SAHETI SCHOOL

PRELIMINARY EXAMINATION 2019

GRADE 12 - ADVANCED PROGRAMME MATHEMATICS

Time: 2 hours **Total: 200**

Examiner: P R Mhuka

Moderators: N Ferreira
E Zachariou

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 6 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. **Answers must be rounded off to two decimal places in SECTION A**
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1:

a) The complex numbers z and w are such that $z = \frac{3a-5i}{1+2i}$ and $w = 1 + 13bi$, where a and b are real numbers. Given $z = w$, find the exact values of a and b. (7)

b) Use your calculator to solve the inequality $\frac{1-2x}{x+8} < e^x$. (5)

c) $log_a(6x^2 + 9x + 2) - log_a x = 4log_a 2$ (6)

d) $\lim_{x \to \frac{3}{2}} \frac{2x^2-3x}{|2x-3|}$ (5)

[23]

QUESTION 2:

a) Express the following function in absolute value notation

$$f(x) = \begin{cases} \frac{-1}{3}x + 1, & x > -3 \\ 3x + 11, & x \leq -3 \end{cases}$$ (5)

b) Sketch the graph of function $f(x) = -|3x - 2| + x + 2$, showing all intercepts with the axis, cusps and indicate each branch of the graph. (7)

[12]

QUESTION 3:

The equation of a curve is $f(x) = ln\left(\frac{3x-7}{x+1}\right)$. The curve cuts the x-axis at the point A.

a) Write down the domain of $f(x)$. (2)

b) Find the x-coordinate of A. (4)

c) Sketch $f(x)$ and $f^{-\prime}(x)$ on the same set of axis showing all intercepts with the axis and asymptotes. (7)

d) Write down the equation of $f^{-\prime}(x)$. (4)

e) Show that $f'(x)$ can be expressed as $\frac{k}{(3x-7)(x+1)}$, where k is an integer to be determined. (6)

[23]

QUESTION 4:

Prove by induction that the sum of the first n terms of the sequence whose r^{th} term is $r(r+1)(2r+1)$ is $\frac{1}{2}n(n+2).(n+1)^2$ **[12]**

QUESTION 5:

a) Given $h(x) = \begin{cases} 2x^2+3x, & x \leq -4 \\ ax+b, & -4 < x \leq 3 \\ -x^3+4x^2-5, & x \geq 3 \end{cases}$

1) Calculate the values of a and b if the function is continuous. (8)

2) Prove if the function is differentiable at $x = 3$ (4)

b) Given $f(x) = k + \frac{1}{x+1}$, where k is a positive constant.

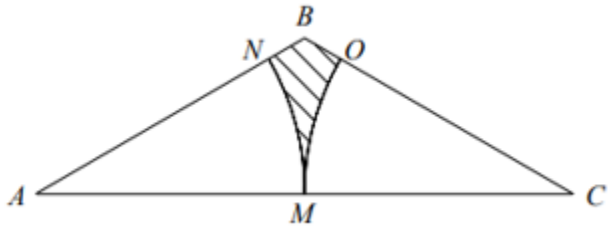
Given that the equation of the tangent to the curve at the point where $x = p$ is given by $y = -\frac{x-p}{(p+1)^2} + \frac{1}{p+1} + k$.

Use the above equation of y to explain why the curve will always have two tangents that pass through the point $(2; 0)$. (7)

[19]

QUESTION 6:

The diagram shows triangle ABC in which $AC = 8\,cm$ and $B\hat{A}C = B\hat{C}A = 30^o$.



a) Find the area of the triangle ABC in the form $k\sqrt{b}$ (5)

b) The point M is the mid-point of AC and the points N and O lie on AB and BC such that MN and MO are arcs of circles with centres A and C respectively. Show that the area of this shaded region $BNMO$ is $\frac{8}{3}\left(2\sqrt{3}-\pi\right)$. (5)

[10]

QUESTION 7:

a) Find $\frac{dy}{dx}$ for the following:

1) $tan(e^{3x-2})$ (4)

2) $x^2 ln(1-x^2)$ (6)

b) Calculate the gradient of the tangent of $y^2x^2 + y + cos(xy) = 0$ (12)

[22]

QUESTION 8:

Given the function $f(x) = \frac{x^2 - \frac{3}{2}x}{(2x+1)}$

a) Find equations of any asymptotes (6)

b) Find the coordinates of the stationary points and intercepts with the axis. (9)

c) Sketch the graph of $f(x)$ (8)

[23]

QUESTION 9:

Evaluate the following integrals without the use of a calculator:

a) $\int_0^3 e^{|x-2|} dx$ (6)

b) $\int \frac{sinx}{(2+3cosx)^2} dx$ (7)

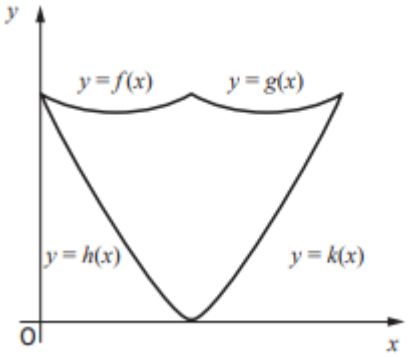
c) $\int (x+1).ln3x\, dx$ (9)

d) $\int \frac{x^2+12x-5}{(x+1)^2(x-7)} . dx$ (9)

[31]

QUESTION 10:

A wall plaque is to be made to commemorate the 150th anniversary of the publication of "Alice's Adventures in Wonderland".
The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.



$f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$ $\quad$ $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$

$h(x) = \frac{3}{8}x^2 - \frac{9}{4}x + 3$ $\quad$ $k(x) = \frac{3}{8}x^2 - \frac{3}{4}x$

a) Find the x-coordinate of the point of intersection of the graphs with equations $y = f(x)$ and $y = g(x)$ (2)

b) The graphs of the functions $f(x)$ and $h(x)$ intersect on the y-axis. The plaque has a vertical line of symmetry

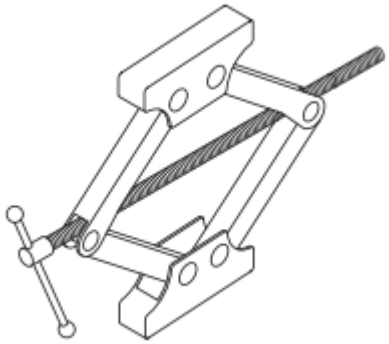
1) Calculate the area of the wall plaque. (7)

2) Find the volume of the wall plaque obtained by rotating the region about the x-axis. (8)

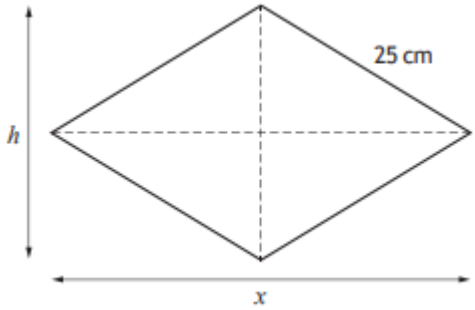
[17]

QUESTION 11:

An engineer has designed a lifting device. The handle turns a screw which shortens the horizontal length and increases the vertical height.



The device is modelled by a rhombus, with each side 25cm. The horizontal length is x cm, and the vertical height is h cm as shown.



a) Show that $h = \sqrt{2500 - x^2}$. (2)

b) The horizontal length decreases at a rate of $0{,}3$ cm per second as the handle is turned. Find the rate of the vertical height when $x = 30$. (6)
[8]