St Andrew's School for Girls

SKILLED FOR LIFE

ADVANCED PROGRAMME MATHEMATICS PAPER 2

Grade 12

Preliminary Examinations

September 2019

Examiner	Lucea Pepper
Moderator/s	Gary Kelly
Marks	100
Time	1 hours
Number of Pages	(4 pages of questions)
Instructions	**1 Show all working.** **2 All answers to 4 dp unless otherwise stated.** **3 Start EACH question on a new page.**

Name:

QUESTION 1 [10 marks]

A and B are two events such that $P(B/A) = \frac{1}{6}$ and $P(A/B) = \frac{1}{5}$.

Given that $P(A \cap B) = m$, express in terms of m:

1.1 $P(A)$ (3)

1.2 $P(B)$ (2)

1.3 Given also that $P(A \cup B) = \frac{2}{5}$, find the value of m. (5)

QUESTION 2 [18 marks]

2.1 Festival lights are arranged in a line. How many different arrangements can be made from 3 purple, 2 green and 4 orange lights if :

(a) all 9 lights are used? (2)

(b) at least 8 lights are used? (6)

2.2 How many odd numbers, greater than 600 000, can be made from the digits 5, 6, 7, 8, 9, 0 if:

(a) repetitions are not allowed. (6)

(b) repetitions are allowed. (4)

QUESTION 3 [9 marks]

3.1 A box of Smarties contains a total of 28 Smarties of which 8 are blue.
If I take out 6 (chosen randomly), find the probability that 3 will be blue. (3)

3.2 How many times do I have to throw an ordinary die in order to be at least 95% sure of obtaining at least one six? (6)

QUESTION 4 [6 marks]

Ridge counts in fingerprints are approximately normally distributed with a mean of 150 and a standard deviation of 35.

4.1 Find the probability that a randomly chosen individual has a ridge count of less than 240. (4)

4.2 A set of finger prints analysed by the forensics department during a crime investigation, is found to have a ridge count of 250. Comment on how the result of 4.1 would be useful in the investigation. (2)

QUESTION 5 [17 marks]

5.1 The probability density function for the lifespan of a certain species is given by :

$$f(x) = \begin{cases} -\frac{3}{16}x^2 + \frac{3}{4} & 0 \le x \le m \\ 0 & \text{elsewhere} \end{cases}$$

where x is the age of the insect in years.

Find m, the maximum lifespan of these insects. (6)

5.2 The probability distribution of the discrete random variable X is shown in the table below.

x	– 3	– 1	0	4
P(X = x)	a	b	0,15	0,4

Given that E [X] = 0,75 ,

(a) Find the values of a and b. (7)

(b) Hence determine Var [X] (to 2 decimal places). (4)

QUESTION 6 [6 marks]

The pH value of water measures the degree of its acidity. The water in a particular dam is known to have pH values with a variance of 0,25. Environmentalists obtain 10 samples of water from the dam and test them. The mean pH of the samples is 8,2.

Obtain a 96% confidence interval for the true population mean pH for the dam (to 2 decimal places). (6)

QUESTION 7 [24 marks]

It is believed that 15% of the population in South Africa have blue eyes.

7.1 If a random sample of South Africans is taken, how large would the sample have to be, to be 95% sure of obtaining an estimate to within 2% ? (8)

7.2 Calculate the probability that at least 2 of a random sample of 8 South Africans have blue eyes. (6)

7.3 From a random sample of 100 South Africans, check if you can use normal approximation to the binomial distribution and then calculate the probability that between 14 and 20 people (inclusive) would have blue eyes. (10)

QUESTION 8 [10 marks]

A machine at a manufacturing plant fills bottles.

A new machine is available and the claim is that this new machine can fill the bottles at an average of at least 1 second faster.

Assume the filling times are normally distributed.

A sample of 10 bottles filled by the old machine gives an average time of 42,14 seconds with a standard deviation of 0,683 seconds.

A sample of 8 bottles filled by the new machine gives an average time of 40,88 seconds with a standard deviation of 0,665 seconds.

8.1 State the null and alternate hypotheses. (3)

8.2 Test the claim at the 4% level of significance. (7)