MEMO – AP MATHS – 2019

PAPER 1: – PRELIM

QUESTION 1

a) $\dfrac{3a-5i}{1+2i} = 1+13bi$

$\dfrac{3a-5i}{1+2i} \times \dfrac{1-2i}{1-2i} = 1+13bi$ ✓

$\dfrac{3a-6ai-5i+10i^2}{1-4i^2} = 1+13bi$ ✓

$\dfrac{3a-10-i(6a+5)}{5} = 1+13bi$ ✓

$\dfrac{3a-10}{5} = 1$ ✓

$a = 5$ ✓

$\dfrac{-(6(5)+5)}{5} = 13b$ ✓

$-\frac{7}{13} = b$ ✓

7

or $3a-5i = (1+13bi)(1+2i)$

$3a-5i = 1+2i+13bi-26b$

$-5 = 2+13b$

$-\frac{7}{13} = b$

$3a = 1-26(-\frac{7}{13})$

$a = 5$

b) $\dfrac{1-2x}{x+8} < e^x$

$x \neq -8$

$x < -8$ ✓✓

$1-2x = e^x(x+8)$ ✓

$x = -0{,}916$

$x > -0{,}916$ ✓✓

5

c) $\log_a\left(\dfrac{6x^2+9x+2}{x}\right) = \log_a 16$ ✓✓

$6x^2+9x+2 = 16x$ ✓

$6x^2-7x+2 = 0$ ✓

$x = \frac{2}{3}$ ✓ $x = \frac{1}{2}$ ✓

6

d) $\displaystyle\lim_{x \to 3/2} \frac{2x^2-3x}{|2x-3|}$

$\displaystyle\lim_{x \to 3/2} \frac{x\cancel{(2x-3)}}{\cancel{2x-3}} = \frac{3}{2}$ ✓✓

$\displaystyle\lim_{x \to 3/2} \frac{-x\cancel{(2x-3)}}{\cancel{2x-3}} = -\frac{3}{2}$ ✓✓

$\therefore \frac{3}{2} \neq -\frac{3}{2}$

DNE ✓

5

QUESTION 2:

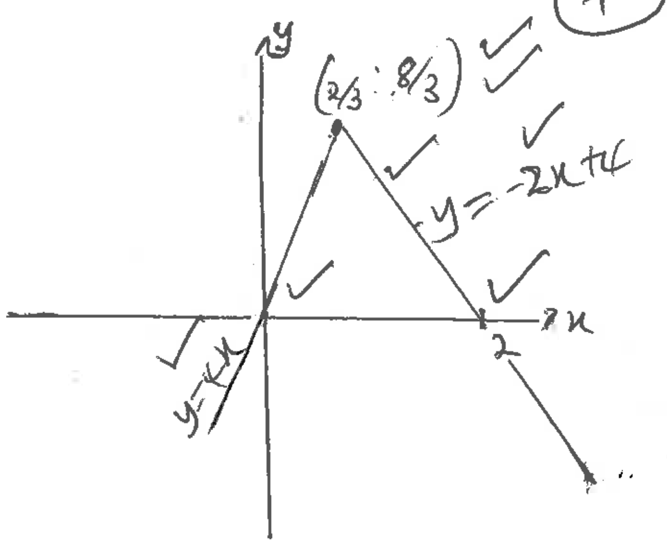
a) $f(x) = -\frac{5}{3}|x+3| + \frac{4x}{3} + 6$ (5)

b) $f(x) = -|3x-2| + x + 2$

$3x - 2 = 0$

$x = 2/3 \quad y = 8/3$

(7)



QUESTION 3

a) $x > 7/3$ (2)

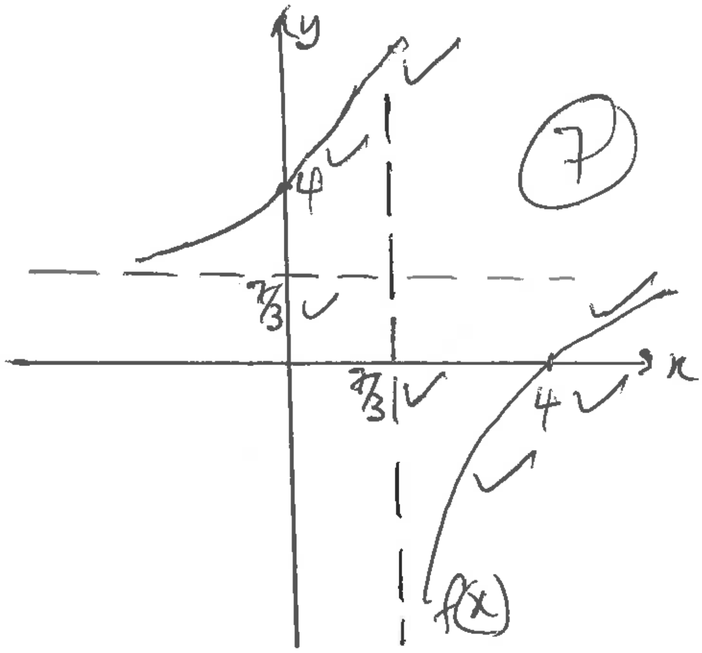
b) $0 = \ln\left(\frac{3x-7}{x+1}\right)$

$1 = \frac{3x-7}{x+1}$

$x+1 = 3x-7$

$4 = x$

(4)



(7)

d) $x = \ln\left(\frac{3y-7}{y+1}\right)$

$e^x y + e^x = 3y - 7$

$e^x + 7 = y(3 - e^x)$

$\frac{e^x+7}{3-e^x} = f^{-1}(x)$

(4)

e) $f(x) = \ln(3x-7) - \ln(x+1)$

$f'(x) = \frac{3}{3x-7} - \frac{1}{x+1}$

$= \frac{3x+3-3x+7}{(x+1)(3x-7)}$

$= \frac{10}{(x+1)(3x-7)}$

(6)

QUESTION 4:

$$\sum_{r=1}^{n} r(r+1)(2r+1) = \frac{1}{2}n(n+2)(n+1)^2$$

Prove true for $n=1$

$LHS = 1(2)(3) = 6$

$RHS = \frac{1}{2}(3)(4) = 6$

$\therefore LHS = RHS$

Assume true for $n=k$

$6 + \ldots + k(k+1)(2k+1) = \frac{1}{2}k(k+2)(k+1)^2$

Prove true for $n=k+1$

$LHS = 6 + \ldots + k(k+1)(2k+1) + (k+1)(k+2)(2k+3)$

$= \frac{1}{2}k(k+2)(k+1)^2 + (k+1)(k+2)(2k+3)$

$= (k+1)(k+2)[\frac{1}{2}k(k+1) + 2k+3)$

$= (k+1)(k+2)(\frac{1}{2}k^2 + \frac{1}{2}k + 2k + 3)$

$= (k+1)(k+2)(\frac{1}{2}k^2 + \frac{5}{2}k + 3)$ 12

$= \frac{1}{2}(k+1)(k+2)(k^2 + 5k + 6)$

$= \frac{1}{2}(k+1)(k+2)(k+2)(k+3)$

By PMI, it is true for all $n \in N$.

QUESTION 5

a 1. $\lim_{x \to -4}(2x^2+3x) = \lim_{x \to -4}(ax+6) = f(-4)$

$20 = -4a + 6$

$\lim_{x \to 3}(3a+b) = \lim_{x \to 3} 4 = f(3)$

$3a + 6 = 4$

$16 = -7a$ 8

$\frac{16}{-7} = a$

$b = 4 - 3(\frac{-16}{7})$

$b = 76/7$

2. $\lim_{x \to 3}(-16/7) = \lim_{x \to 3}(-3x^2 + 8x)$

$-16/7 \neq -3$ 4

$\therefore$ Not differentiable

b. $0 = -\frac{2-p}{(p+1)^2} + k + \frac{1}{p+1}$

$-(2-p) + k(p+1)^2 + p+1 = 0$

$-2 + p + kp^2 + 2pk + 1 + p + 1 = 0$

$kp^2 + (2+2k)p + (k-1) = 0$ 7

$\therefore b^2 - 4ac$

$= (2k+2)^2 - 4k(k-1)$

$= 12k + 4 > 0$ for all $k > 0$

$\therefore$ Hence 2 distinct values of p implies 2 tangents at p.

QUESTION 6

a) $\tan 30 = \frac{BM}{4}$

$4\tan(\pi/6) = BM$

$\frac{4\sqrt{3}}{3} = BM$ (5)

$\therefore \text{Area} = \frac{1}{2}(8)\left(\frac{4\sqrt{3}}{3}\right)$

$= \frac{16\sqrt{3}}{3}$

b) Area of shaded

$= \text{area of } \triangle - 2(\text{area of sector})$

$= \frac{16\sqrt{3}}{3} - 2\left(\frac{1}{2}(4)^2 \cdot \frac{\pi}{6}\right)$

$= \frac{16\sqrt{3}}{3} - \frac{8\pi}{3}$ (5)

$= \frac{8}{3}\left(2\sqrt{3} - \pi\right)$

QUESTION 7:

a1. $y = \tan\left(e^{3x-2}\right)$

$f'(x) = \sec^2\left(e^{3x-2}\right) \cdot e^{3x-2} \cdot 3$ (4)

2. $y = x^2 \ln(1-x^2)$

$f'(x) = x^2 \cdot \frac{1}{1-x^2} \cdot -2x + 2x\ln(1-x^2)$ (6)

$= \frac{-2x^3}{1-x^2} + 2x\ln(1-x^2)$

b) $y^2x^2 + y + \cos(xy) = 0$

$y^2 \cdot 2x + x^2 \cdot 2y\frac{dy}{dx} + \frac{dy}{dx}$ (12)

$-\sin(xy)\left[x\frac{dy}{dx} + y\right] = 0$

$\frac{dy}{dx}\left[2yx^2 + 1 - x\sin(xy)\right] = y\sin(xy) - 2xy^2$

$\frac{dy}{dx} = \frac{y\sin(xy) - 2xy^2}{2x^2y + 1 - x\sin(xy)}$

QUESTION 8

a) $VA: x = -\frac{1}{2}$

$\frac{1}{2}\left(x+\frac{1}{2}\right)(x-2) + R = x^2 - \frac{3}{2}x$

$\therefore OA: y = \frac{1}{2}x - 1$ (6)

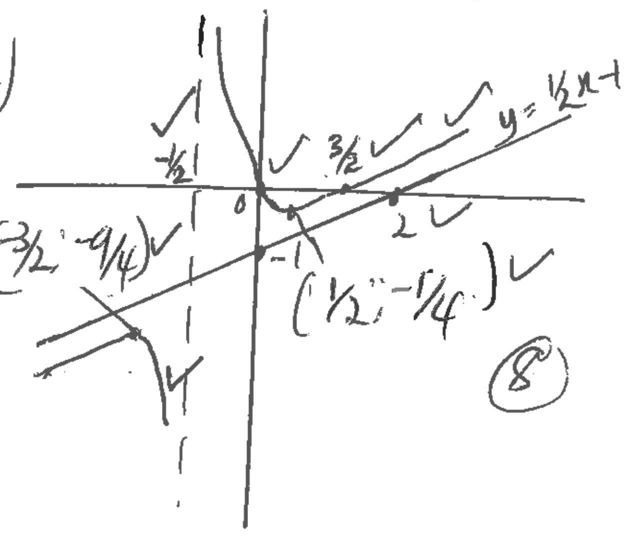
b) $f'(x) = \frac{(2x+1)\left(2x-\frac{3}{2}\right) - 2\left(x^2 - \frac{3}{2}x\right)}{(2x+1)^2} = 0$

$2x^2 + 2x - \frac{3}{2} = 0$ (9)

$x = \frac{1}{2}$ or $x = -\frac{3}{2}$

$y = -\frac{1}{4}$ $\quad$ $y = -\frac{9}{4}$

c)



(8)

QUESTION 4

a) $\int_0^3 e^{|x-2|}\,dx$

$= \int_0^2 e^{-x+2}\,dx + \int_2^3 e^{x-2}\,dx$

$= -e^{-x+2}\Big|_0^2 + e^{x-2}\Big|_2^3$

$= (-1+e^2) + (e^1-1)$

$= \underline{e^2+e-2}$ (8, 10)

(6)

b) $\int \frac{\sin x}{(2+3\cos x)^2}\,dx$

let $u = 2+3\cos x$

$du = -3\sin x\,dx$

$= \int u^{-2} \cdot \frac{du}{-3}$

$= \frac{1}{3}u^{-1} + C$

$= \frac{1}{3}(2+3\cos x)^{-1} + C$

(7)

c) $\int (x+1)\ln 3x\,dx$

$f(x) = \ln 3x \quad g'(x) = x+1$ (9)

$f'(x) = \frac{1}{x} \quad g(x) = \frac{(x+1)^2}{2}$

$= \frac{(x+1)^2}{2}\ln 3x - \int \frac{(x+1)^2}{2}\cdot\frac{1}{x}\,dx$

$= \frac{(x+1)^2}{2}\ln|3x| - \frac{1}{2}\int\left(x+2+\frac{1}{x}\right)dx$

$= \frac{(x+1)^2}{2}\ln 3x - \frac{1}{2}\left(\frac{x^2}{2}+2x+\ln|x|\right) + C$

d) $\int \frac{x^2+12x-5}{(x+1)^2(x-7)}\,dx$

$x^2+12x-5 = A(x+1)(x-7) + B(x-7) + C(x+1)^2$

$x=-1: \; -16 = -8B$

$\underline{2 = B}$

subs $x=7 \quad 128 = 64C$

$\underline{2 = C}$

$x=0: \; -5 = -7A - 7(2) + 2$ (9)

$\underline{A = -1}$

$\therefore \int \frac{-1}{x+1}\,dx + \int \frac{2}{(x+1)^2}\,dx + \int \frac{2}{x-7}\,dx$

$= -\ln|x+1| + 2\ln|x-7| - 2(x+1)^{-1} + C$

OR $\left(\frac{x^2}{2}+x\right)\ln 3x - \left(\frac{x^2}{4}+x\right) + C$

if $g(x) = \frac{x^2}{2} + x$

QUESTION 10:

a) $\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$

$\underline{x = 2}$ ✓ (2)

b) $\text{Area} = \int_0^2 \left(\frac{1}{4}x^2 - \frac{1}{2}x + 3\right) dx$

$- \int_0^2 \left(\frac{3}{8}x^2 - \frac{9}{4}x + 3\right) dx +$

$\int_2^4 \left(-\frac{1}{8}x^2 - \frac{3}{4}x + 5\right) dx$

$= \int_0^2 \left(-\frac{x^2}{8} + \frac{7}{4}x\right) dx + \left.\frac{-x^3}{24} - \frac{3x^2}{8} + 5x\right|_2^4$ ✓✓✓✓

$= \left.\frac{-x^3}{24} + \frac{7x^2}{8}\right|_0^2 + \left.\frac{-x^3}{24} - \frac{3x^2}{8} + 5x\right|_2^4$ ✓✓

$= \frac{19}{6} + \left(\frac{34}{3} - \frac{49}{6}\right)$ (7)

$= \frac{19}{3}$ ✓

or $2\int_0^2 \left(-\frac{x^2}{8} + \frac{7}{4}x\right) dx = \frac{19}{3}$

c) $V = \pi\int_0^2 (f(x))^2 - \pi\int_0^2 (h(x))^2$ ✓✓

$+ \left(\pi\int_2^4 (g(x))^2 - \int_2^4 (k(x))^2 dx\right.$ ✓✓

$= \pi\left(\frac{241}{15} - \frac{93}{20}\right) \times 2$ ✓✓ (8)

$= \frac{137\pi}{6}$ ✓✓ $71,7$

QUESTION 11

a) $\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}h\right)^2 = 25^2$ ✓

$\frac{x^2}{4} + \frac{h^2}{4} = 625$ ✓

$x^2 + h^2 = 2500$ (2)

$h = \sqrt{2500 - x^2}$

b) $\frac{dx}{dt} = -0,3$ ✓

$\frac{dh}{dx} = -2x \cdot \frac{1}{2} \cdot (2500 - x^2)^{-\frac{1}{2}}$ ✓✓

$= -x\,(2500 - x^2)^{-\frac{1}{2}}$

$\frac{dh}{dt} = \frac{dh}{dx} \times \frac{dx}{dt}$

$= \frac{0,3x}{(2500 - x^2)^{\frac{1}{2}}}$ ✓✓ (6)

$= \frac{0,3 \times 30}{(2500 - 900)^{\frac{1}{2}}}$

$= \frac{9}{40}$ ✓

MEMO – PRELIM : 2019

STATS AP PAPER

QUESTION 1

a) $P(X > 170)$

$= P\left(Z > \frac{170-160}{8}\right)$

$= 0{,}5 - 0{,}4944$

$= \underline{0{,}1056}$ (3)

b) 1. $P(X > 180) = P\left(Z > \frac{180-160}{8}\right)$

$= 0{,}5 - 0{,}4938$

$= \underline{0{,}0062}$ (5)

$\therefore P(X > 180 / X > 170) = \frac{0{,}0062}{0{,}1056}$

$= \underline{0{,}0587}$

2. $P(X > x / X > 170) = 0{,}5$

$\frac{P(X > x)}{P(X > 170)} = 0{,}5$

$P(X > x) = 0{,}5 \times 0{,}1056$

$P\left(Z > \frac{x-160}{8}\right) = 0{,}0528$

$H(0{,}4472) = 1{,}62$ (5)

$\therefore \frac{x-160}{8} = 1{,}62$

$\underline{x = 172{,}96 = 173}$

QUESTION 2:

a) $\int_0^1 kx^n \, dx = 1$

$\left.\frac{kx^{n+1}}{n+1}\right|_0^1 = 1$ (3)

$\underline{k = n+1}$

b) $E(X) = \int_0^1 k\, x^{n+1} \, dx$

$= \left.\frac{k\, x^{n+2}}{n+2}\right|_0^1$

$= \frac{k}{n+2} = \underline{\frac{n+1}{n+2}}$

$E(X^2) = \int_0^1 kx^{n+2} \, dx$

$= \left.\frac{k\, x^{n+3}}{n+3}\right|_0^1$ (6)

$= \underline{\frac{n+1}{n+3}}$

c) $Var(3X)$ if $n = 2$

$= 9\,Var(X)$

$= 9\left[3/5 - (3/4)^2\right]$ (4)

$= \underline{27/80}$

QUESTION 3:

a) $3^{10} = 59049$ (2)

b) 1. ${}^{10}C_4 \cdot {}^{6}C_3 \cdot {}^{3}C_3 = 4200$

or $\dfrac{10!}{4!\,3!\,3!} = 4200$ (4)

2. $\dfrac{{}^{4}C_2\,{}^{6}C_1 + {}^{4}C_3\,{}^{6}C_0}{{}^{10}C_3}$ (5)

$= \dfrac{40}{120} = \dfrac{1}{3}$

QUESTION 4:

a) $\bar{x} = 0,488$

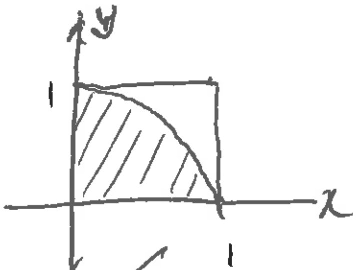
$0,488 + 1,65 \cdot \dfrac{\sigma}{10} = 0,5127$

$\sigma = 0,1497$

$0,479 \pm 1,96\,\dfrac{0,1497}{\sqrt{150}}$ (8)

$[0,455 ; 0,503]$

b) 1.



1. $\dfrac{\pi(1)^2}{4} \div 1$ (3)

$= \dfrac{\pi}{4}$

2. $p = \dfrac{784}{1000}$

$0,784 \pm 1,65\sqrt{\dfrac{0,784 \cdot 0,216}{1000}}$

$= [0,763 ; 0,805]$ (5)

3. $1,65\sqrt{\dfrac{0,784 \cdot 0,216}{n}} \leq 0,0025$

$n > 73766,24$ (5)

$n = 73767$

QUESTION 5

a) $n > 30$ ✓ (2)

p is close to $0,5$ ✓

b1. $p = \frac{3}{200}(90-40)$

$= \frac{3}{4}$ ✓

$P(X \leq 8) = 1 - \left({}^{10}C_9 \left(\frac{3}{4}\right)^9 \left(\frac{1}{4}\right)^1 + {}^{10}C_{10} \left(\frac{3}{4}\right)^{10} \left(\frac{1}{4}\right)^0\right)$ ✓✓✓

$= 0,7560$ ✓✓ (6)

2. $P(X \geq 1) = 0,8$

$1 - P(X=0) = 0,8$ ✓

$1 - {}^{20}C_0 \left(\frac{3}{200}(90-x)\right)^0 \left(1 - \frac{3}{200}(90-x)\right)^{20}$ ✓

$= 0,8$

$1 - \left(1 - \frac{27}{20} + \frac{3x}{200}\right)^{20} = 0,8$

$\left(1 - \frac{27}{20} + \frac{3x}{200}\right)^{20} = 0,2$ ✓

$\left(1 - \frac{27}{20} + \frac{3x}{200}\right) = 0,9227$ ✓

$\frac{3x}{200} = 0,9227 + \frac{7}{20}$

$x = 84,8$ ✓ (6)

$\therefore x = 85$ ✓

or $1 - P(0) = 0,8$

$P(0) = 0,2$

$(1-p)^{20} = 0,2$

$p = 0,0773$

$\frac{3}{200}(90-x) = 0,0773$

$x = 84,8$

$x = 85$

c) $X \sim N(84 ; 48,72)$ ✓✓

$P(25 \leq X \leq 75)$

$= P\left(\frac{24,5-84}{\sqrt{48,72}} \leq Z \leq \frac{75,5-84}{\sqrt{48,72}}\right)$ ✓✓

$= P(-8,52 \leq Z \leq -1,2178)$ ✓

$= 0,5 - 0,3888$ ✓ (7)

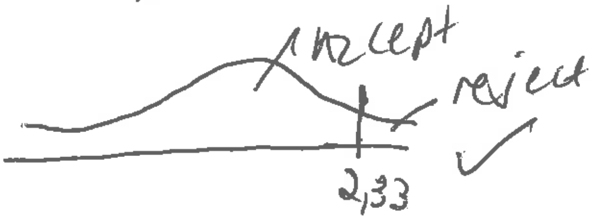
$= 0,1112$ ✓

QUESTION 6:

a) x = group y = on their own

$H_0: \mu_x = \mu_y + 1{,}5$

$H_1: \mu_x - \mu_y > 1{,}5$



$$z = \frac{8{,}7 - 6{,}6 - 1{,}5}{\sqrt{\frac{2{,}1^2}{80} + \frac{1{,}4^2}{65}}} = 2{,}0546$$

Accept H_0 at 1% and conclude that there is insufficient evidence that using plan as part of a group leads to weight loss of more than 1,5 kg than using plan on one's own.

"or researcher's belief not supported"

(8)

b). Since sample is large (CLT applies) no need to assume normal distribution (1)

QUESTION 7

a) $3C_1(0{,}6)^2(0{,}4)^1$

$= 0{,}432$ (3)

or $(0{,}6^2 \times 0{,}4)3 = 0{,}432$

b) $1 - (0{,}216 + 0{,}432 + 0{,}064)$ (1)

$= 0{,}288$

$3(0{,}6 \times 0{,}4^2) = 0{,}288$

c) $[30:0]$; $(0:30)$ or $(15:15)$

$= 0{,}216 \times 0{,}288 + 0{,}288 \times 0{,}216$

$+ 0{,}432 \times 0{,}432$ (4)

$= 0{,}311$

d) Let y be number of points scored in bonus round

y	60	35	10	−15
$P(Y=y)$	0,216	0,432	0,288	0,064

$E(Y) = 60 \times 0{,}216 + 35 \times 0{,}432$

$+ 10 \times 0{,}288 + -15 \times 0{,}064$

$= 30$ (4)