# AP Mathematics 2019 Paper 1 Prelim

KJ

## Question 1

1.1 $\ln x - 4\log_x e + 3 = 0$

$\ln x - \frac{4}{\ln x}$ ✓ $+ 3 = 0$

let $k = \ln x$

$k - \frac{4}{k} + 3 = 0$ ✓

$k^2 + 3k - 4 = 0$ ✓

$(k+4)(k-1) = 0$

$k = -4 \quad k = 1$ ✓

$\ln x = -4 \quad \ln x = 1$

$e^{-4} = x$ ✓ $\quad e = x$ ✓ (6)

1.2 $t = 100 \ln\left(\frac{4}{3} - \frac{P}{60000}\right)$

(a) $\ln 1 = 0$

$\therefore \frac{4}{3} - \frac{P}{60\,000} = 1$ ✓✓

$\frac{1}{3} = \frac{P}{60\,000}$

$\therefore P = 20\,000$ ✓✓ (4)

(b) $t = 100 \ln\left(\frac{4}{3}\right)$ ✓✓

$= 28{,}7 \quad \therefore 29$ years ✓ (3)

c) $t = 100 \ln\left(\frac{4}{3} - \frac{P}{60000}\right)$

$\frac{t}{100} = \ln\left(\frac{4}{3} - \frac{P}{60000}\right)$ ✓

$e^{\frac{t}{100}} = \frac{4}{3} - \frac{P}{60000}$ ✓

$\frac{P}{60000} = \frac{4}{3} - e^{\frac{t}{100}}$ ✓

$P = 80000 - 60000e^{\frac{t}{100}}$ ✓ (4)

d) $\frac{dP}{dt}$ ✓ $= -60000e^{t/100} \cdot \frac{1}{100}$

$= -600e^{t/100}$ ✓✓

$\therefore$ at $t = 0 \quad \frac{dP}{dt} = -600$ ✓✓ people per years (5)

[22]

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## Question 2

2.1 $P(x) = 4x^4 - 8x^3 + 33x^2 - 8x + 29 = 0$

a) $x = 1 - \frac{5}{2}i^\circ$

$(x-1)^2 = -\frac{5}{2}i^\circ)^2$

$x^2 - 2x + 1 = -\frac{25}{4}$ ✓✓

$4x^2 - 8x + 4 + 25 = 0$ ✓

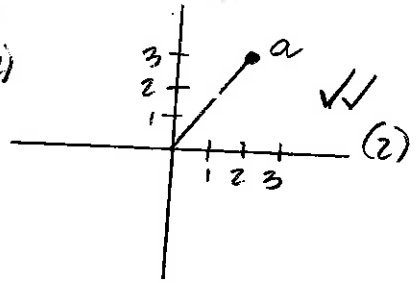
$4x^2 - 8x + 29 = 0$ ✓ ✓ (4)

b) $(4x^2 - 8x + 29)(x^2 + 1) = 0$

$x = 1 \pm \frac{5}{2}i^\circ$ ✓ $x = \pm i^\circ$ ✓✓ (4)

2

a)




✓✓ (2)

b) $|a| = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$ ✓

$\arg(a) = \tan^{-1}\left(\frac{3}{2}\right)$ ✓

$= 0,98^c$ ✓ (3)

c) $a = \sqrt{13}(\cos 0,98^c + i \sin 0,98^c)$ ✓✓ (2)

d) $b = \sqrt{13}(\cos 2,03 + i \sin 2,03)$ ✓✓

$= -1,99 + 3,24i$ ✓✓ (4)

[19]

## Question 3

3.1 $C(-2; 1)$ ✓✓ (2)

3.2 $B(0; 3)$ ✓ (1)

3.3 $y = -(x+2) + 1$

$y = -x - 2 + 1$

$y = -x - 1$ ✓✓

$y = \frac{-3}{x+1}$ ✓

$\therefore -x - 1 = \frac{-3}{x+1}$ ✓

$-(x+1)^2 = -3$

$(x+1)^2 = 3$

$x + 1 = \pm\sqrt{3}$

$x = 1 \pm \sqrt{3}$ ✓

$\therefore x = 1 - \sqrt{3}$ ✓ (6)

[9]

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## Question 4

RTP; $(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})\times\ldots\times(1-\frac{1}{n+1}) = \frac{1}{n+1}$

**Prove true for n = 1**

LHS $= 1-\frac{1}{2}$ RHS $= \frac{1}{1+1}$

$= \frac{1}{2}$ $= \frac{1}{2}$

∴ true for $n=1$ ✓✓

**Assume true for n = k**

ie assume $(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})\times\ldots\times(1-\frac{1}{k+1}) = \frac{1}{k+1}$ ✓✓

**Prove true for n = k+1**

ie RTP $(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})\times\ldots\times(1-\frac{1}{k+1})(1-\frac{1}{k+2}) = \frac{1}{k+2}$ ✓

LHS $= (1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})\times\ldots(1-\frac{1}{k+1})(1-\frac{1}{k+2})$ ✓

$= \frac{1}{k+1}(1-\frac{1}{k+2})$ ✓ from assumption

$= \frac{1}{k+1}\left[\frac{k+2-1}{k+2}\right]$ ✓

$= \frac{(k+1)}{(k+1)(k+2)}$ ✓

$= \frac{1}{k+2}$ ✓

= RHS

By the principle of mathematical induction, the statement is true for all $n \in N$ ✓✓ (12)

[12]

JS

## Question 5

5.1 a) $x=-3$ ✓ jump disc. ✓

$x=-1$ ✓ removeable disc ✓

$x=1$ ✓ removeable disc ✓ (6)

b) (1) $x=-1$ not continuous ✓

(2) $x=2$ continuous ✓ but $\lim_{x\to 2^-} f'(x) \neq \lim_{x\to 2^+} f'(x)$ ✓ (1)

5.2 $$f(x) = \begin{cases} m\cdot\ln(x+2), & x>-1 \\ e^{x+1}+k, & x\leq -1 \end{cases}$$ (2)

$\lim_{x\to -1^-} f(x) = m\ln 1 = 0$ ✓

$\lim_{x\to -1^+} f(x) = e^0 + k = 1+k$ ✓

continuous ∴ $0 = 1+k$

$k=-1$ ✓ (5)

$$f'(x) = \begin{cases} \frac{m}{x+2}, & x>-1 \\ e^{x+1}, & x\leq -1 \end{cases}$$ ✓ ✓

$\lim_{x\to -1^-} f'(x) = \frac{m}{1} = m$ ✓

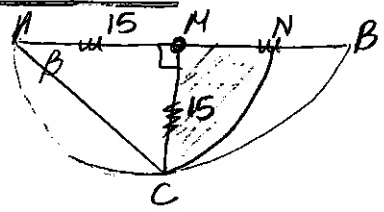
$\lim_{x\to -1^+} f'(x) = e^0 = 1$ ✓

differentiable ∴ $m=1$ ✓ (5)

[19]

JS

## Question 6




6.1 $\beta = \frac{\pi}{4}$ sum ∠ isos △, AM = MC radii ✓ (1)

6.2 $AC = \sqrt{15^2 + 15^2}$ Pyth ✓✓
$= 15\sqrt{2}$

area shaded = ◡ − △

$= \frac{1}{2}r^2\theta - \frac{1}{2}(15)(15)$

$= \frac{1}{2}(15\sqrt{2})^2\frac{\pi}{4} - \frac{1}{2}(15)^2$ ✓ ✓ ✓

$= 64{,}21\ cm^2$ ✓ (6)

[7]

JS

## Question 7

$x^2 - 3xy + 25y^2 = 91$

7.1 $2x - 3\left[1.y + 1\frac{dy}{dx}.x\right] + 25.2y\frac{dy}{dx} = 0$ ✓ ✓ ✓ ✓

$2x - 3y - 3x\frac{dy}{dx} + 50y\frac{dy}{dx} = 0$

$\frac{dy}{dx}[50y - 3x] = 3y - 2x$ ✓

$\frac{dy}{dx} = \frac{3y - 2x}{50y - 3x}$ ✓ (6)

7.2 $m_t = \frac{dy}{dx} = \frac{3(2) - 2(3)}{50(2) - 3(3)} = \frac{0}{91} = 0$ ✓✓

∴ $y = c$ ie $y = 2$ ✓ (3)

[9]

JS

## Question 8

$$f(x) = \frac{2x^2+4x+6}{2x+1} = \frac{2[x^2+2x+3]}{2x+1}$$

8.1 vertical asymptote $x = -\frac{1}{2}$ ✓

oblique asymptote

$$\begin{array}{r|l} & x + \frac{3}{2} \\ \hline 2x+1 & 2x^2+4x+6 \\ & 2x^2+x \quad \checkmark \\ \hline & 3x+6 \\ & 3x+\frac{3}{2} \\ \hline \end{array}$$

$\therefore y = x + \frac{3}{2}$ ✓✓ (4)

2a) $f'(x) = (4x+4)(2x+1)^{-1} - 2(2x^2+4x+6)^{-1} = 0$ ✓

$8x^2+12x+4-4x^2-8x-12 = 0$

$4x^2+4x-8 = 0$

$x^2+x-2 = 0$ ✓

$(x+2)(x-1) = 0$

$x = -2$ ✓ $\quad x = 1$ ✓

$y = -2$ ✓ $\quad y = 4$ ✓

$(-2;-2) \qquad (1;4)$ (6)

b) range: $y \leq -2$ or $y \geq 4$ ✓✓

$y \in (-\infty;-2] \cup [4;\infty)$ (2)

[12]

## Question 9

JS

9.1 a) $y = 4x(\ln x)^{-3}$

$$\frac{dy}{dx} = 4^{\checkmark}(\ln x)^{-3} + -3(\ln x)^{-4}\cdot\frac{1}{x}^{\checkmark}\cdot 4x$$

$$= \frac{4}{(\ln x)^3} - \frac{12}{(\ln x)^4} \checkmark\checkmark$$ (5)

b) $y = e^{-3x}\cos^5 4x$

$$\frac{dy}{dx} = e^{-3x}\cdot -3\cdot\cos^5 4x + 5\cos^4 4x\cdot(-\sin 4x)\cdot 4\cdot e^{-3x}$$

$$= \frac{-3\cos^5 4x}{e^{3x}} - \frac{20\cos^4 4x \sin 4x}{e^{3x}} \checkmark\checkmark$$ (6)

9.2 $f(x) = \cosh x$

$= \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

a) $f'(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}\cdot -1$ ✓

$= \frac{1}{2}(e^x - e^{-x})$ ✓

$= \sinh x$ (2)

b) $m = f'(x) = \sinh x = \frac{1}{2}(e^x - e^{-x})$

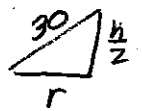
at $x = \ln a$ $\quad m = \frac{1}{2}[e^{\ln a} - e^{-\ln a}]$ ✓✓

$= \frac{1}{2}\left[a - \frac{1}{a}\right]$ ✓✓ $= \frac{a^2-1}{2a}$ ✓

(5) [8]

## Question 10

LP

10.1 


$30^2 = \left(\frac{h}{2}\right)^2 + r^2$ ✓ P4 th

$\frac{h^2}{4} + r^2 = 900$ (1)

10.2 $V = \pi r^2 h$

$= \pi\left[900 - \frac{h^2}{4}\right]h$ ✓

$= 900\pi h - \frac{\pi h^3}{4}$ ✓✓ (3)

10.3 $\frac{dV}{dh} = 900\pi - \frac{3\pi h^2}{4} = 0$ ✓✓ ∴ max

$\frac{3\pi h^2}{4} = 900\pi$

$h^2 = 1200$ ✓

$h = 20\sqrt{3}$ ✓

$V_{max} = 900\pi(20\sqrt{3}) - \frac{\pi(20\sqrt{3})^3}{4}$

$= 65\,296,78\ cm^3$ ✓✓ (6)

[10]

## QUESTION 11

LP

$Area = \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{p=1}^{n} f(x_p)$

11.1 $b - a = 3$ but $a = 2$ ✓

$\therefore b - 2 = 3$

$b = 5$ ✓ (2)

11.2 $f(x) = x^2 - 2x + 2$ ✓✓ (2)

11.3 $\int_2^5 (x^2 - 2x + 2)\,dx = 24$ ✓✓ (2)

[6]

## QUESTION 12

12.1 a) $\int \sqrt{\sin 4x}\cos 4x\,dx$

let $u = \sin 4x$ ✓

$du = 4\cos 4x\,dx$ ✓

$\frac{1}{4}\int u^{\frac{1}{2}}\,du$ ✓ $= \frac{1}{4}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$ ✓

$= \frac{u^{\frac{3}{2}}}{6} + c$

$= \frac{\sin^{\frac{3}{2}} 4x}{6}$ ✓ $+ c$ ✓

(6)

b) $\int \sin(x+1)\cos(3x-2)\,dx$

$= \frac{1}{2}\int \left(\sin[(x+1)+(3x-2)] + \sin[(x+1)-(3x-2)]\right)dx$ ✓

$= \frac{1}{2}\int \left(\sin(4x-1) + \sin(-2x+3)\right)dx$ ✓

$= \frac{1}{2}\left[\frac{-\cos(4x-1)}{4} - \frac{\cos(-2x+3)}{-2}\right] + c$ ✓

$= \frac{-\cos(4x-1)}{8} + \frac{\cos(-2x+3)}{2} + c$ ✓

(8)

c) $\int xe^x\,dx$ let $f(x) = x$ ✓ and $g'(x) = e^x$

then $f'(x) = 1$ ✓ $g(x) = e^x$

$\therefore \int xe^x dx = xe^x - \int e^x dx + c$ ✓

$= xe^x - e^x + c$ ✓✓

(8)

d) Consider $\frac{3}{2x^2-x-1} = \frac{3}{(x-1)(2x+1)}$

$= \frac{1}{x-1}$ ✓ $+ \frac{-2}{2x+1}$ ✓

$\int \frac{3}{2x^2-x-1}\,dx = \int \frac{1}{x-1}\,dx - 2\int \frac{1}{2x+1}\,dx$

$= \ln|x-1|$ ✓ $- 2\,\frac{\ln|2x+1|}{2}$ ✓ $+ c$

$= \ln|x-1| - \ln|2x+1| + c$ ✓

$= \ln\left|\frac{x-1}{2x+1}\right| + c$

(8)

12.2 area $= \int_0^4 (4e^{\frac{1}{2}x} - 6x + 3)\,dx$ LP

$= \left[\frac{4e^{\frac{1}{2}x}}{\frac{1}{2}} - \frac{6x^2}{2} + 3x\right]_0^4$ ✓✓✓✓

$= \left[8e^{\frac{1}{2}x} - 3x^2 + 3x\right]_0^4$

$= 8e^2 - 3(4)^2 + 3(4) - 0$

$= 8e^2 - 36$ ✓✓ $= 8e^2 + k$

$\therefore k = -36$ ✓✓

(8)

[38]

## QUESTION 13

LP

13.1 $f(x) = e^{-2x}$ $\quad A(0;1)$ ✓

$m_t = f'(x) = -2e^{-2x}$ ✓

at $x = 0$, $m_t = -2e^0 = -2$ ✓

$\therefore$ Eqn of tangent: $y = -2x + 1$ ✓

$\therefore$ B is $x$-int : put $y = 0$ $\therefore -2x + 1 = 0$

$x = \frac{1}{2}$

$B(\frac{1}{2};0)$ ✓

(5)

13.2 $e^{-2b} - 1 + b = 0$

let $f(b) = e^{-2b} - 1 + b$

$f'(b) = -2e^{-2b} + 1$ ✓✓

$b_{r+1} = b_r - \left[\frac{e^{-2b_r} - 1 + b_r}{-2e^{-2b_r} + 1}\right]$ ✓✓

$b_0 = 0,5$

$b_1 = 1$

$b_2 = 0,814438$

$b_3 = 0,79701..$

$b_4 = 0,7968..$

$b_5 = 0,7968$ ✓✓

(6)

[11]

## QUESTION 14

14.1 $\frac{2}{\sqrt{x+1}} = 1$

$\sqrt{x+1} = 2$

$x+1 = 4$ ✓

$x = 3$ ✓ $\therefore A(3;1)$ (2)

14.2 $V = \pi \int_a^b y^2 dx$

$= \pi \int_0^3 \left(\frac{2}{\sqrt{x+1}}\right)^2 dx - \pi \int_0^3 (1)^2 dx$

$= \pi \int_0^3 \frac{4}{x+1} dx - \pi \int_0^3 dx$ ✓✓

$= \pi \left[4 \ln|x+1| - x\right]_0^3$ ✓✓

$= \pi \left[(4 \ln 4 - 3) - (4 \ln 1 - 0)\right]$

$= 7,9959$

$\simeq 8,00$ ✓✓ (6)

[8]