SECTION B - STATISTICS (100 marks)

Question 1 (20 marks)

(a) In a soccer practice, the probability that my shot is saved by the goalkeeper is 0,3.

(i) If I take 10 shots, what is the probability that at least two shots are saved? (7)

$P(X \geq 2)$

$= 1 - P(X = 0) - P(X = 1)$

$= 1 - {}^{10}C_0 (0,3)^0 (0,7)^{10} - {}^{10}C_1 (0,3)^1 (0,7)^9$

$= \underline{0,8507}$

(ii) How many shots should I take to be at least 99% certain that at least 1 goal will be saved? (6)

$P(X \geq 1) > 0,99$

$\therefore 1 - {}^{n}C_0 (0,3)^0 (0,7)^n > 0,99$

$\therefore 0,01 > (0,7)^n$

$\therefore \log_{0,7} 0,01 < n$

$\therefore 12,911 < n$

$\therefore \underline{n = 13}$

(b) An international committee to effect climate change strategies is to be set up. There are 8 US nominees, 6 from the UK and 5 from France. The final committee must consist of 10 and include at least three from each country. How many different committees are possible? (7)

$${}^{8}C_3 \times {}^{6}C_3 \times {}^{5}C_4 + {}^{8}C_4 \times {}^{6}C_3 \times {}^{5}C_3 + {}^{8}C_3 \times {}^{6}C_4 \times {}^{5}C_3$$

$$= \underline{28\ 000}$$

Question 2 (25 marks)

(a) Consider two non-independent events, A and B.

$p(A' \cap B') = 0,2 \qquad p(A) = 0,7 \qquad p(A \mid B) = 0,75$

Calculate:

(i) $p(B)$ (7)

$P(A \mid B) = 0,75$

$\therefore \dfrac{P(A \cap B)}{P(B)} = 0,75$

$\therefore P(A \cap B) = 0,75\, P(B)$

$P(A' \cap B') = P(A \cup B)'$

$\therefore P(A \cup B) = 0,8$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0,8 = 0,7 + P(B) - 0,75\, P(B)$

$\therefore 0,1 = 0,25\, P(B)$

$\therefore 0,4 = P(B)$

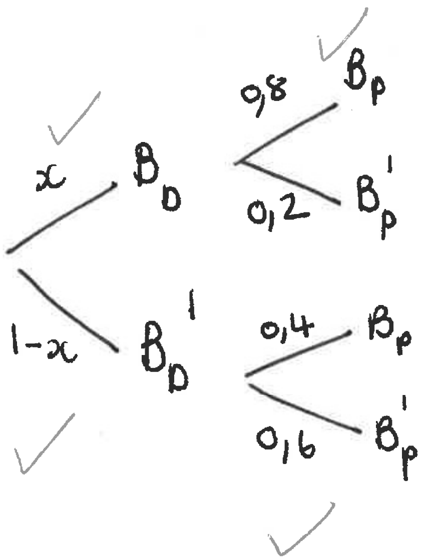
(ii) $p(A \cap B)$ (3)

$= 0,75(0,4)$

$= 0,3$

(b) Daryll and Piet like going to the beach. If Daryll goes to the beach, then the probability that Piet goes is 0,8. If Daryll does not go, the probability that Piet goes to the beach is 0,4. The total probability that Piet goes to the beach is 0,65.

(i) Calculate the probability that Daryll goes to the beach (9)



$$x \times 0{,}8 + (1-x)0{,}4 = 0{,}65$$

$$\therefore 0{,}8x + 0{,}4 - 0{,}4x = 0{,}65$$

$$\therefore 0{,}4x = 0{,}25$$

$$\therefore x = 0{,}625$$

(ii) Given that Piet went to the beach, calculate the probability that Daryll was also there. (6)

$$P(D|P) = \frac{P(D \cap P)}{P(P)}$$

$$= \frac{0{,}625 \times 0{,}8}{0{,}625 \times 0{,}8 + 0{,}375 \times 0{,}4}$$

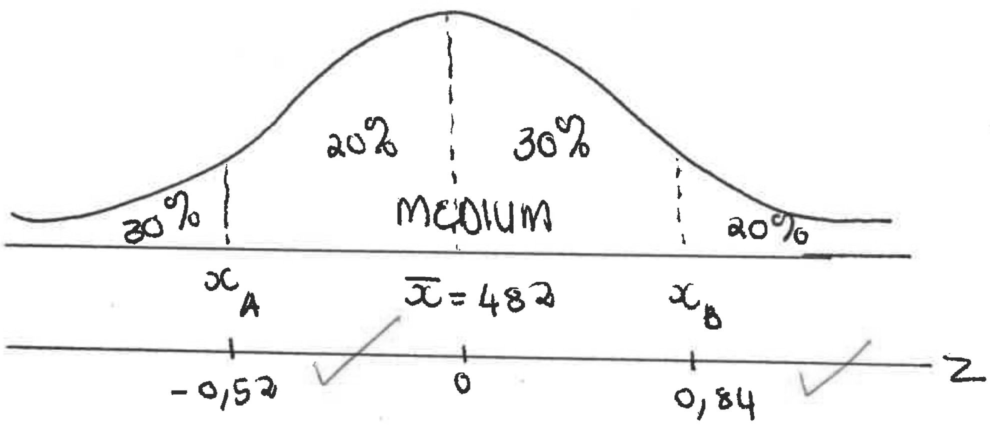
$$= \frac{10}{13} \; / \; 0{,}7692$$

Question 3 (31 marks)

(a) The students in Grade 12 are being punished for bunking a mathematics revision lecture. They are required to run a 1 500 metre race dressed as their favourite super hero.

Their times were normally distributed with mean 482 seconds and standard deviation of 54 seconds. The runners were divided into three categories. The top 20% were regarded as "Fast", the bottom 30% were regarded as "Slow" and the remainder were regarded as "Medium".

Determine the interval of times that would be regarded as medium. (8)



$$-0,52 = \frac{x_A - 482}{54} \qquad 0,84 = \frac{x_B - 482}{54}$$

$$\therefore x_A = 453,92 \qquad \therefore x_B = 527,36$$

Medium interval = $(453,92 ; 527,36)$

(b) A random sample of 100 cars passing an intersection is investigated and the colour noted. 28 of the cars are white.

(i) Calculate a 92% confidence interval for the true proportion of white cars. (6)

$$p \pm z\sqrt{\frac{p(1-p)}{n}}$$

$$= 0{,}28 \pm 1{,}75\sqrt{\frac{0{,}28 \times 0{,}72}{100}}$$

$$\therefore \; 92\%\ CI = (0{,}2014;\ 0{,}3586)$$

(ii) How could the experiment be improved to narrow the interval? (2)

- Lower the confidence level
- Increase the sample size
- Reduce variability

(iii) How many cars would need to be sampled in order that the true proportion would not deviate from the sample proportion by more than 3%? (7)

$$1{,}75\sqrt{\frac{0{,}28 \times 0{,}72}{n}} < 0{,}03$$

$$\therefore \sqrt{\frac{0{,}28 \times 0{,}72}{n}} < \frac{3}{175}$$

$$\therefore \frac{0{,}28 \times 0{,}72}{n} < \left(\frac{3}{175}\right)^2$$

$$\therefore \frac{0{,}28 \times 0{,}72 \times 175}{3} < n$$

$$\therefore 686 < n$$

(c) A random variable, X, is defined as follows:

$X \sim B(50; 0{,}4)$

By using a suitable approximation, calculate P(X > 25) (8)

$\mu = np = 20 \quad > 5$

$\sigma^2 = npq = 12 \quad > 5$

$P(X > 25)$

$= P(X > 25{,}5)$

$= P\left(Z > \dfrac{25{,}5 - 20}{\sqrt{12}}\right)$

$= P(Z > 1{,}59)$

$= 0{,}5 - 0{,}4441$

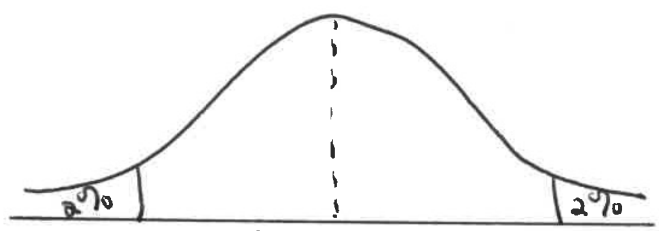
$= \underline{0{,}0559}$

Question 4 (12 marks)

The volume of coffee dispensed by the staff room coffee dispenser is normally distributed with standard deviation 8 ml. It is supposed to dispense 150 ml of coffee. In order to test the machine, Mrs van Rooy volunteers to drink a sample of 50 cups of coffee. She finds that the average amount of coffee is 147 ml. Test at a 4% level of significance whether one could regard the machine as faulty.

$H_0 : \mu = 150$

$H_1 : \mu \neq 150$



CV = $-2{,}05$ $\quad$ $2{,}05$

$$z = \frac{147 - 150}{8/\sqrt{50}}$$

$$= -2{,}65$$

Conclusion:

Reject H_0 at a 4% level of significance.

Suggest sufficient evidence to support the claim that the machine is faulty.

Question 5 (12 marks)

A probability density function is defined as:

$$f(x) = \begin{cases} 1 - kx^2 & \text{if} \quad 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate the value of k. (8)

$$\int_0^2 1 - kx^2 = 1$$

$$\therefore \left[x - \frac{kx^3}{3} \right]_0^2 = 1$$

$$\therefore 2 - \frac{8k}{3} - 0 = 1$$

$$\therefore 1 = \frac{8k}{3}$$

$$\therefore \frac{3}{8} = k$$

(b) Hence calculate $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$ (4)

$$\int_{1/3}^{1/2} 1 - \frac{3}{8}x^2 \, dx$$

$$= \left[x - \frac{x^3}{8} \right]_{\frac{1}{3}}^{\frac{1}{2}}$$

$$= \frac{1}{2} - \frac{(\frac{1}{2})^3}{8} - \frac{1}{3} + \frac{(\frac{1}{3})^3}{8}$$

$$= \frac{269}{1728} \; / \; 0,1557$$