Our Lady of Fatima
Dominican Convent School

ADVANCED PROGRAMME MATHEMATICS
TRIAL EXAMINATION

PAPER 1 – ALGEBRA & CALCULUS

Grade 12 August 2019

Time: 2 hours Marks: 200

EXAMINER: Mrs. D. Fell **MODERATOR: Mrs. S. Batley**

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This examination paper consists of 8 pages including this coversheet.
2. Please refer constantly to the formula sheet supplied.
3. Answer in the examination booklet provided except for questions **3.2(h) and 5.2(c)** for which you must use the **annexure supplied**.
   Write your name on this.
4. Round off to **2 decimal places** unless otherwise stated.
5. Calculators may be used unless otherwise stated. Please assess the extent to which you may use the calculator based on the mark allocation and the advice given by Mrs Fell.
6. Ensure that your calculator is in RADIAN MODE.
7. Number your answers exactly as the questions are numbered.
8. Write only in black or blue ink and draw graphs in pencil.
9. Diagrams and graphs are not drawn to scale unless otherwise stated.
10. All necessary working as per mark allocation must be clearly shown. However, answers that are too lengthy will impact negatively on your finishing time.
11. It is in your own interest to **write legibly** and to present your work neatly.
    RELAX and do your best!

QUESTION 1

1.1 Solve for x :

$|x-5| + 4x = 10$ (5)

1.2

a.) Prove that $\log_x e = \frac{1}{lnx}$. (2)

b.) Hence solve for x :

$lnx + 2\log_x e = 3$ (7)

1.3 Given $f(x) = ln(x-e)^3$,

a.) Give the domain of f. (3)

b.) Calculate the X intercept/s of f. (4)

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QUESTION 2

2.1 Given the complex number, $z = 2 - 1{,}5i$ in rectangular form,

a.) Represent z in an argand diagram. (2)

b.) Find the value of $|z|$, the mod of z. (2)

c.) In polar form, $z = r(cos\theta + isin\theta)$. Find θ in radian measure. (2)

2.2 Prove, using mathematical induction, that

$(cos\theta + i.sin\theta)^n = \cos(n\theta) + i.\sin(n\theta)$ for all $n \epsilon Natural\ numbers, N$,

where $cos\theta + i.sin\theta$ is a complex number. (14)

2.3 Given the complex number, $m = (a + 2i)(b + 3i)$, write in terms of a and b,

a.) $\text{Re}(m)$
b.) $\text{Im}(m)$ (5)

2.4 If one root of the quadratic equation, $x^2 + px + q = 0$, is $3 - i$,

a.) Write down the other root. (1)

b.) Hence find the values of p and of q. (5)

c.) Hence find the value for a and for b, if two of the roots of the cubic equation,

$ax^3 - 15x^2 + 38x + b = 0$, are $3 - i$ and $\frac{3}{2}$. (4)

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QUESTION 3

$f(x) = \frac{x^2+3x-4}{x-p}$

3.1 Give two values for p such that f has a removable discontinuity.
Show reasoning. (5)

3.2 **If $p = 2$,**

a.) Write down the equation of the vertical asymptote. (1)

b.) Calculate the values of x such that $f(x) \leq 0$. (6)

c.) Write $f(x)$ in the form, $ax + b + \frac{k}{x-2}$. (3)

d.) Hence give the equation of the oblique asymptote for the graph of f. (2)

e.) Explain briefly why there is no horizontal asymptote. (2)

f.) Show that f has stationary points (4,45; 11,90) and (-0,45; 2,10). (5)

g.) Show, using the second derivative, that f does not have any points of inflection. (3)

h.) Sketch the graph of f, showing the asymptote/s, x and y intercept/s, stationary points and indicate the nature of the concavities (i.e. up or down). Use the annexure supplied and do NOT try to draw the graph to scale. (8)

i.) If you did not have an integral function on your calculator, state which form of $f(x)$ would you use, to find the area bounded by the curve of f and the X axis, between $x = -4$ and $x = 1$. (1)

j.) Calculate this area using your calculator. (2)

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QUESTION 4

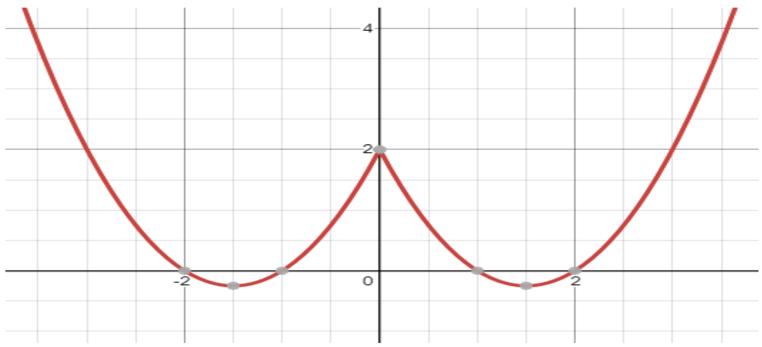
4.1 Use implicit differentiation to find the gradient of the tangent to the curve with equation,

$x^2 + y^2 = 9 - 3xy^2$ at the point (0;3). (9)

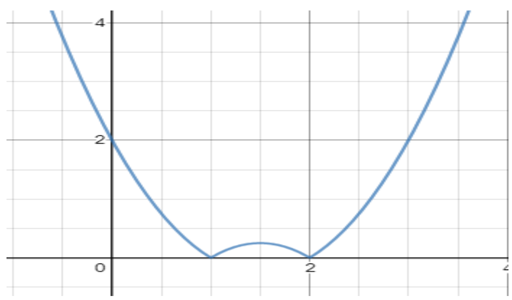
4.2 $f(x) = x^2 - 3x + 2$ and $g(x) = |x|$

a.) Given the two graphs below, drawn to scale, justify mathematically, which is $f(g(x))$ and which is $g(f(x))$. (6)

A.



B.



b.) Use the graphs to find the value/s of x for which, $f(g(x)) = g(f(x))$. (4)

c.) If $k(x) = f(g(x))$, for which value/s of x is k continuous but not differentiable. Show reasoning. (6)

d.) If $h(x) = f(x),\ for\ \ x \geq 1$
$= g(x),\ for\ \ x < 1$

(i) Sketch h. (4)

(ii) Hence, use the graph to state the x value/s at which h is discontinuous, as well as what type of discontinuity/ies they are.
There is no need to show reasoning here. (2)

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QUESTION 5

5.1 An engineer is required to create a metallic part of machinery that has a volume of 20 000 cm^3 .
In calculating the height, x, that she must make this shape, she arrives at the following equation:

$$20000 = x^3 + 2x^2 + 15x$$

She decides to use the Newton-Raphson method to solve for x to 2 decimal places.

a.) She calculates that an accurate solution must be in the interval (25; 30). Explain mathematically how she arrived at this decision. (2)

b.) Hence use the Newton-Raphson method to solve for x to 2 decimal places, using 25 as your initial estimate. (5)

5.2 The area under the curve, $f(x) = x^3$, above the X axis and in the interval [1;5] is calculated using Riemann sums. Using right hand boundaries the area of n rectangles simplifies to ,

$$A_n = 156 + \frac{248}{n} + \frac{96}{n^2}$$

a.) Calculate the area if 6 rectangles are used. (2)

b.) Use the Riemann Sum to find the exact area. (2)

c.) Add to the sketch on the annexure to show why the answers to (a) and (b) differ in value. (2)

d.) Calculate the percentage error if 6 rectangles are used. (2)

5.3

a.) Resolve $\frac{3}{(x-1)(2x-3)}$ into partial fractions. (4)

b.) Hence find $\int \frac{3}{(x-1)(2x-3)}\, dx$ (4)

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QUESTION 6

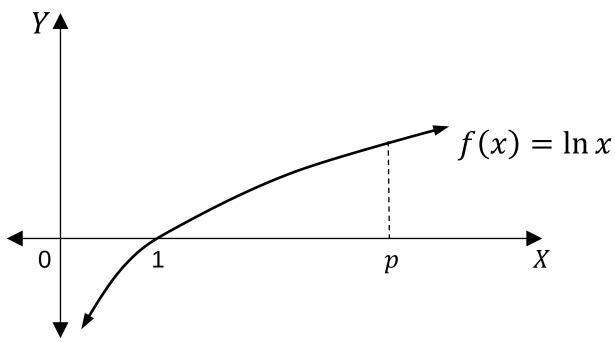
6.1 Find the following integrals:

a.) $\int (6x-9)(x^2-3x)^6\,dx$ (4)

b.) $\int sin4x.sin3x\ \ dx$ (4)

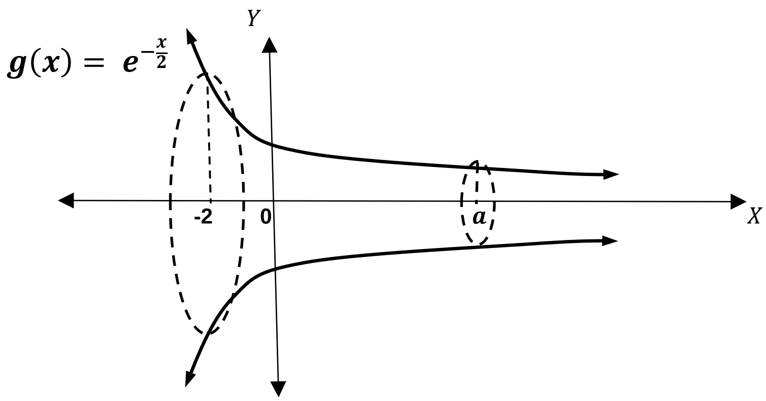
c.) $\int sin^3x\ +\ sec^2x\,dx$ (8)

6.2



If the area shaded above is, A = $(10ln10) - 9$, find the value of p. (8)

6.3 If the shaded area below is revolved about the X axis, the volume generated is 23,1922 cm^3 when rounded to 4 decimal places.



Find the value of a.
Round your answer to 2 decimal places. (9)

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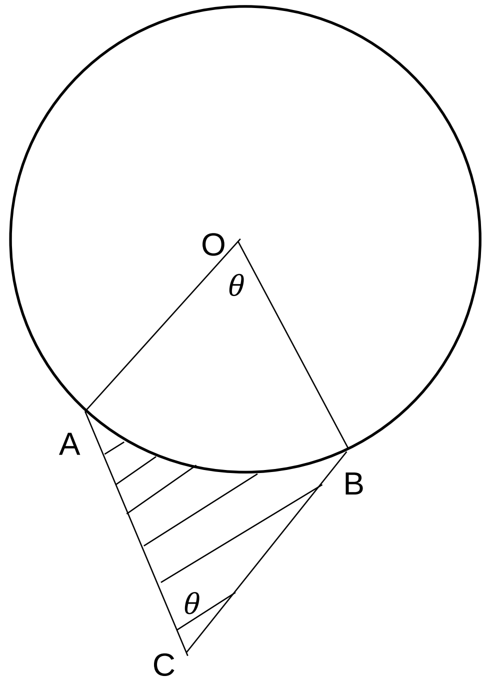
QUESTION 7

7.1



In the diagram above, circle centre O, has radii OA = OB = r. AOBC is a rhombus.

$A\hat{O}B = A\hat{C}B = \theta$. The arc length from A to B is 10 cm.

a.) Find r in terms of θ. (1)

b.) Hence show that the shaded area, $A = \frac{100}{\theta^2}.sin\theta - \frac{50}{\theta}$ (5)

c.) Hence find the shaded area in cm^3 , if $\theta = \frac{\pi}{5}$. (2)

7.2 The mass, $m\ (in\ kg)$, of a newly born baby after t months, is given by

$$m = \frac{1}{28}(6t^3 - 45t^2 + 108t + 84)$$

a.) Calculate what the baby's mass was at birth? (2)

b.) The baby's mass reached a peak after h months and then it decreased to a minimum at an age of k months, after which it started gaining again.
Find h and k. (6)

c.) After the baby's mass starts decreasing, how old is the baby when the rate of decrease starts slowing down? (3)

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