ROEDEAN SCHOOL (SA)

INSPIRING A LIFE OF SIGNIFICANCE

ADVANCED PROGRAMME MATHEMATICS
PAPER II
PRELIMINARY EXAMINATION
SEPTEMBER 2019

Time: 1 HOUR 10 MINUTES
Reading Time: 10 MINUTES

Marks: 100
Examiner: R. Bourquin
Moderator: M. Brown

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 9 pages, and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2. This question paper consists of TWO modules:

   Choose **ONE** of the **TWO** modules:

   MODULE 2: STATISTICS (100 marks)
   OR
   MODULE 3: FINANCE AND MODELLING (100 marks)

3. Non-programmable and non-graphical calculators may be used.

4. All necessary calculations must be clearly shown and writing should be legible.

5. Diagrams have not been drawn to scale.

6. **Rounding of final answers:**

   MODULE 2 (STATISTICS): Four decimal places, unless otherwise stated.

   MODULE 3 (FINANCE AND MODELLING): Two decimal places, unless otherwise stated.

MODULE 2 STATISTICS

QUESTION 1

1.1 A rugby coach has 12 new rugby balls and 10 used rugby balls.
If he selects 7 balls at random to use for a practice, what is the probability that there will be at least 2 new balls. (6)

1.2 In South Africa, 29% of adults are unemployed. Twenty adults are chosen at random. Find the probability that fewer than 2 adults are unemployed.

Give your answer as a percentage, correct to 4 decimal places. (6)

1.3 Debbie is about to write a multiple-choice examination, where each question has 4 possible answers, of which only one is correct. Debbie knows 70% of the work being assessed. She decides that she will randomly guess if she does not know an answer.

(a) What is the total probability that she would choose the correct answer in a particular question? (4)

(b) Given that Debbie selected the correct answer, what is the probability that she guessed? (4)

[20]

QUESTION 2

2.1 Consider the function defined by:

$P(X = x) = \frac{1}{24}(x + 6)$ for $x \in \{1; 2; 3\}$

(a) Determine the probability distribution for X.

Give your answer in table form. (3)

(b) Is $P(X = x)$ a probability mass function?
Justify your answer. (3)

(c) Calculate the expected value of X. (3)

2.2 The speed of a bus, x km/h, on a certain journey, is a continuous random variable X with a probability function given by:

$$f(x) = \begin{cases} kx^{-2} & 20 < x < 28 \\ 0 & otherwise \end{cases}$$

(a) Prove that $k = 70$, showing **all** your working. (6)

(b) Determine the median speed of the bus for this journey. (6)

[21]

QUESTION 3

3.1 The mass of adult males in America is normally distributed.
The mean mass of adult males in America is 88,8kg.
The standard deviation of their mass is 12,6kg.

120 adult American males are chosen at random.

(a) Approximately how many of the chosen males will have a mass between 80kg and 100kg? (8)

(b) 27% of American males are classified as obese.
Based on the information given above, determine what the minimum mass an adult male must be in order to be classified as obese. (6)

3.2 It is known that 38% of university students in South Africa fail Physics I.
800 students are expected to do Physics I next year.

(a) Can a normal approximation be used for calculations of this binomial distribution? Justify your answer. (3)

(b) Determine μ and σ for the normal approximation. (2)

(c) Determine the probability that at least 275 students will fail Physics I next year. (6)

[25]

QUESTION 4

4.1 A random sample of 80 bottles of Jameson Whiskey showed the mean volume to be 750ml and the variance to be $20ml^2$.

Determine a 97% confidence interval for the population mean volume of Jameson Whiskey given that the volume is normally distributed. (6)

4.2 A survey of random people produces a 95% confidence interval for the proportion of people who use WhatsApp as (0,86; 0,98).

(a) Determine the estimated proportion of people who use WhatsApp. (2)

(b) Determine the size of the sample that was taken to obtain this confidence interval. (8)

[16]

QUESTION 5

A machine has produced bolts over a long period of time, where the length, X, in mm was normally distributed with a mean of 22 and a variance of 0,64.

It is believed that recently the mean length has changed.

To test this claim, a random sample of 28 bolts is taken and the mean length is found to be 21,7mm.

Test the claim at the 7% level of significance.

[10]

QUESTION 6

A palindrome is a sequence of letters that reads the same forwards as backwards. For example, DCBEBCD.

6.1 How many 9 letter palindromes can be formed from our alphabet of 26 letters? (4)

6.2 How many of the palindromes from (a) will have more than one pair of letters repeated? (4)

[8]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

Tassy won R1 000 in a raffle. She decided to invest this money in a fund paying *i*% p.a. compounded monthly.
After *n* years the value of the fund was R1 427, 96 and after another 18 months the value of the fund was R1 541, 23.

1.1 Calculate *i.* (6)

1.2 Calculate *n*. (6)

[12]

QUESTION 2

David borrows R3 000 000 from the bank to buy a house.

The bank agrees to defer his first repayment to 6 months after he takes out the loan.

The bond agreement stipulates that David will make 240 monthly payments to amortise the loan.

The bank charges an interest rate of 9% p.a compounded quarterly.

Determine the value of his monthly payments.

[12]

QUESTION 3

Lawrence borrows R35 000 at 15% p.a. compounded monthly and agrees to pay back R800 per month for the first year (first payment to be made 1 month after loan taken), R900 per month for the second year and R1 000 per month for the remaining time until the loan is paid back in full (including interest).

3.1 How much will he still owe after the first two years? (12)

3.2 If the balance outstanding after two years is R23 640,80, how many monthly payments of R1 000 are needed? (6)

3.3 What will be the amount of his final payment, which is less than R1 000? (8)

[26]

QUESTION 4

The number of yeast cells in a new laboratory culture grows from 12 to 85 over the course of 3 hours.

4.1 Determine the hourly intrinsic growth rate of the culture.
Give the answer as a percentage correct to 2 decimal places. (4)

4.2 Write a Malthusian difference-equation to represent the population growth on an hourly basis for the population and use it to determine the yeast population after 8 hours to the nearest integer. (4)

[8]

QUESTION 5

A population of 25 zebra are kept in a protected reserve.

Their population increases before levelling off to a stable population.

The table below records the zebra population at yearly intervals.

n	P_n	ΔP	$\frac{\Delta P}{P_n}$
0	25		
1	31	9	**A**
2	43	12,5	0,290698
3	56	**B**	**C**
4	59	2	0,033898
5	**D**		

5.1 Calculate the values of A, B, C and D in the table above.
Give your answers correct to 6 decimal digits and show your working. (4)

5.2.

$\boldsymbol{n}$	$\boldsymbol{P_n}$	$\boldsymbol{\Delta P}$	$\frac{\boldsymbol{\Delta P}}{\boldsymbol{P_n}}$
0	25		
1	31	9	0,2903
2	43	12,5	0,2907
3	56	8	0,1429
4	59	2	0,0339
5	60		

Using the completed table above:

(a) Determine the equation of the straight line representing the relationship between the growth rate $\left(\frac{\Delta P}{P_n}\right)$ and the population (P_n).
Give numeric values correct to 6 decimal digits. (4)

(b) Write down, as a percentage correct to 2 decimal places, the annual intrinsic growth rate for this model. (2)

(c) Determine the carrying capacity of this population of zebra.
Give your answer correct to the nearest whole number. (2)

(d) Using your answers for (b) and (c), express the growth of the zebra population as a recursive formula.
Use your formula to calculate the zebra population after 10 years. (5)

(e) How many of the zebra should be sold each year if it is decided to maintain a stable population of 63 at the reserve? (4)

[21]

QUESTION 6

A farmer is concerned that the number of lynxes (L) will destroy the population of rabbits (R) on his 20-acre farm.

The farm can hold up to 30 rabbits per acre and there are currently 500 rabbits on the farm. There are currently 35 lynxes on the farm.

The population indicators of the rabbits on the farms are:

Rabbits have an average of 7 kits per litter and only one litter per year.
Assume that 65% of the rabbit population is female and that 70% of the kits survive to maturity.

The population indicators for the lynxes on the farm are as follows:

Lynxes live for an average of 5 years. Each lynx kills about 1 rabbit per week.
A female lynx has an average of 3 kittens per year.
On average, 54% of lynx populations are female.

$$L_{n+1} = L_n + f.b.R_n.L_n - c.L_n$$
$$R_{n+1} = R_n + a.R_n.\left(1 - \frac{R_n}{K}\right) - b.R_n.L_n$$

6.1 Using a population cycle of years, determine:

(a) the value of K. (2)

(b) the value of c. (2)

(c) how many rabbits were killed by lynxes in the first population cycle. (2)

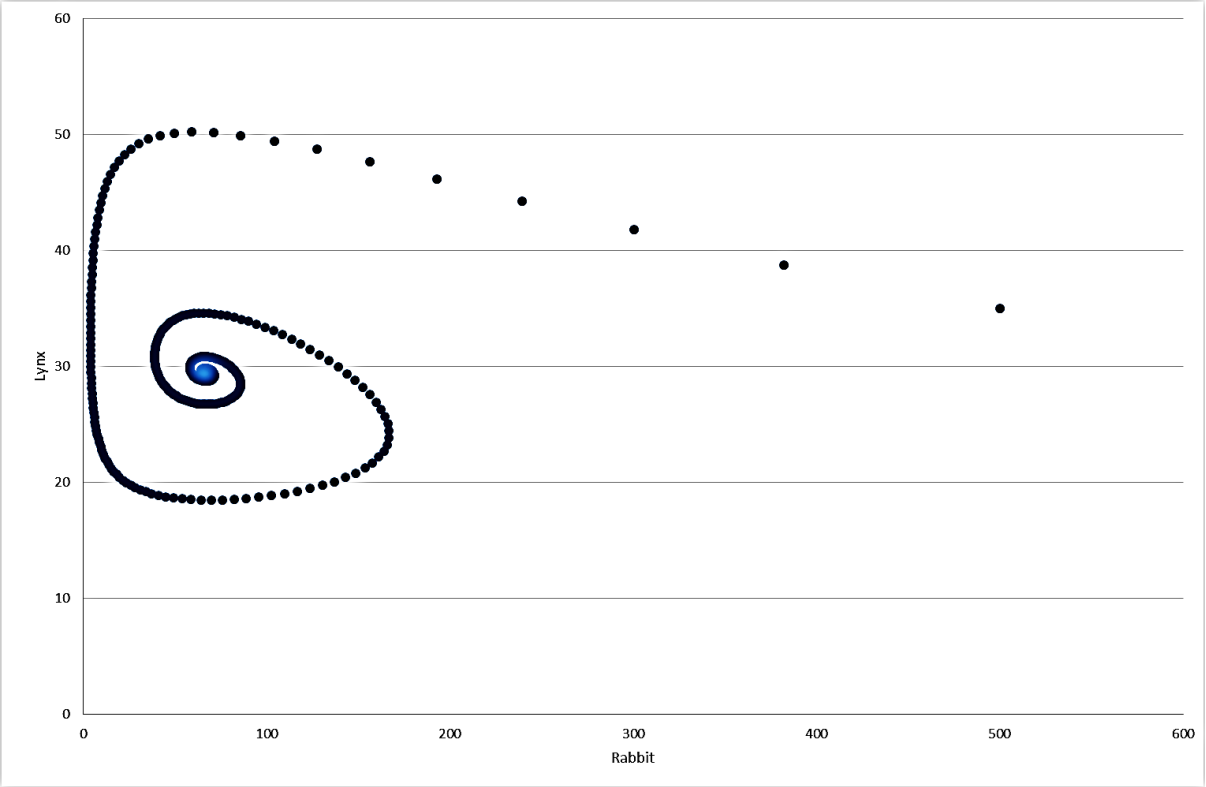
(d) the value of b. (2)

(e) the number of lynx kittens born in the first population cycle. (2)

(f) the value of f, correct to 4 decimal digits. (2)

(g) the value of a. (2)

6.2 Given below is a phase plot for the lynx and rabbit populations, in annual cycles.



Use the phase plot above to answer the following questions:

(a) Determine the maximum and minimum lynx population. (2)

(b) During which years were the most rapid changes in the lynx and rabbit populations? Justify your answer by referring to the phase plot. (2)

(c) Will the lynxes drive the rabbits to extinction? Justify your answer by referring to the equilibrium values of both populations. (3)

[21]

Total for Module 3: 100 marks