PARKLANDS COLLEGE

PRELIM EXAMINATION 2019

GRADE 12:

AP MATHEMATICS
PAPER 1

MARKS: 200

TIME: 2 hours

EXAMINER: A Rossouw Signature: …………….

MODERATOR: S Loseby Signature: …………….

This paper consists of 13 pages.

NAME: ____________________________

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
22	18	14	20	15	18	13	12	35	9	18	6	200

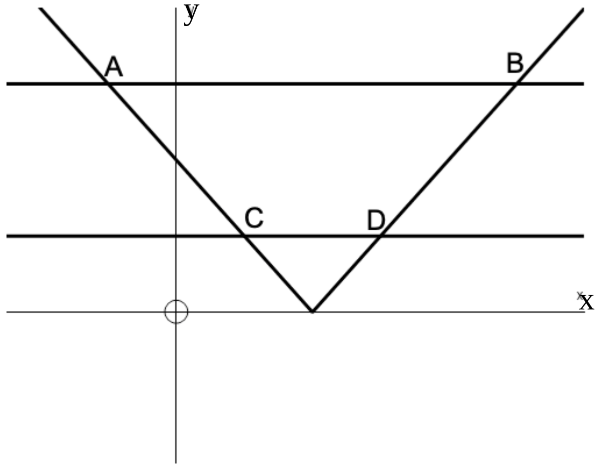
INSTRUCTIONS AND INFORMATION :

1. **Write in blue or black pen only.**
2. **Correction fluid, highlighters and erasable pens may not be used.**
3. **Answer all the questions.**
4. **Leave a line open between each question.**
5. **Draw a 2 cm marking margin on the right hand side of each page.**
6. **Rule a line after each section.**
7. **This paper consists of 12 QUESTIONS and 13 PAGES, including an INFORMATION (FORMULA) SHEET.**
8. **All calculations must be shown.**
9. **Unless stated otherwise, calculators (non-programmable) may be used, in which case answers must be correctly approximated to two decimals.**
10. **Where applicable, answers must be left with positive exponents.**
11. **The diagrams are not necessarily drawn to scale, unless stated otherwise.**
12. **Number your answers correctly according to the numbering system in this paper.**

QUESTION 1

1.1 The sketch below shows the graphs of $f(x) = |x - 2|$ as well as the lines $y = 1$ and $y = 3$.

These lines intersect the graph at the points A, B C and D.



(a) Determine the x-values of A,B,C and D. (6)

(b) Hence write down the solutions for $1 \leq |x - 2| \leq 3$ (4)

1.2 Newton's law of cooling a liquid, like soup, is given by $T(t) = 20 + 60e^{-0,054t}$ where T is the temperature, measured in ℃, of the soup after t minutes.

(a) Determine the initial temperature of the soup. (2)

(b) Determine the temperature of the soup after 10 minutes, correct to the nearest integer. (2)

(c) How long will it take the soup to reach a temperature of 40℃? (3)

(d) The asymptote of the graph of $y = T(t)$ is $T = 20$. What does this mean? (1)

1.3 Solve for x:

(a) $ln|1-x| = 3 - ln5$ (4)

[22]

QUESTION 2

2.1 Solve for x if $x \in C$ and $x^4 + 5x^3 + 27x^2 + 5x - 174 = 0$ and if one of the solutions is $x = -2 - 5i$. (8)

2.2 (a) Show, without the use of a calculator, that $\frac{1+i}{1-i} = i$. Do your calculations in rectangular form. Show all your working. (3)

(b) Hence determine , without the use of a calculator, the value of $\frac{(1+i)^{17}}{(1-i)^{16}}$. Show all your working. (3)

2.3 Write $(1+i)$ and $(1-i)$ in polar form. Leave your answer in root form and in terms of π. (4)

[18]

QUESTION 3

Use Mathematical Induction to prove that
$x^n - y^n$ is divisible by $x - y$. (14)

[14]

QUESTION 4

4.1 Given: $f(x) = |\ln(x+2)|$ and $g(x) = -3x - 3$

(a) The graphs of f and g intersect at $(-1; 0)$ and another point. Show that this point's x-value can be calculated by solving the equation $e^{3x+3} = x + 2$. (3)

(b) Hence use Newton's method and determine this x-value correct to four decimal places. Use $x = -1{,}8$ as first approximation. (5)

4.2 Given

$$f(x) = \begin{cases} e^x & if \quad x < 0 \\ x^2 + 1 & if \quad 0 \leq x < 2 \\ \ln(x-1) & if \quad x \geq 2 \end{cases}$$

Determine whether $f(x)$ is:

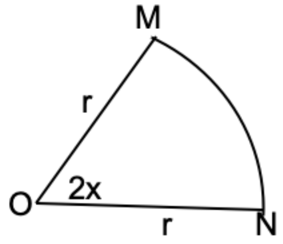
(a) Differentiable at $x = 0$. Show all your working. (8)

(b) Continuous at $x = 2$. Show all your working. (4)
If discontinuous, state the type of discontinuity.

[20]

QUESTION 5

The sketch shows a sector OMN of a circle with centre O. The radius is r and $M\hat{O}N = 2x$.



(a) Determine an expression, in terms of r and x, for the perimeter P and the area A of the sector. (4)

(b) Hence, show that if P=20, then $A = \frac{100x}{(1+x)^2}$ (4)

(c) For which value of x will the area be a maximum? (7)

[15]

QUESTION 6

Given: The rational function $f(x) = \frac{x^2-3}{e^x}$

6.1 Give a reason why the graph of f has no vertical asymptotes. (2)

6.2 The graph has a maximum turning point at $(a; 0{,}3)$ and a minimum turning point at $(b; -5{,}4)$ with $a > 0$ and $b < 0$. Determine the values of a and b. (8)

6.3 Determine the $x-$ and $y-$intercepts of the graph. (2)

6.4 Hence draw a sketch of $y = f(x)$ if it is also given that the graph has a horizontal asymptote at $y = 0$. Clearly indicate all intercepts with the axes as well as all asymptotes. (6)

[18]

QUESTION 7

Given: $x^2 - 3xy + 25y^2 = 91$

7.1 Determine $\frac{dy}{dx}$. (5)

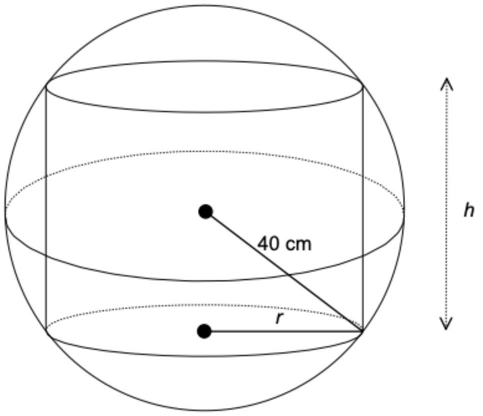
7.2 Show that $\frac{dy}{dx} = 0$ at $(3; 2)$. (1)

7.3 Determine $\frac{d^2y}{dx^2}$ and hence determine the nature of the turning point at $(3; 2)$. (7)

[13]

QUESTION 8

A cylinder is fitted into a sphere of radius 40 cm. The cylinder has a height of h and a radius of r.



(a) Express the volume of the cylinder in terms of r. (5)

(b) Determine the value of r that will maximise the volume. (7)
[12]

QUESTION 9

9.1 Determine the following integrals:

(a) $\int \sin(3x+4)\,dx$ (4)

(b) $\int (sin^2 4x)(cos 4x)dx$ (6)

9.2

(a) Show that $\frac{d}{dx}(tan^3 x) = 3sec^4 x - 3sec^2 x$. (5)

(b) Hence determine $\int sec^4 x\, dx$ (6)

9.3 Given: $y = ln\left(\frac{x-2}{x}\right)$

(a) Show that $y' = \frac{2}{x(x-2)}$ (5)

(b) Decompose $\frac{2}{x(x-2)}$ into partial fractions. (6)

(c) Hence determine $\int \frac{2}{x(x-2)} dx$ (3)

[35]

QUESTION 10

Consider $\int_a^b f(x)dx = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} [3\left(2 + \frac{i}{n}\right)^2 - 6]$

(a) Determine the values of a and b. (5)

(b) Write down the function $f(x)$. (2)

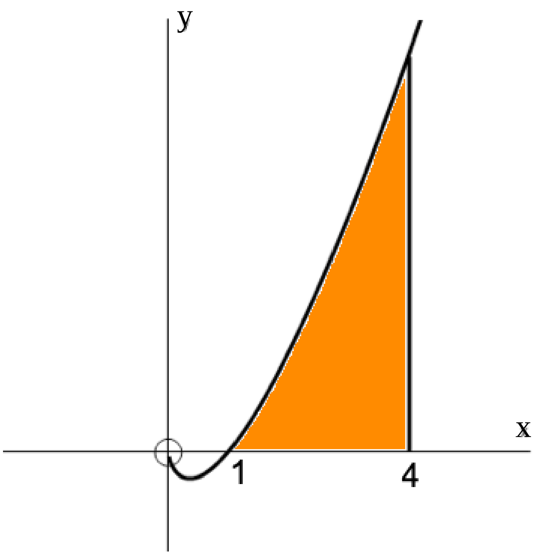
(c) Calculate the area enclosed by the graph of f, the x-axis and the lines $x = a$ and $x = b$. (2)

[9]

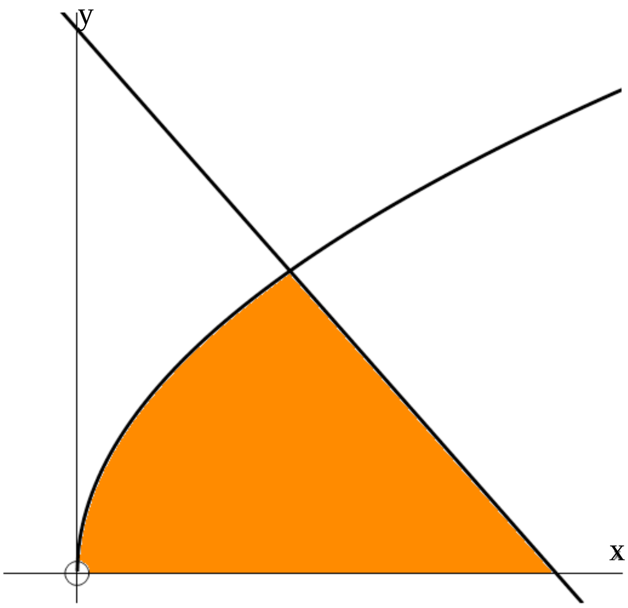
QUESTION 11

(a) Determine $\int x.lnxdx$ (7)

(b) The sketch shows the graphs of $f(x) = x.lnx$ and $x = 4$. (2)
Calculate the shaded area.



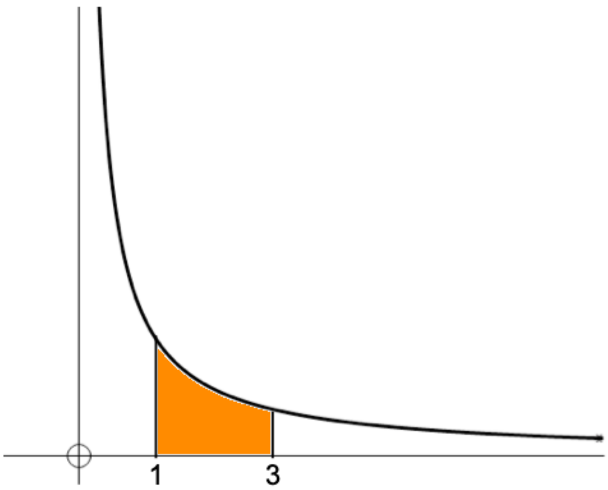
(c) Given the graphs of $y = 18 - 2x$ and $y = 5\sqrt{x}$. Calculate (9)
the area of the shaded region.



[18]

QUESTION 12

The diagram shows part of the curve $y = \frac{a}{x}$ where a is a positive constant . Determine the value of a if it is given that the volume obtained when the shaded region is rotated about the x-axis is $24\pi\ units^3$. (6)



[6]

TOTAL 200

INFORMATION SHEET

General Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$z = a + bi$ $\qquad$ $z^* = a - bi$

$\ell n\,A + \ell n\,B = \ell n\,(AB)$ $\qquad$ $\ell n\,A - \ell n\,B = \ell n\left(\frac{A}{B}\right)$

$\ell n\,A^n = n\,\ell n\,A$ $\qquad$ $\log_a x = \frac{\log_b x}{\log_b a}$

Calculus

$$Area = \lim_{n \to \infty} \left(\frac{b-a}{n}\right) \sum_{i=1}^{n} f(x_i)$$

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\int f'(g(x)).g'(x)\,dx = f(g(x)) + c$$

$$\int f(x).g'(x)dx = f(x).g(x) - \int g(x).f'(x)\,dx + c$$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$V = \pi \int_a^b y^2 dx$$

Function	**Derivative**
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x.\cot x$
$f(g(x))$	$f'(g(x)).g'(x)$
$f(x).g(x)$	$g(x).f'(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

Trigonometry

$A=\frac{1}{2}r^2\theta$ $\qquad$ $s=r\theta$

In $\triangle ABC$:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc.\cos A$$

$$Area=\frac{1}{2}ab.\sin C$$

$\sin^2 A+\cos^2 A=1$ $\qquad$ $1+\tan^2 A=\sec^2 A$ $\qquad$ $1+\cot^2 A=\text{cosec}^2 A$

$\sin(A\pm B)=\sin A.\cos B\pm\cos A\sin B$ $\qquad$ $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$\sin 2A=2\sin A\cos A$ $\qquad$ $\cos 2A=\cos^2 A-\sin^2 A$

$$\sin A.\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

$$\sin A.\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A.\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$