the word "DAZZLEDUDE".

a.) How many different "words" can be formed using all the letters? (3)

b.) Calculate the probability that a randomly selected word from the ones described above, start and end with a vowel. (7)

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QUESTION 2

2.1 A probability mass function is defined as follow:

$$P(X = x) = 1 - \frac{2x}{k} \quad for\ x = 2; 3; 4; 5$$

$$= 0 \qquad otherwise.$$

a.) Prove that $k = \frac{28}{3}$. (3)

b.) Hence calculate P(X > 3). (3)

c.) Calculate the value of E[X] , the expected value of the random variable, X. (3)

d.) Write the value of the mode of this probability mass function. (1)

2.2 In a random sample of 8 customers, at the Triple Five Café,

a.) Find the probability that exactly 6 customers buy a cappuccino, if it is known that 60% of their customers buy a cappuccino. (7)

b.) Find the probability that at least 2 people buy coffee, if the Triple Five Café offers 5 types of coffee, 5 types of tea and 5 types of milkshakes. (8)

2.3 The masses of the vegetables, X, delivered to a market are distributed Normally with a mean of 110 grams and a standard deviation of 20 grams.

a.) Find the probability that a randomly chosen vegetable from the deliveries has a mass that is greater than 150 grams. (6)

b.) If 15% of the vegetables delivered, have less than a certain mass, m grams, the market will not buy them.

(i) Draw a rough sketch of the Normal curve, indicating the meaning of 15% and m. (1)

(ii) Copy and complete $P\left(Z < \text{————}\right) = \text{\_\_\_\_\_}$ and hence find the mass, m. (5)

c.) Of the 600 vegetables that Farmer Joe delivers to the market, 400 are bought by the market.
Find a 95% confidence interval for the **proportion** of vegetables that are bought by the market. (5)

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QUESTION 3

3.1 a.) For which of the following Binomially distributed random variables, CAN the Normal approximation be applied? Show reasoning.

(3)

A. $X \sim B(28;\ 0{,}1)$
B. $X \sim B(8;\ 0{,}7)$
C. $X \sim B(20;\ 0{,}3)$
D. $X \sim B(7;\ 0{,}6)$

b.) Find the mean and variance of the distributions in 3.2 that CAN be Normally approximated.

(3)

3.2 If a random variable Y, binomially distributed, has been approximated to a Normal distribution, for each of the following probabilities, state the appropriate **continuity correction** (there is no need to calculate the probabilities):

a.) $P(Y = 3)$
b.) $P(Y \leq 3)$
c.) $P(Y > a)$
d.) $P(4 \leq Y < 8)$ (4)

3.3 35% of households in a certain municipality do not have hot water.
25 people from different households are selected randomly.
Let X = the number of households without hot water.

a.) Copy and complete with the values of the parameters :

$X \sim B($___ ; _____$) \rightarrow X \sim$ N(_____;_____) (2)

b.) Hence, find the probability that more than 12 in this sample of people do not have hot water. (5)

3.4 a.) Complete in your answer booklet:

The Central Limit Theorem states that for samples of size n, drawn from ANY distribution with mean μ and finite variance σ^2, the distribution of the _____ is approximately Normal, i.e $\bar{X} \sim N\left(\mu; \frac{\sigma^2}{n}\right)$ only if n_____ (2)

b.) Explain, for each of the questions below, whether the CENTRAL LIMIT THEOREM applies or not. Give brief reasons:

A. The heights of fourteen year old girls is Normally distributed with $\mu = 1{,}62\ m$ and $\sigma = 0{,}1\ m$ find the probability that 10 randomly selected girls will have a mean height of more than 1,7 m.

B. Studies show that the mean age of students at a graduation ceremony is 23 years and the standard deviation is 2 years.
20 students are randomly selected at the graduation ceremony.
What is the probability that their mean age is less than 22 years?

C. The mean mass of a certain type of bird is 150 grams with a standard deviation of 20 grams.
35 birds are randomly selected from an aviary.
What is the probability that their mean mass is greater than 160 grams? (6)

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QUESTION 4

A municipality claims that the mean pollution level in storm water is only 5mg/100ml water with a standard deviation of 3,2mg/100ml from their past experience.
An environmental company claim that the pollution level is more than this and so they collect storm water from a sample of 50 different outlets.
They subsequently find that the mean pollution level in this sample is 5,88mg/100ml.

4.1 Conduct a hypothesis test at the 1% level of significance to determine whether the mean pollution level has increased. (10)

4.2 What minimum level of significance (to the nearest percent), would the environmental company need to use, in order to alter their conclusion? Show reasoning using the statistics of the scenario. (3)

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