SAHETI SCHOOL

PRELIMINARY EXAMINATION 2019

GRADE 12 - ADVANCED PROGRAMME MATHEMATICS

Time: 1 hour   **Total: 100**

Examiner: P R Mhuka   Moderators: N Ferreira
                                                                                        E Zachariou

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 4 pages. Please check that your paper is complete.

2.   Read the questions carefully.

3.   Answer all the questions.

4.   Number your answers exactly as the questions are numbered.

5.   You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6.   **Answers must be rounded off to four decimal places.**

7.   All the necessary working details must be clearly shown.

8.   It is in your own interest to write legibly and to present your work neatly.

Page 1 of 4

QUESTION 1:

The heights of adult females are normally distributed with mean 160cm and standard deviation of 8cm.

a) Find the probability that a randomly selected adult female has a height greater than 170cm. (3)

b) An adult female whose height is greater than 170cm is defined as tall.

   1) An adult female is chosen at random. Given that she is tall, find the probability that she has a height greater than 180cm. (5)

   2) Half of tall adult females have a height greater than h, find the value of h. (5)

   [13]

QUESTION 2:

A continuous random variable X has a probability density function given by:

$$f(x) = \begin{cases} kx^n & 0 \leq x \leq 1 \\ \\ 0 & otherwise \end{cases}$$

where k and n are positive integers

a) Find k in terms of n (3)

b) Calculate $E(X)$ and $E(X^2)$ in terms of n. (6)

c) Given that $n = 2$, find $Var(3X)$ (4)

   [13]

QUESTION 3:

a) An art collector, who owns 10 paintings by famous artists, is preparing her will. In how many different ways can she leave these painting to her three sons? (2)

b) A supervisor at a production plant works with a team of 10 workers.

   1) He wants to form three production teams with four, three and three members respectively. In how many different ways can this be done? (4)

   2) Four of the ten workers are younger than 30 years. Calculate the probability of choosing a team of three members in such a way that at least two of the members are younger than 30 years. (5)

   [11]

QUESTION 4:

a) The weights of bags of rice, *X kg*, have a normal distribution with unknown mean $\mu\ kg$ and known standard deviation $\sigma\ kg$. A random sample of 100 bags of rice gave a 90% confidence interval for μ of $[0,4633;\ 0,5127]$.

A second random sample, of 150 of these bags of rice, had a mean weight of 0,479. Calculate a 95% confidence interval for μ based on this second sample. (8)

b) A point whose coordinates are $(x; y)$ with respect to rectangular axes is chosen at random where $0 < x < 1$ and $0 < y < 1$.

1) What is the probability that the point lies inside the circle whose equation is $x^2 + y^2 = 1$. (3)

In a computer simulation 1000 such points were generated and 784 of them lay inside the circle.

2) Obtain an estimate for the proportion and give an approximate 90% confidence interval for your estimate. (5)

3) Calculate the number of points need to be selected in order to be 90% certain of obtaining a value for the proportion which will be in error by less than 0,0025. (5)

[21]

QUESTION 5:

a) State the conditions under which the normal distribution may be used as an approximation to the binomial distribution. (2)

b) A cadet fires shots at a target at distances ranging from 25m to 90m. the probability of hitting the target with a single shot is p. When firing from a distance d m, $p = \frac{3}{200}(90 - d)$. Each short is fired independently.

1) The cadet fires 10 shots from a distance of 40 m. Calculate, the probability that at most 8 shots hit the target. (6)

2) The cadet fires 20 shots from a distance of x m. Find, to the nearest integer, the value of x if the cadet has an 80% chance of hitting the target at least once. (6)

c) A recent study found that cedar trees by indigenous settlements grow taller than cedar trees not by indigenous settlements. The probability of a cedar tree being over 90m tall by an indigenous settlement is 0.42. If we take a random sample of 200 cedar trees growing near indigenous settlements, what is the probability that between 25 and 75 trees (exclusive) will be over 90m tall? (7)

[21]

QUESTION 6:

A researcher believes that the mean weight loss of those people using a slimming plan as part of a group is more than 1,5 *kg* a year greater than the mean weight loss of those using the plan on their own. The mean weight loss of a random sample of 80 people using the plan as part of a group is 8,7 *kg* with a standard deviation of 2,1 *kg*. The mean weight loss of a random sample of 65 people using the plan on their own is 6,6 *kg* with a standard deviation of 1,4 *kg*.

a) Stating your hypotheses clearly, test the researcher's claim. Use a 1% level of significance. (8)

b) For the test in part (a), state whether or not it is necessary to assume that the weight loss of a person using this plan has a normal distribution. Give a reason for your answer (1)

[9]

QUESTION 7:

In a quiz, a team gains 10 points for every question it answers correctly and loses 5 points for every question it does not answer correctly. The probability of answering a question correctly is 0,6 for each question. One round of the quiz consists of 3 questions.

The discrete random variable *X* represents the total number of points scored in one round. The table shows the incomplete probability distribution of *X*.

x	30	15	0	−15
$P(X=x)$	0.216			0.064

a) Show that the probability of scoring 15 points in a round is 0,432 (3)

b) Find the probability of scoring 0 points in a round. (1)

c) Find the probability of scoring a total of 30 points in 2 rounds. (4)

d) In a bonus round of 3 questions, a team gains 20 points for every question it answers correctly and loses 5 points for every question it does not answer correctly. Find the expected number of points scored in the bonus round. (4)

[12]