Memo AP P1

1.1 a $-x+2=3$ ✓ $-x+2=1$ $x-2=3$ ✓ $x-2=1$

$x=-1$ $x=1$ $x=5$ $x=3$ 6

(A) ✓ (C) ✓ (B) ✓ (D) ✓

b $x \in [-1;1]$ ✓✓ or $x \in [3;5]$ ✓✓ 4

1.2 $T(t) = 20+60$ ✓ $= 80^{\circ}C$ 2

b $T(10) = 20 + 60e^{-0,054\times 10}$ ✓

$= 54,96\ldots$

$\approx 55^{\circ}C$ ✓ 2

c 40 ✓ $= 20 + 60e^{-0,054t}$

$20 = 60e^{-0,054t}$

$\frac{1}{3} = e^{-0,054t}$

$\ln \frac{1}{3} = -0,054t$ ✓

$20,34 = t$ ✓ 3

d Temp won't drop below $20^{\circ}C$ ✓ (room temp.) 1

1.3 $\ln|1-x| = 3 - \ln 5$

$\ln(1-x) = 1,39\ldots$ ✓ or $\ln(x-1) = 1,39\ldots$ ✓

$e^{1,39\ldots} = 1-x$ $e^{1,39} = x-1$

$-3,02 = x$ ✓ or $5,02 = x$ ✓ 4

Question 2

2.1 $x = -2 + 5i$ is also a solution

$\therefore$ sum $= -4$

product $= 29$

$x^4 + 5x^3 + 27x^2 + 5x - 174 = (x^2 + 4x + 29)(x^2 + mx - 6)$

$-24 + 29m = 5$

$m = 1$

8

$\therefore x^4 + 5x^3 + 27x^2 + 5x - 174 = (x^2 + 4x + 29)(x^2 + x - 6)$

$= (x + 2 - 5i)(x + 2 + 5i)(x + 3)(x - 2)$

$\therefore x = -2 \pm 5i$ or $x = -3$ or $x = 2$

2.2a LHS

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2} = \frac{1+i^2+2i}{2} = \frac{2i}{2} = i$$

$= RHS$

$\therefore LHS = RHS$

3

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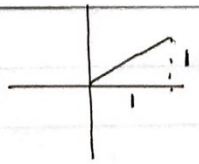
b $$\left[\frac{(1+i)}{(1-i)}\right]^{16} \cdot (1+i) = i^{16}(1+i)$$

$= (i^2)^8(1+i)$

$= 1 + i$

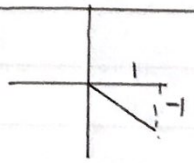
3

2.3



$z = r^2 \therefore \sqrt{2} = r$

$\tan\theta = 1 \therefore \theta = \pi/4$



4

$z = \sqrt{2}\,\text{cis}\,\frac{\pi}{4}$

$(1-i) = \sqrt{2}\,\text{cis}(-\pi/4)$

Question 3

For $x=1$ ✓

$x-y$ is divisible by $x-y$ ✓

∴ statement true for $n=1$

Assume the statement is true for $n=k$ ✓

∴ x^k-y^k is divisible by $x-y$ .... (1) ✓

Now prove the statement true for $n=k+1$ ✓

LHS

$x^{k+1}-y^{k+1}$ ✓

$= (x^k-y^k)\cdot x$ ✓ $+ xy^k - y^{k+1}$ ✓

$= (x^k-y^k)x + y^k(x-y)$ ✓

↑ Divisible by $(x-y)$ } ✓

↑ Divisible by $(x-y)$ from (1) ✓

∴ statement true for $n=k+1$ ✓ 14

By the principle of Mathematical Induction ✓✓ the statement is true for $n \in N$.

Question 4

4.1 a $|\ln(x+2)| = -3x-3$ 3

$-\ln(x+2) = -3x-3$ ✓ or $\ln(x+2) = -3x-3$

$\ln(x+2) = 3x+3$ ✓✓ $e^{-3x-3} = x+2$

$e^{3x+3} = x+2$ $x = -1$

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b $x_0 = -1,8$ (answ) $x_{n1} = answ - \frac{f(answ)}{f'(answ)}$

$x_1 = -1,95014$ ✓

$x_2 = -1,94050$ ✓ $f(x) = e^{3x+3} - x - 2$

$x_3 = -1,94047$ $f'(x) = 3e^{3x+3} - 1$ ✓

$\approx -1,9405$ ✓ 5

4.3 a $\lim_{x\to 0^-} e^x = 1$ ✓ $\lim_{x\to 0^+} x^2+1 = 1$ ✓ $f(0) = 1$ ✓

$\therefore \lim_{x\to 0} f(x) = f(0)$ $\therefore$ Continuous

8

$\lim_{x\to 0^-} e^x = 1$ ✓ $\lim_{x\to 0^+} 2x = 0$ ✓ $\therefore \lim_{x\to 0} f'(x)$ ✓ ∄

$\therefore$ Not differentiable ✓

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b $\lim_{x\to 2^-} x^2+1 = 5$ ✓ $\lim_{x\to 2^+} \ln(x-1) = 0$ ✓

$\therefore \lim_{x\to 2} f(x)$ ∄ ✓ 4

$\therefore$ Not continuous ✓

Jump Discontinuity

Question 5

a $S = \theta r$

$S = 2\pi r$ ✓

$P = 2r + 2\pi r$ ✓

$A = \frac{1}{2}(r^2)(2\pi) = \pi r^2$ ✓ ✓ ✓

4

---

b $20 = 2r + 2\pi r$

$20 = 2r(1+\pi)$ ✓ ✓

$\frac{10}{1+\pi} = r$ ✓

$A = \pi\left(\frac{10}{1+\pi}\right)^2 = \frac{100\pi}{(1+\pi)^2}$ ✓

4

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c $\frac{100(1+\pi)^2 - 2(1+\pi)(100\pi)}{(1+\pi)^4} = 0$ ✓ ✓ ✓

$100(1+\pi)((1+\pi) - 2\pi) = 0$ ✓ ✓

$\therefore \pi = 1$ ✓

7

Question 6

6.1 $e^x = 0$ ✓

$\ln|0| = x$ ✓

undef

2

6.2 $\frac{2x(e^x) - e^x(x^2-3)}{(e^x)^2} = 0$ ✓ ✓ ✓

$2x(e^x) - e^x(x^2-3) = 0$

$e^x(2x - x^2 + 3) = 0$ ✓

$e^x(x-3)(x+1) = 0$ ✓

$x = 3$ or $x = -1$

$\therefore a = 3$ ✓ $b = -1$ ✓

8

6.3 x-int

$x^2 - 3 = 0$

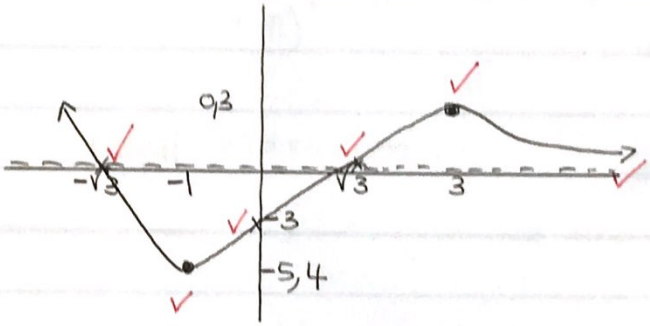
$x = \pm\sqrt{3}$ ✓

y-int

$y = -3$ ✓

2

6.4



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Question 7

7.1 $2x - (3y + 3x\frac{dy}{dx}) + 50y\frac{dy}{dx} = 0$

$2x - 3y - 3x\frac{dy}{dx} + 50y\frac{dy}{dx} = 0$

5

$\frac{dy}{dx} = \frac{3y - 2x}{50y - 3x}$

7.2 At $(3;2)$

$\frac{dy}{dx} = \frac{3(2) - 2(3)}{50(2) - 3(3)} = 0$

1

7.3 $\frac{d^2y}{dx^2} = \frac{(3\cdot\frac{dy}{dx} - 2)(50y - 3x) - (50\frac{dy}{dx} - 3)(3y - 2x)}{(50y - 3x)^2}$

At $(3;2)$

7

$= \frac{(-2)(91) + 3(0)}{(91)^2} = \frac{-2}{91}$

$\therefore$ Local Maximum

Question 8

a. $\frac{1}{2}h = \sqrt{40^2 - r^2}$ ✓✓ pythagoras

$h = 2\sqrt{1600 - r^2}$ ✓

$\therefore$ Volume $= \pi r^2$ ✓ $\cdot 2\sqrt{1600 - r^2}$ ✓ 5

b $2\pi r$ ✓ $\cdot 2\sqrt{1600 - r^2} + (1600 - r^2)^{-\frac{1}{2}}$ ✓ $\cdot \pi r^2 \cdot -2r$ ✓ $= 0$

$2\pi r \cdot 2\sqrt{1600 - r^2} = \frac{\pi r^3 \cdot 2}{\sqrt{1600 - r^2}}$

$4\pi r(1600 - r^2) = 2\pi r^3$ ✓

$2(1600 - r^2) = \frac{2\pi r^3}{2\pi r}$ 7

$2(1600 - r^2) = r^2$

$3200 = 3r^2$ ✓

$1066,\ldots = r^2$

$r = \frac{40\sqrt{6}}{3}$ ✓

Question 9

9.1a $\int \sin(3x+4)\,dx$ 4

$= -\frac{\cos(3x+4)}{3}$ ✓ ✓ ✓ ✓ $+ c$ ✓

b $\int (\sin^2 4x)(\cos 4x)\,dx$ 6

$= \int (\sin 4x)^2 (\cos 4x)\,dx$ ✓

$= \frac{(\sin 4x)^3}{3 \cdot 4}$ ✓ ✓ ✓ ✓ $+ c$ ✓

or let $u = \sin 4x$

$\frac{du}{dx} = 4\cos 4x$

$dx = \frac{du}{4\cos 4x}$

$\therefore \int u^2 \cdot \cos 4x \cdot \frac{du}{4\cos 4x}$

$= \frac{1}{4}\int u^2\,du$

$= \frac{1}{4}\frac{u^3}{3} + c$

9.2a $\frac{d}{dx}(\tan^3 x) = 3\sec^4 x - 3\sec^2 x$

LHS

$3(\tan x)^2 \cdot \sec^2 x$ 5

$= 3(\sec^2 x - 1) \cdot \sec^2 x$

$= 3\sec^4 x - 3\sec^2 x$

$=$ RHS

$\therefore$ LHS $=$ RHS

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b $\frac{d}{dx}(\tan x)^3 = 3\sec^4 x - 3\sec^2 x$

$\therefore \int \frac{d}{dx}(\tan x)^3 dx = \int (3\sec^4 x)\, dx - \int 3\sec^2 x\, dx$

$\tan^3 x + 3\tan x = 3\int \sec^4 x\, dx$ 6

$\frac{1}{3}\tan^3 x + \tan x = \int \sec^4 x\, dx$

9.3a $y = \ln\left(\frac{x-2}{x}\right)$

$y' = \frac{x}{x-2}\left[\frac{1(x) - (1)(x-2)}{x^2}\right]$ 5

$= \frac{x[x - x + 2]}{x^2(x-2)} = \frac{2}{x(x-2)}$

---

b $\dfrac{2}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$ ✓

$2 = A(x-2) + B(x)$ ✓

$x=0$ ✓

$2 = A(-2) + 0$

$-1 = A$ ✓

$x = 2$ ✓

$2 = B(2)$

$1 = B$ ✓

6

$\therefore \dfrac{2}{x(x-2)} = \dfrac{-1}{x} + \dfrac{1}{x-2}$

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c $\int \frac{2}{x(x-2)}dx = -\int \frac{1}{x}\,dx + \int \frac{1}{x-2}\,dx$

$= -\ln|x| + \ln|x-2| + c$ ✓ ✓ ✓

3

Question 10

a $a + i\Delta x_i = 2 + \frac{i}{n}$ ✓

$\therefore a = 2$ ✓

$\frac{b-a}{n} = \frac{1}{n}$ ✓

$b - a = 1$ ✓ but $a = 2$

$b - 2 = 1$

$b = 3$ ✓

5

b $f(x) = 3x^2 - 6$ ✓✓

2

c $\int_2^3 3x^2 - 6\,dx = 13$ ✓✓

2

Question 11

$f \frac{1}{2}x^2$

a $\int^{f1} x \ln^{g} x\,dx$

$g1$: $\frac{1}{x}$

$= \frac{1}{2}x^2 . \ln x - \int \frac{1}{2}x^2 . \frac{1}{x}\,dx$ ✓ ✓

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$ ✓ ✓

$= \frac{1}{2}x^2 \ln x - \frac{\frac{1}{2}x^2}{2}$ ✓ ✓ $+c$ ✓

7

b $\int_1^4 x \ln x\,dx = 7{,}34$ ✓✓

2

c $18-2x = 5\sqrt{x}$

$(18-2x)^2 = 25x$ ✓

$324 - 72x + 4x^2 = 25x$ ✓²

$4x^2 - 97x + 324 = 0$

$x \neq \frac{81}{4}$ or $x = 4$ ✓✓

$\int_0^4 5\sqrt{x}\,dx + \int_4^9 (18-2x)\,dx$ ✓✓ 9

$= \frac{155}{3}$ ✓

Question 12

$\pi \int_1^3 a^2 x^{-2} = 24\pi$ ✓

$\left.\frac{\pi a^2 x^{-1}}{-1}\right|_1^3 = 24\pi$

✓✓ 6

$\left.-\frac{\pi a^2}{x}\right|_1^3 = 24\pi$

$-\frac{\pi a^2}{3} + \pi a^2 = 24\pi$ ✓

$\frac{2a^2}{3} = 24$ ✓

$a^2 = 36$

$a = 6$ ✓