QUESTION

1.1. Let X = # new balls (12 new, 10 used, choose 7)

$P(X \geq 2)$ m

$= 1 - (P(X=0) + P(X=1))$

$= 1 - \left(\frac{\binom{12}{0}\binom{10}{7} + \binom{12}{1}\binom{10}{6}}{\binom{22}{7}} \right)$ √a √a √a m using combination

$= 0,9845$ √a

(6)

1.2. X = # adults unemployed

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$ √m

$n = 20; \quad p = 0,29$

$P(X<2) = P(X=0) + P(X=1)$ √m

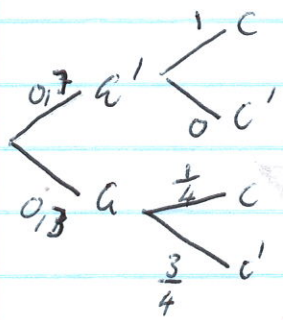
$= \binom{20}{0}(0,29)^0(1-0,29)^{20}$ √a

$+ \binom{20}{1}(0,29)^1(1-0,29)^{19}$ √a

$= 0,9716$ √a

(6)

1.3.



(a) $P(C) = (0,7 \times 1) + (0,3 \times 0,25)$ √a √m √a

$= \frac{31}{40}$

$= 0,775.$ √a

(4)

(b) $P(a/C) = \frac{P(a \cap C)}{P(C)}$ √m

$= \frac{0,3 \times 0,25}{0,775}$ √a √a

$= \frac{3}{31}$

$= 0,0968$ √a

(4)

[20]

QUESTION 2

2.1 $P(X=x) = \frac{1}{24}(x+6)$; $x \in \{1;2;3\}$

(a) $P(X=1) = \frac{1}{24}(1+6) = \frac{7}{24}$

$P(X=2) = \frac{1}{24}(2+6) = \frac{8}{24} = \frac{1}{3}$

$P(X=3) = \frac{1}{24}(3+6) = \frac{9}{24} = \frac{3}{8}$

x	1	2	3
$P(X=x)$	$\frac{7}{24}$	$\frac{8}{24}$ or $\frac{1}{3}$	$\frac{9}{24}$ or $\frac{3}{8}$
	✓a	✓a	✓a

(3)

(b) $x \in \{1;2;3\}$ ∴ discrete.

$P(X=x) \geq 0$ for all x ✓a

$\sum_{x=1}^{3} P(x=x) = \frac{7}{24} + \frac{8}{24} + \frac{9}{24}$

$= 1$ ✓a

∴ $P(X=x)$ is a PMF ✓a (3)

(c) $E(X) = \sum x \cdot P(X=x)$ ✓m ✓a

$= (1 \times \frac{7}{24}) + (2 \times \frac{1}{3}) + (3 \times \frac{3}{8})$

$= \frac{25}{12} = 2,0833$ ✓a

(3)

(2)

2.2 $f(x) = \begin{cases} kx^{-2} & ; 20 < x < 29 \\ 0 & ; \text{otherwise} \end{cases}$

(a) $\int_{20}^{28} kx^{-2}\,dx = 1$ ✓a ✓m

∴ $k \int_{20}^{28} x^{-2} = 1$

∴ $k \left[-x^{-1}\right]_{20}^{28} = 1$ ✓a ✓m ✓a

∴ $-k \left[\frac{1}{x}\right]_{20}^{28} = 1$

∴ $-k \left[\frac{1}{28} - \frac{1}{20}\right] = 1$ ✓a ✓m (6)

∴ $k = \frac{-1}{\left[\frac{1}{28} - \frac{1}{20}\right]}$

∴ $k = 70$

(b) $\int_{20}^{m} 70x^{-2}\,dx = 0,5$ ✓a ✓a

∴ $70 \int_{20}^{m} x^{-2}\,dx = 0,5$

∴ $70 \left[-x^{-1}\right]_{20}^{m} = 0,5$ ✓a

∴ $-70 \left[\frac{1}{x}\right]_{20}^{m} = 0,5$

∴ $\frac{1}{m} - \frac{1}{20} = -\frac{1}{140}$

∴ $\frac{1}{m} = \frac{1}{20} - \frac{1}{140}$

∴ $m = \frac{70}{3} = 23,33$ km/h. ✓a

(6)

[21]

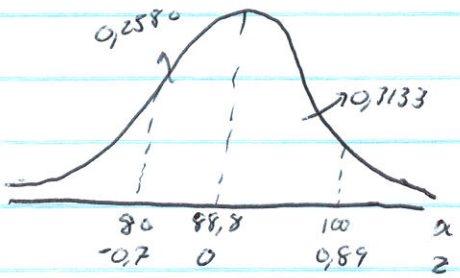
QUESTION 3

3.1 $\mu = 88,8\ kg$

$\sigma = 12,6\ kg$

Sample of 120

(a)



$x = 80$: | $x = 100$

$z = \frac{x-\mu}{\sigma}$ | $z = \frac{x-\mu}{\sigma}$

$\therefore z = \frac{80-88,8}{12,6}$ ✓a | $\therefore z = \frac{100-88,8}{12,6}$ ✓a

$z = -0,70$ ✓a | $z = 0,89$ ✓a

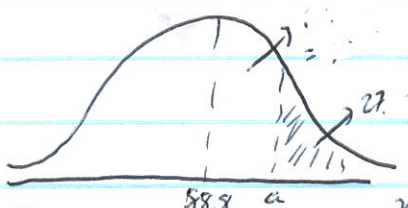
$H(0,70) = 0,2580$ ✓a | $H(0,89) = 0,3133$ ✓a

$\therefore P(80 < X < 100)$

$= 0,2580 + 0,3133 = 0,5713$ ✓a

$\#men = 0,5713 \times 120 = 68,556 \approx 69\ men$ ✓ca (8)

(b)



$P(X > a) = 0,27$

$H(a) = 0,23$ ✓a

$\therefore z = a = 0,61$ ✓a | $z = \frac{x-\mu}{\sigma}$

$\therefore z\sigma + \mu = x$ ✓m

$\therefore (0,61)(12,6) + 88,8 = x$ ✓a

$\therefore x = 96,49\ kg$ ✓a (6)

(3)

3.2

Let X = # students fail physics

$p = 0,38$

$n = 800$

(a) $np = 800(0,38) = 304 > 5$ ✓a

and

$n(1-p) = 800(1-0,38) = 496 > 5$ ✓a

$\therefore$ can use normal approximation ✓a (3)

(b) $\mu = np = 800(0,38) = 304$ ✓a

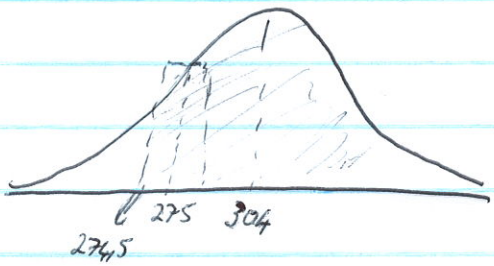
$\sigma^2 = np(1-p)$

$= 304(1-0,38)$

$= 188,48$

$\therefore \sigma = \sqrt{188,48} \approx 13,7288$ ✓a (2)

(c) $P(X \geq 275) = ?$



$z = \frac{x-\mu}{\sigma}$

$\therefore z = \frac{274,5-304}{\sqrt{188,48}}$ a ✓a ✓ca

$\therefore z = -2,15$ ✓a

$H(2,15) = 0,4842$ ✓a

$\therefore P(X \geq 274,5) = 0,5 + 0,4842$

$= 0,9842$ ✓ca (6)

[25]

④

QUESTION 4

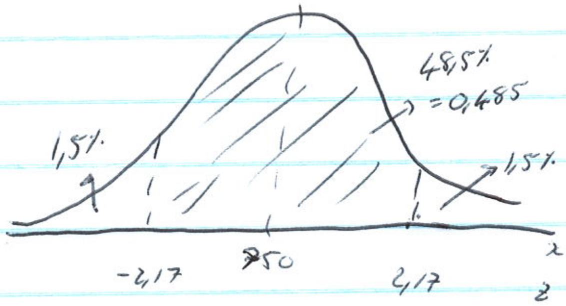
4.1. Sample: $\bar{x} = 750$

$n = 80$

Pop: $\sigma^2 = 20\,ml^2$

$\therefore \sigma = \sqrt{20} = 2\sqrt{5} \approx 4,4721$

$\mu = ? \Rightarrow 97\%$ C.I.



$H(Z) = 0,485$ √a

97% CI $\Rightarrow z = \pm 2,17$ √a

$$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$$

$$\mu = 750 \pm \frac{(2,17)(\sqrt{20})}{\sqrt{80}}$$ √m √a

$\mu = (748,915;\ 751,085)$

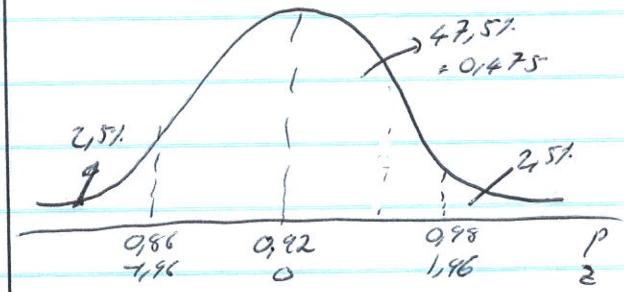
$(748,92;\ 751,09)$ √a √a

(6)

4.2. (a) $\hat{p} = \frac{0,86 + 0,98}{2}$ √m

$\therefore \hat{p} = 0,92$ √a (2)

(b) 95% C.I



$n = ?$

$H(z) = 0,475 \Rightarrow z = 1,96$ √a √a

$$p = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$ √m

Using upper boundary:

$$p = \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0,98 = 0,92 + 1,96\sqrt{\frac{(0,92)(1-0,92)}{n}}$$ √a √m √a

$n = 78,53$ √a

$\therefore$ need 79 people. √ca

OR Using lower boundary:

$$0,86 = 0,92 - 1,96\sqrt{\frac{(0,92)(0,08)}{n}}$$ √a √m √a

$n = 78,53$ √a

$\therefore$ need 79 people. √a (8)

[16]

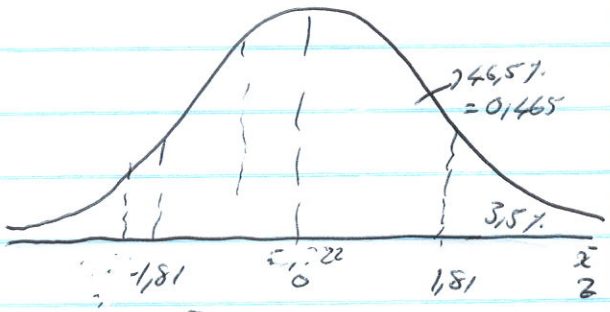
QUESTION 5

$\mu = 22$

$\sigma = \sqrt{0,64} = 0,8$ √a

$n = 28$

$\bar{x} = 21,7$



$H_0: \mu = 22$
$H_1: \mu \neq 22$
} Two tail test a√√a

7% significance $\Rightarrow z^{crit} = \pm 1,81$ a√√a

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ √m

$$z = \frac{21,7 - 22}{\frac{0,8}{\sqrt{28}}}$$ √a √a

$\therefore z = -1,98$ √a

∴ There is sufficient evidence to reject H_0 √ca
ie mean length of bolts has changed. [10]

(5)

QUESTION 6

(a) 26 25 24 23 22 1 1 1 1

c√√a a√

$\therefore 26^5 = 11881376$ (4)

(b) ~~26~~ not repeated

$\Rightarrow 26 \times 25 \times 24 \times 23 \times 22 \times 1 \times 1 \times 1 \times 1$

$= 7\,893\,600$ i√√a

∴ Repeated = 11881376 − 7893600 √m

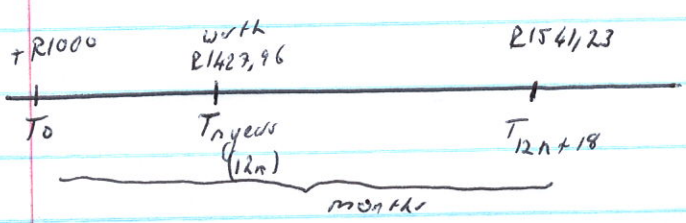
$= 3\,987\,776$ √a (4)

[8]

Total Module 2 : 100

FINANCE AND MODELLING

QUESTION 1



1.1 $A = P(1+i)^n$ ✓a

$\therefore 1541,23 = 1427,96\left(1+\frac{i}{12}\right)^{18}$ ✓a ✓m ✓a

$\therefore i = 5,1\%$ p.a ✓a ✓a compound monthly (6)

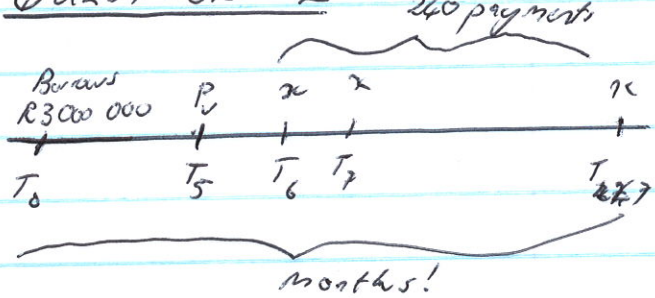
1.2 $A = P(1+i)^n$

$1427,96 = 1000\left(1+\frac{5,1\%}{12}\right)^{12n}$ ✓a ✓a ✓m ✓a

$\therefore n = 7$ years ✓a ✓a (6)

[12]

QUESTION 2



$i^{(4)} = 9\%$ } ✓m
$i^{(12)} = ?$ }

$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{i^{(4)}}{4}\right)^{4}$ ✓m

$\therefore \left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{9\%}{4}\right)^{4}$ ✓a

$\therefore \frac{i^{(12)}}{12} = \left(1+\frac{9\%}{4}\right)^{\frac{4}{12}} - 1$ } ✓m

$\therefore i^{(12)} = \left[\left(1+\frac{9\%}{4}\right)^{\frac{1}{3}} - 1\right](12)$ }

$\therefore i^{(12)} = 8,93\%$ ✓a

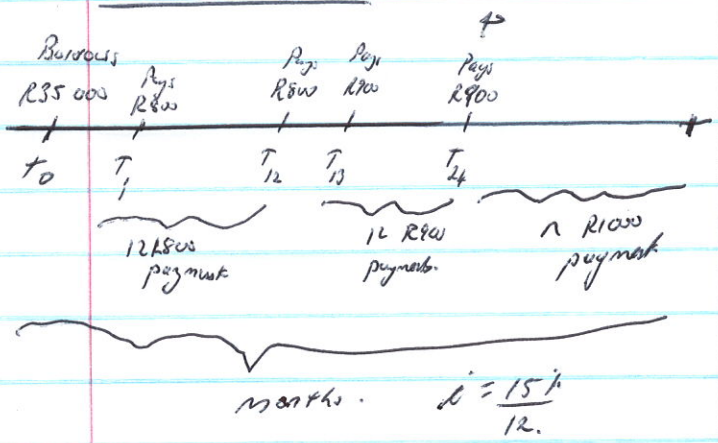
$P = \frac{x\left(1-(1+i)^{-n}\right)}{i}$

$\therefore 3000\,000\left(1+\frac{8,93\%}{12}\right)^{5} = \frac{x\left(1-\left(1+\frac{8,93\%}{12}\right)^{-24}\right)}{\frac{8,93\%}{12}}$ ✓a ✓a ✓a ✓m ✓a

$\therefore x = R27\,878,19$ ✓a

[12]

QUESTION 3



3.1 Balance outstanding after 2 years:

B_0 = Amount Borrowed with interest √√m
− Amount paid back with interest

$$B_0 = 35\,000\left(1+\frac{15\%}{12}\right)^{24} \quad \text{√a √a}$$

$$- \underset{15\%/12}{800\left(\left(1+\frac{15\%}{12}\right)^{12}-1\right)}\left(1+\frac{15\%}{12}\right)^{12} \quad \text{√a √a √m}$$

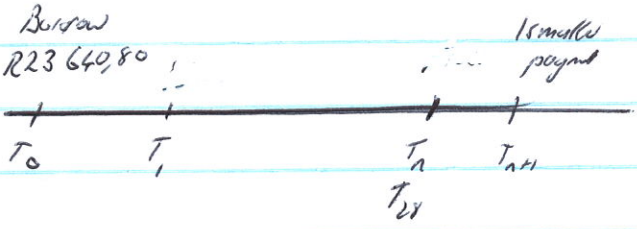
$$- \underset{\frac{15\%}{12}}{900\left(\left(1+\frac{15\%}{12}\right)^{12}-1\right)} \quad \text{√a √a}$$

$$= 47157,287 - 11942,178 - 11574,325$$

$$= \underline{R23\,640,78} \quad \text{√a} \qquad (12)$$

(7)

3.2



At T_n:

$$P = \frac{x\left(1-(1+i)^{-n}\right)}{i} \quad \text{√m}$$

$$23\,640,80 = \underset{\frac{15\%}{12}}{1000\left(1-\left(1+\frac{15\%}{12}\right)^{-n}\right)} \quad \text{√a √a √a}$$

$\therefore n = 28,19\ldots$ √a

∴ 28 payments of R1000 and one smaller one. √a (6)

At T_{28}:

3.3 $B_0 = BWI - PBWI$ √m

$$B_0 = 23\,640,80\left(1+\frac{15\%}{12}\right)^{28} \quad \text{√a √a}$$

$$- \underset{\frac{15\%}{12}}{1000\left(\left(1+\frac{15\%}{12}\right)^{28}-1\right)} \quad \text{√a √a}$$

* (Alt method)

$$= \underline{R195,81} \quad \text{√a}$$

$$\therefore \text{Final payment} = 195,81\left(1+\frac{15\%}{12}\right)^{1} \quad \text{√a}$$

$$= \underline{2198,25} \quad \text{√a}$$

(8)

* Alt method

[26]

QUESTION 4

4.1 $A = P(1+i)^n$

$85 = 12(1+i)^3$ a/√m

$\therefore i = 92,805\%$ √a√a (4)

4.2 $P_{n+1} = P_n(1+r)$;

$P_{n+1} = P_n(1+0,9205)$; $P_0 = 12$

$P_{n+1} = P_n(1,9205)$; $P_0 = 12$ √a √a

$P_1 = 23,046$ √a

$P_8 = 2220,72$

$\approx$ 2221 cells after 8 hours. √a (4)

[8]

QUESTION 5

5.1 $A = \frac{9}{31} = 0,290323$ √a

$B = \frac{59-43}{2} = 8$ √a

$C = \frac{4}{56} = 0,142857$ √a

$\frac{D-56}{2} = 2$ √a ; $D = 60$

(4)

5.2.(a) $\frac{\Delta P}{P_n} = 0,598853 - 0,00866P_n$ √a √a √a substitution

(4)

(b) $r = 59,89\%$ √a √a (2)

(c) $0 = 0,598853 - 0,00866P_n$ √m

$\therefore k = \frac{0,598853}{0,00866}$

$k = 69,19$

$\therefore k = 69$ √a (2)

(d) $P_{n+1} = P_n + rP_n\left(1 - \frac{P_n}{k}\right)$

$P_{n+1} = P_n + 0,5989P_n\left(1 - \frac{P_n}{69}\right)$ a/√m ;

$P_0 = 25$ √a

$P_1 = 34,54$

$P_{10} = 68,95$ √a

$\therefore 69$ zebra after 10 years. √a (5)

(e) $P_{n+1} = P_n = 63$

$\therefore 63 = 63 + 0,8989(63)\left(1 - \frac{63}{69}\right) - x$ m/a

$\therefore x = 3,28$ √a

$\therefore x = 3$ √a

3 sold each year. (4)

[21]

QUESTION 6

6.1 (a) $k = 30 \times 20$ ✓a
$\therefore k = 600$ rabbits ✓a (2)

(b) $c = \frac{1}{5}$ ✓a
$c = 0,2$ ✓a (2)

(c) $35 \times 1 \times 52$ ✓a $= 1820$ rabbits ✓a (2)

(d) $1820 = 6 R_0 L_0$
$\therefore 1820 = 6(500)(35)$ ✓a
$\therefore 6 = \frac{1820}{(500)(35)}$
$\therefore 6 = 0,104$ ✓a (2)

(e) $35 \times 54\% \times 3$ ✓a
$= 56,7 = 57$ ✓a (2)

(f) $f.6.R_0.L_0 = 56,7$
$f = \frac{56,7 \text{ or } (57)}{(0,104)(500)(35)}$ ✓a
$\therefore f = 0,0312$ ✓a or $0,0313$ ✓a (2)

(g) $a = 7 \times 1 \times 65\% \times 70\%$ ✓a
$\therefore a = 3,185 = 3,19$ ✓a (2)

6.2 (a) max = 50 lynx ✓a
min = 18 or 19 ✓a lynx (2)

(b) During the first 10 ✓a (or 9 or 11) years.
Large gaps ✓a between each population cycle. (2)

(c) no. ✓a
Population of lynx stabilise at about (28/29) ✓a while population of rabbits stabilise about (60 to 70) ✓a (3)

[21]

Total mark 2 = 100 marks.