AP Mathematics 2019 Paper 2 Prelim

QUESTION 1

1.1 $P(B/A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{1}{6}$ ✓ $\therefore P(B\cap A) = \frac{1}{6}P(A)$ ✓

ie $m = \frac{1}{6}P(A)$

$\therefore \underline{P(A) = 6m}$ ✓ (3)

1.2 $P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{1}{5}$ $\therefore P(A\cap B) = \frac{1}{5}P(B)$ ✓

ie $m = \frac{1}{5}P(B)$

$\therefore \underline{P(B) = 5m}$ ✓ (2)

1.3 $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{2}{5} = 6m + 5m - m$ ✓✓✓

$\frac{2}{5} = 10m$

$\therefore \underline{m = \frac{1}{25}}$ ✓✓ (5)

[10]

QUESTION 2

2.1 3P 2G 4O

a) $\dfrac{9!}{3!\,2!\,4!}$ ✓ $= \underline{1260}$ ✓ (2)

b) (at least 8) = 8 or 9

$= \dfrac{8!}{2!\,2!\,4!}$ ✓ $+ \dfrac{8!}{3!\,1!\,4!}$ ✓ $+ \dfrac{8}{3!\,2!\,3!}$ ✓ $+ \dfrac{9}{3!\,2!\,1!}$ ✓

$= 420 + 280 + 560 + 1260$

$= \underline{2520}$ ✓✓ (6)

2.2 ^ ^ ^ ^ ^ ^ 5; 6; 7; 8; 9; 0

a)

6	4!	5 7 9	
7	4!		5 9
8	4!	5 7 9	
9	4!		5 7

$\therefore 2\times 4!\, 3 = 144$ ✓✓

$\therefore 2\times 4!\, 2 = 96$ ✓✓

$\underline{240 \text{ numbers}}$ ✓✓ (6)

b)

6	6^4	5 7 9
7	6^4	5 7 9
8	6^4	5 7 9
9	6^4	5 7 9

$4\times 6^4\times 3$ ✓✓✓ $= \underline{15552}$ ✓ (4)

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QUESTION 3

.1 $P(X=3) = \dfrac{\binom{8}{3}\binom{20}{3}}{\binom{28}{6}} = 0{,}1695$ (3)

Hypergeometric

.2 $P(X \geq 1) = 1 - P(X=0) \geq 0{,}95$ Binomial

$1 - \binom{n}{0}\left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^n \geq 0{,}95$

$1 - \left(\frac{5}{6}\right)^n \geq 0{,}95$

$0{,}05 \geq \left(\frac{5}{6}\right)^n$

$\log_{\frac{5}{6}} 0{,}05 \leq n$

$n \geq 16{,}43$

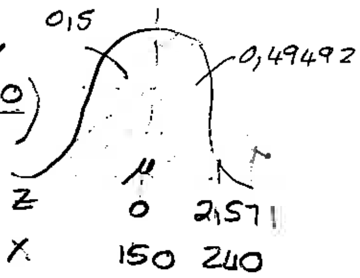
$\therefore$ at least 17 times (6)

[9]

QUESTION 4.

$X \sim N(150; 35^2)$

4.1 $P(X<240) = P\left(Z < \dfrac{240-150}{35}\right)$



$= P(Z < 2{,}571)$

$= 0{,}5 + 0{,}49492$

$= 0{,}99492$ (4)

4.2 As 99,5% of the population has a ridge count of less than 240, only 0,5% have a ridge count of above 240 ∴ it will be easy to confirm if a suspect is guilty (2)

[6]

QUESTION 5

1) $\displaystyle\int_0^m \left(-\frac{3}{16}x^2 + \frac{3}{4}\right) dx = 1$ ✓

$\left[-\frac{3}{16}\frac{x^3}{3} + \frac{3}{4}x\right]_0^m = 1$ ✓✓

$\left(-\frac{m^3}{16} + \frac{3m}{4}\right) - 0 = 1$ ✓

$-m^3 + 12m - 16 = 0$

$m^3 - 12m + 16 = 0$ ✓

$\cancel{m=-4}$ $\quad m = 2$ ✓

not valid

(6)

2 a) $a + b + 0{,}15 + 0{,}4 = 1$ ✓

$a + b = 0{,}45$ ① ✓

$E[X] = -3a - 1b + 0 + 4(0{,}4) = 0{,}75$ ✓

$-3a - b = -0{,}85$

$3a + b = 0{,}85$ ② ✓

sim solve ✓

$\therefore\ a = \frac{1}{5} \qquad b = \frac{1}{4}$ ✓✓

(7)

b) $Var[X] = (-3)^2\frac{1}{5} + (-1)^2\frac{1}{4} + (0)^2 0{,}15 + (4)^2 0{,}4 - (0{,}75)^2$ ✓✓

$= 7{,}89$ ✓✓

(4)

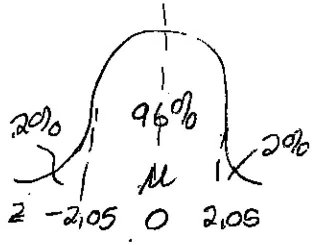
[17]

QUESTION 6.

$\sigma^2 = 0{,}25 \quad n = 10 \quad \bar{x} = 8{,}2 \quad 96\%$ CI

$\mu = \bar{x} \pm \frac{z\sigma}{\sqrt{n}}$

$= 8{,}2 \pm \frac{2{,}055\,(0{,}5)}{\sqrt{10}}$ ✓ ✓



$\mu \in [7{,}88\ ;\ 8{,}52]$ ✓✓ ✓✓

(6)

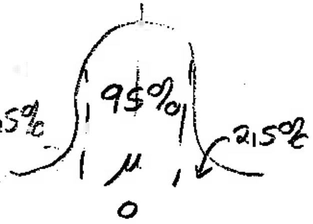
[6]

QUESTION 7

Binomial $p = 0{,}15$ $q = 0{,}85$

7.1 $n = ?$ 95% within 2%

$$p = \hat{p} \pm z\sqrt{\frac{\hat{p}\hat{q}}{n}}$$



$$p - \hat{p} = \pm z\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0{,}02 = \pm 1{,}96\sqrt{\frac{0{,}15(0{,}85)}{n}}$$

ie $n \geq \dfrac{z^2\hat{p}\hat{q}}{d^2}$

$$\geq \frac{1{,}96^2(0{,}15)(0{,}85)}{(0{,}02)^2}$$

$$\geq 1224{,}51$$

ie at least 1225 people (8)

7.2 $P(X \geq 2) = 1 - [P(X=1) + P(X=0)]$

$$= 1 - \binom{8}{1}0{,}15^1\,0{,}85^7 - \binom{8}{0}0{,}15^0\,0{,}85^8$$

$$= 1 - 0{,}3846 - 0{,}2724$$

$$= \underline{0{,}3428}$$

(6)

7.3 $n = 100$ $p = 0{,}15$ $q = 0{,}85$

check $np = 100(0{,}15) = 15$; both > 5
$nq = 100(0{,}85) = 85$

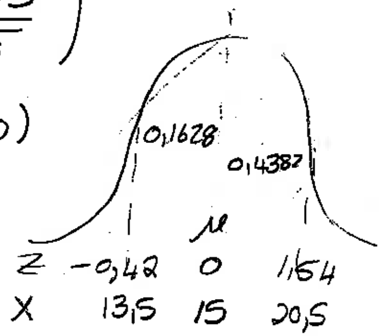
∴ can approx to normal distr

$X \sim Bin(100; 0{,}15) \sim N(15; 12{,}75)$

$P(14 \leq X \leq 20) = P(13{,}5 \leq X \leq 20{,}5)$ after cont. corr

$$= P\left(\frac{13{,}5 - 15}{\sqrt{12{,}75}} \leq Z \leq \frac{20{,}5 - 15}{\sqrt{12{,}75}}\right)$$

$$P(-0{,}420 \leq Z \leq 1{,}540)$$



$$= 0{,}1628 + 0{,}4382$$

$$= \underline{0{,}601}$$

(10)

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QUESTION 8

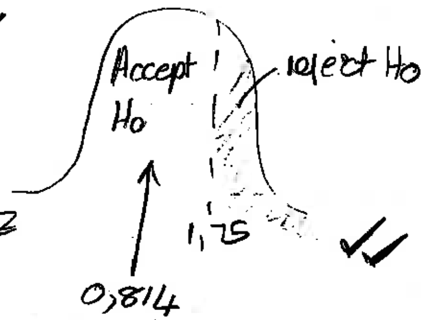
8.1 $H_0 : \mu_O - \mu_N = 1$
$H_1 : \mu_O - \mu_N > 1$ ∴ One sided test (3)

8.2 $\alpha = 4\%$ ∴ $z = 1{,}75$

Critical value $z = \dfrac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x} + \dfrac{\sigma_y^2}{n_y}}}$

$$= \frac{(42{,}14 - 40{,}88) - 1}{\sqrt{\dfrac{0{,}683^2}{10} + \dfrac{0{,}665}{8}}}$$

$$= 0{,}814$$



∴ Fail to reject Ho and say there is insufficient evidence to support the claim at the 4% level of significance

(7)

[10]