ST BENEDICT'S

SUBJECT	**AP Mathematics**	**PAPER**	**Paper 1**
GRADE	**12**	**DATE**	11 July 2019
EXAMINER	Mr Benecke	**MARKS**	200
NAME	**Memo**	**MODERATOR**	Mrs Povall + Mrs Serafino
TEACHER		**DURATION**	2 hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Solving for x	27	
2	Limits	12	
3	Inverse Functions	12	
4	Differentiation	39	
5	Continuity and Differentiability	13	
6	Trigonometry	8	
7	Drawing Functions	15	
8	Interpretation and Newton-Raphson	22	
9	Integration	32	
10	Solids of Revolution	10	
11	Min/Max Problem	10	
TOTAL		200	

INSTRUCTIONS:

1. This paper consists of 11 questions and 18 pages.
2. Read the questions carefully.
3. Answer all questions.
4. Number your answers clearly and use the same numbering as in the question paper.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to **four** decimal digits where necessary.
7. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks. Equations may not be solved solely with a calculator.
8. It is in your interest to write legibly and present your work neatly.

QUESTION 1 **27 MARKS**

Solve for x in each of the following:

a) $|x|^2 - 5|x| = 14$ (6)

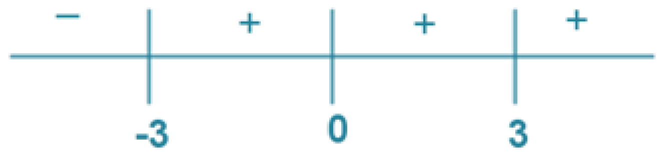
$x^2 - 5x - 14 = 0$ ✓ or $x^2 + 5x - 14 = 0$ ✓

$(x-7)(x+2) = 0$ or $(x+7)(x-2) = 0$

$x = 7$ ✓ or $x = -2$ or $x = -7$ ✓ or $x = 2$

N/A ✓ N/A ✓

b) $\frac{|x^2-3x|(x+3)}{|x-3|} \leq 0$ (5)



✓✓✓

$x \leq -3$ ✓ or $x = 0$ ✓

c) $\ln(e^{2x} - 12) - x = 0$ (7)

$\ln(e^{2x} - 12) = x$ ✓

$e^{2x} - 12 = e^x$ ✓

$e^{2x} - e^x - 12 = 0$ ✓

$(e^x - 4)(e^x + 3) = 0$ ✓

$e^x = 4$ ✓ or $e^x = -3$ (n/a) ✓

$x = \ln 4$

$x = 1{,}3863$ ✓

d) Given $f(x) = x^4 + 4x^3 + 3x^2 + 4x + 2$ and $f(i) = 0$, solve for x if $f(x) = 0$ (9)

Since $x = i$ is a root, $x = -i$ must also be a root ✓

$(x - i)(x + i) = x^2 + 1$ ✓✓

$x^4 + 4x^3 + 3x^2 + 4x + 2 = (x^2 + 1)(ax^2 + bx + c)$ ✓

$= (x^2 + 1)(x^2 + 4x + 2)$ ✓✓

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$ ✓

$x = -2 \pm \sqrt{2}$ ✓

$\therefore x = i$ or $x = -i$ or $x = -2 + \sqrt{2}$ or $x = -2 - \sqrt{2}$ ✓

QUESTION 2 **12 MARKS**

Evaluate:

a) $\lim_{x \to 4} \frac{x^2-2x-8}{x-4}$ (3)

$= \lim_{x \to 4} \frac{(x-4)(x+2)}{x-4}$ ✓

$= \lim_{x \to 4} x + 2$ ✓

$= 6$ ✓

b) $\lim_{x \to 0} \frac{\sin 4x}{2\sin 2x}$ (4)

$= \lim_{x \to 0} \frac{2 \sin 2x \cos 2x}{2\sin 2x}$ ✓✓

$= \lim_{x \to 0} \cos 2x$ ✓

$= 1$ ✓

c) $\lim_{x \to \infty} \frac{\sqrt{4x^2-1}}{2x-3}$ (5)

$\lim_{x \to \infty} \frac{x\sqrt{4-\frac{1}{x^2}}}{x(2-\frac{3}{x})}$ ✓✓

$= \frac{\sqrt{4-0}}{2-0}$ ✓✓

$= 1$ ✓

QUESTION 3 **12 MARKS**

Given $f(x) = \ln(x + 4)$

a) State the domain and range of $f(x)$. (2)

Domain: $x > -4$ ✓

Range: $y \in R$ ✓

b) Determine $f^{-1}(x)$, the inverse of $f(x)$ in the form $f^{-1}(x) = ...$ (3)

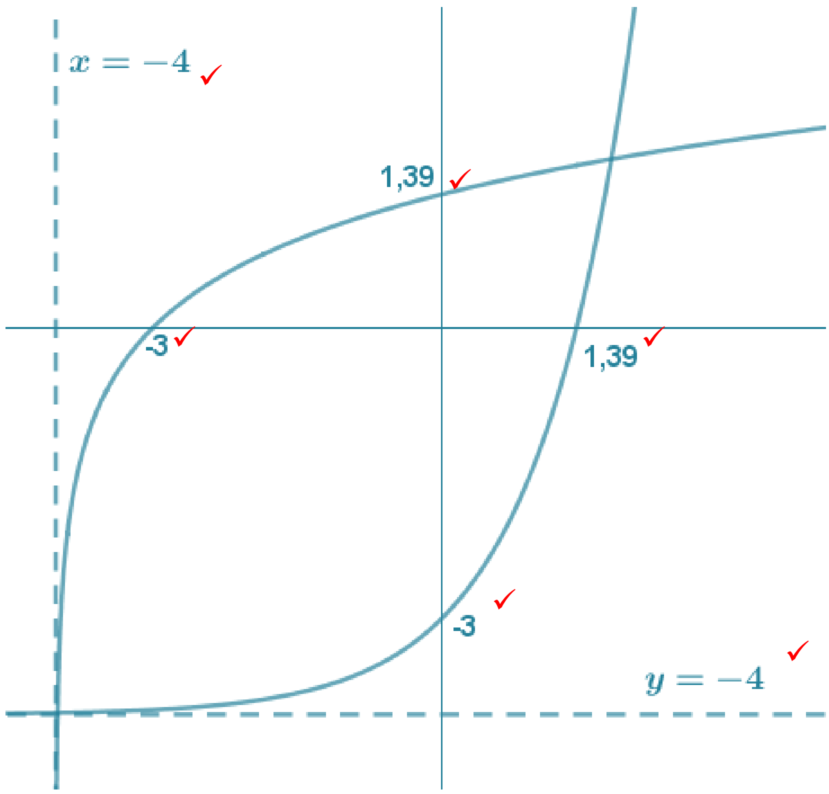
$x = \ln(y + 4)$ ✓

$y + 4 = e^x$ ✓

$f^{-1}(x) = e^x - 4$ ✓

c) Sketch the graphs of $f^{-1}(x)$ and $f(x)$ on the same axes, clearly labelling intercepts with the axes and asymptotes. (7)

1 mark for shape of the graphs ✓



QUESTION 4 **39 MARKS**

Determine:

a) $f'(x)$ if $f(x) = \sqrt{5x}$ by first principles. (8)

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{5x+5h}✓ - \sqrt{5x}✓}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{5x+5h} - \sqrt{5x}}{h} \times \frac{\sqrt{5x+5h} + \sqrt{5x}}{\sqrt{5x+5h} + \sqrt{5x}}✓$$

$$f'(x) = \lim_{h \to 0} \frac{5x+5h-5x ✓}{h(\sqrt{5x+5h} + \sqrt{5x})✓}$$

$$f'(x) = \lim_{h \to 0} \frac{5h}{h(\sqrt{5x+5h} + \sqrt{5x})} ✓$$

$$f'(x) = \frac{5}{\sqrt{5x} + \sqrt{5x}} ✓$$

$$f'(x) = \frac{5}{2\sqrt{5x}} ✓$$

b) $\frac{dy}{dx}$ if $y = (4x^2 + 2)^9$ (3)

$$\frac{dy}{dx} = 9(4x^2 + 2)^8✓.(8x)✓$$

$$\frac{dy}{dx} = 72x(4x^2 + 2)^8✓$$

c) $D_x\left[e^{x^3}\right]$ (3)

$$= 3x^2.e^{x^3}✓✓✓$$

d) $f'(x)$ if $f(x) = \ln\left(\frac{x}{x^2-1}\right)$ (7)

$$f'(x) = \frac{1(x^2-1)✓ - 2x(x)✓}{(x^2-1)^2 ✓} \times \frac{x^2-1}{x} ✓$$

$$f'(x) = \frac{x^2-1-2x^2}{(x^2-1)^2} \times \frac{x^2-1}{x} ✓$$

$$f'(x) = \frac{-x^2-1}{(x^2-1)^2} \times \frac{x^2-1}{x} ✓$$

$$f'(x) = \frac{-x^2-1}{x^3-x} ✓$$

e) $D_x[2x^4.\cos(x^3-1)]$ (6)

$= 8x^3 ✓ .\cos(x^3-1) ✓ - ✓ 2x^4 ✓ .\sin(x^3-1).3x^2 ✓$

$= 8x^3.\cos(x^3-1) - 6x^6.\sin(x^3-1)$ ✓

f) $\frac{dy}{dx}$ if $x^2 + xy + y^2 = 9$ (6)

$2x ✓ + y ✓ + \frac{dy}{dx}x ✓ + 2y\frac{dy}{dx} ✓ = 0$

$\frac{dy}{dx}x + 2y\frac{dy}{dx} = -2x - y$ ✓

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y} ✓$$

g) $f^n(x)$ if $f(x) = \frac{1}{x}$ (6)

$$f^1(x) = -1\left(\frac{1}{x^2}\right) \checkmark$$

$$f^2(x) = 1.2\left(\frac{1}{x^3}\right) \checkmark$$

$$f^3(x) = -3.2.1\left(\frac{1}{x^4}\right) \checkmark$$

$$f^n(x) = (-1)^n \checkmark .\, n! \checkmark \left(\frac{1}{x^{n+1}}\right) \checkmark$$

QUESTION 5 **13 MARKS**

Given:

$$f(x) = \begin{cases} x^2 & if\ \ x < 2 \\ |x-6| & if\ \ x \geq 2 \end{cases}$$

a) Write $f(x)$ as a split function without the absolute value notation. (4)

$$f(x) = \begin{cases} x^2 & if\ \ x < 2 \\ -x+6 ✓ & if\ 2 \leq\ x < 6 ✓ \\ x-6 ✓ & if\ \ x \geq 6\ ✓ \end{cases}$$

b) Determine the value of $f'(1)$ and $f'(6)$. (3)

$f'(x) = 2x$ ✓ for $x < 2$

$f'(1) = 1$✓

$f'(6)$ doesn't exist✓

c) Determine if $f(x)$ is continuous and differentiable at $x = 2$. Justify your answer fully. (6)

$\lim_{x \to 2^-} x^2 = 4$ ✓

$\lim_{x \to 2^+} -x + 6 = 4$ ✓

$\therefore f(x)$ is continuous ✓

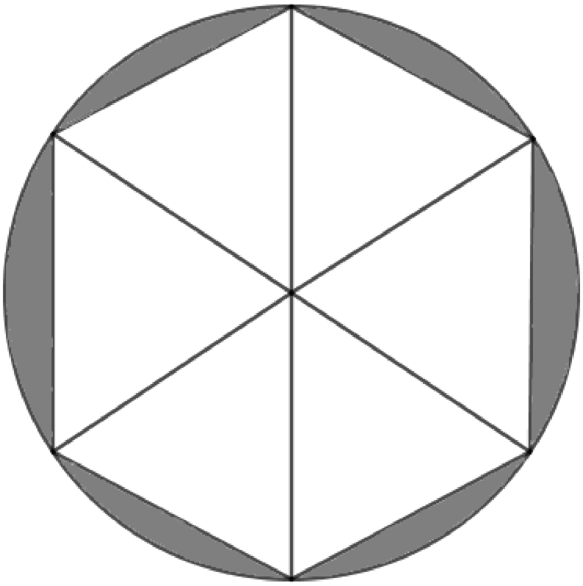
$\lim_{x \to 2^-} 2x = 4$ ✓

$\lim_{x \to 2^+} -1 = -1$ ✓

$\therefore f(x)$ is not differentiable ✓

QUESTION 6 **8 MARKS**

Six identical triangles are inscribed in a circle. Area of the total shaded region is $10\ units^2$.
Determine the radius of the circle. (8)



$Area\ of\ Circle = 10 + 6 \times \frac{1}{2} r^2 sin\theta$ ✓✓

$\pi r^2 = 10 + 3r^2 sin\theta$

$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$ ✓

$\pi r^2 = 10 + 3r^2 sin\frac{\pi}{3}$ ✓

$r^2\left(\pi - 3sin\frac{\pi}{3}\right) = 10$ ✓

$r^2 = \frac{10}{\pi - 3sin\frac{\pi}{3}}$ ✓

$r^2 = 18{,}3989$ ✓

$r = 4{,}2894$ ✓

QUESTION 7 **15 MARKS**

Consider the function $f(x) = \frac{x^2+x-12}{x+3}$

a) Determine the equation of the vertical asymptote. (2)

$x = -3$ is the vertical asymtote✓✓

b) Determine the equation of the oblique asymptote. (4)

$\frac{x^2+x-12}{x+3} = x - 2 + R$ ✓✓ so oblique asymptote is $y = x - 2$✓✓

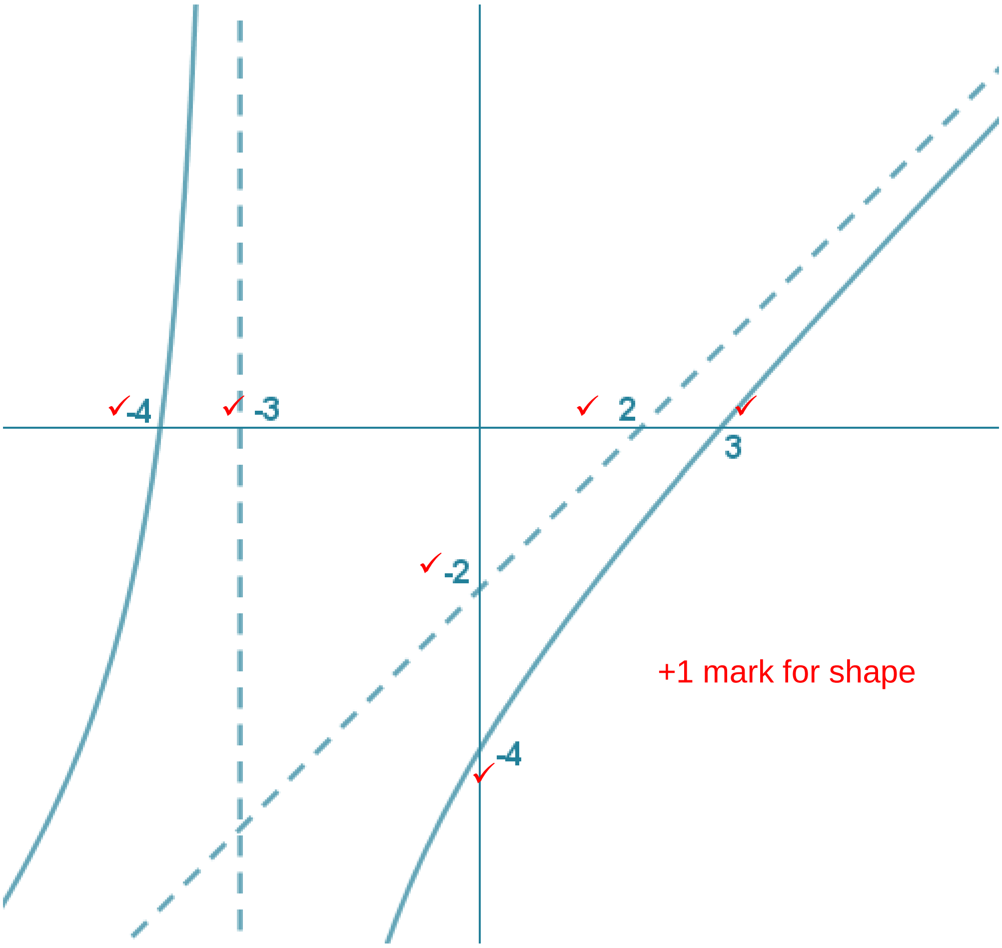
c) Sketch the graph of f. (9)

$y = \frac{0^2+0-12}{0+3} = -4$ $(y - int)$ ✓

$0 = x^2 + x - 12$

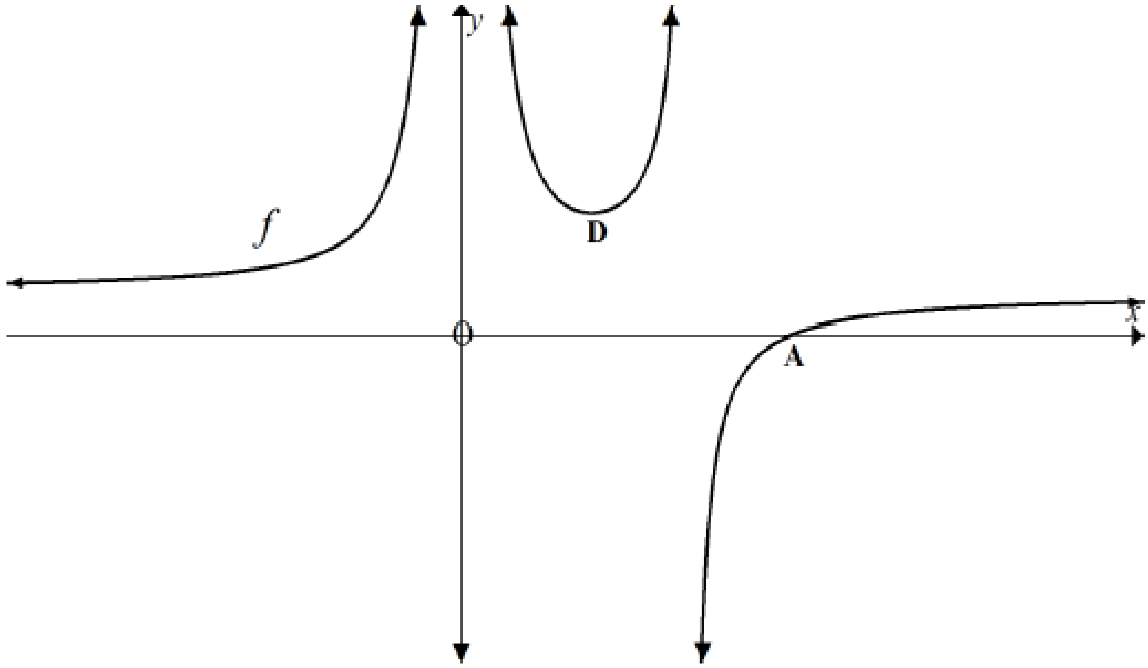
$(x + 4)(x - 3) = 12$✓

$x = -4$; $x = 3$ ✓ $(x - int)$



QUESTION 8 **22 MARKS**

The graph of $f(x) = \frac{1}{x^2} - \frac{1}{x-2} + 1$ is shown, with a turning point at D(1,14; 2,93) and an x-intercept at the point indicated by A



a) Give the equations of all the vertical and horizontal asymptotes of the graph of f. (4)

$x = 0,$ ✓ $x = 2,$ ✓ $y = 1$✓✓

b) Use Newton's method to determine the coordinates of A, the x-intercept, correct to 4 decimal places. Use $x = 3$ as your initial value. (You must find x_1 manually, the rest can be done on the calculator, do not use the "solve function"). (8)

$f(x) = x^{-2} - (x-2)^{-1} + 1$ ✓

$f'(x) = -2x^{-3} + (x-2)^{-2}$ ✓

$x_0 = 3$

$x_1 = x_0$✓ $- \frac{{x_0}^{-2} - (x_0 - 2)^{-1} + 1 ✓}{-2{x_0}^{-3} + (x_0 - 2)^{-2} ✓}$

$x_1 = 2{,}88$✓

$x_2 = 2{,}8931$

$x_3 = 2{,}8933$

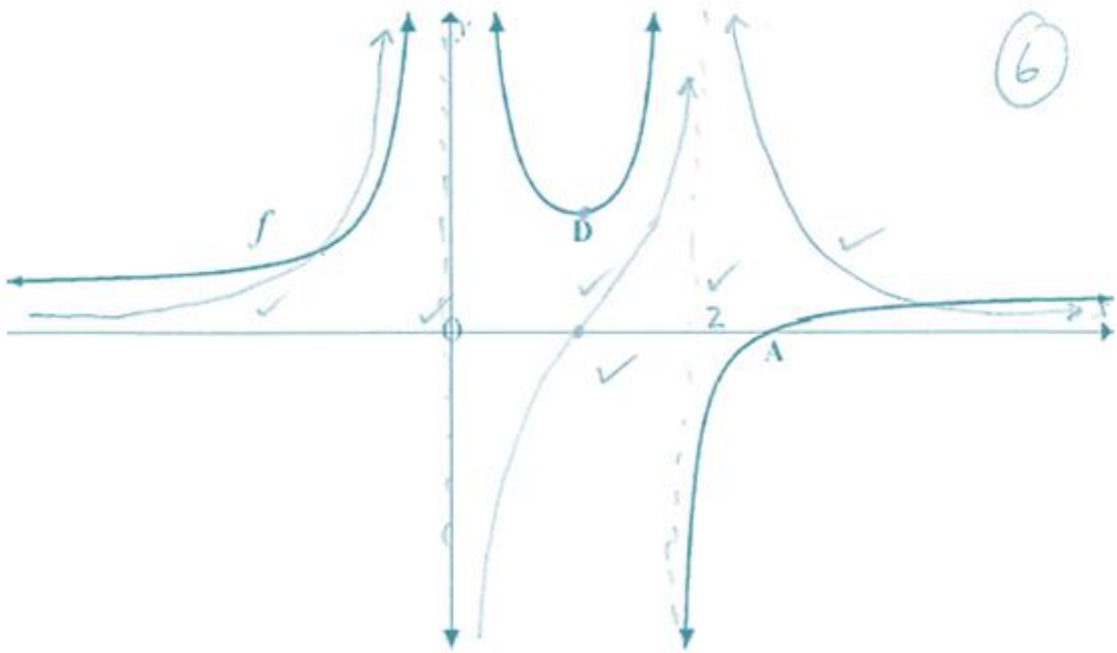
$x_4 = 2{,}8933$✓

$A(2{,}8933; 0)$ ✓

c) For which values of x is the function f strictly increasing? (4)

$x < 0$✓ and $1{,}14$✓ $< x < 2$✓ and $x > 2$✓

d) Draw a rough sketch of f' on the same system of axes above. Show clearly where f' has asymptotes and intercepts with the axes, if any. (6)



QUESTION 9 **32 MARKS**

a) Determine the following integrals:

1) $\int e^{2x+3}\,dx$ (3)

$= e^3 \int e^{2x}\,dx$ ✓

$= e^3\left(\frac{1}{2}e^{2x}\right) + C$✓

$= \frac{e^{2x+3}}{2} + C$✓

2) $\int \frac{2x^7-x^3}{x^8-x^4}\,dx$ (4)

$= \frac{1}{4}\int \frac{8x^7-4x^3}{x^8-x^4}\,dx$ ✓✓

$= \frac{1}{4}\ln|x^8 - x^4|\,C$✓✓

3) $\int 3x(x+5)^8\,dx$ (7)

$u = x + 5$ ✓

$\frac{du}{dx} = 1$ ✓

$x = u - 5$ ✓

$\int 3x(x+5)^8\ \,dx$

$= \int 3(u-5)(u)^8\ \,du$ ✓

$= \int 3u^9 - 15u^8\ \,du$ ✓

$= \frac{3u^{10}}{10} - \frac{15u^9}{9} + C$ ✓

$= \frac{3(x+5)^{10}}{10} - \frac{5(x+5)^9}{3} + C$ ✓

4) $\int (2x+3)\sin 4x \; dx$ (9)

$let \quad f(x) = 2x+3 \quad and \quad g'(x) = \sin 4x$ ✓

$then \; f'(x) = 2 \quad and \quad g(x) = -\frac{1}{4}\cos 4x$ ✓

$$= (2x+3)✓\left(-\frac{1}{4}\cos 4x\right)✓ - \int 2✓ \times -\frac{1}{4}\cos 4x \, dx✓ + C$$

$$= -\frac{(2x+3)\cos 4x}{4}✓ + \frac{\sin 4x}{8}✓✓ + C$$

b) Find k if $\int_0^k 3x\sqrt{x^2+5} \; dx = 10$ (9)

$u = x^2 + 5$ ✓

$\frac{du}{dx} = 2x$ ✓

$= \int \frac{3}{2}\sqrt{u} \; du$ ✓

$= \frac{3}{2}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$ ✓

$= u^{\frac{3}{2}}$ ✓

$\left[(x^2+5)^{\frac{3}{2}}\right]_0^k = 10$✓

$(k^2+5)^{\frac{3}{2}} - 5^{\frac{3}{2}} = 10$ ✓

$(k^2+5)^{\frac{3}{2}} = 21{,}18033989$

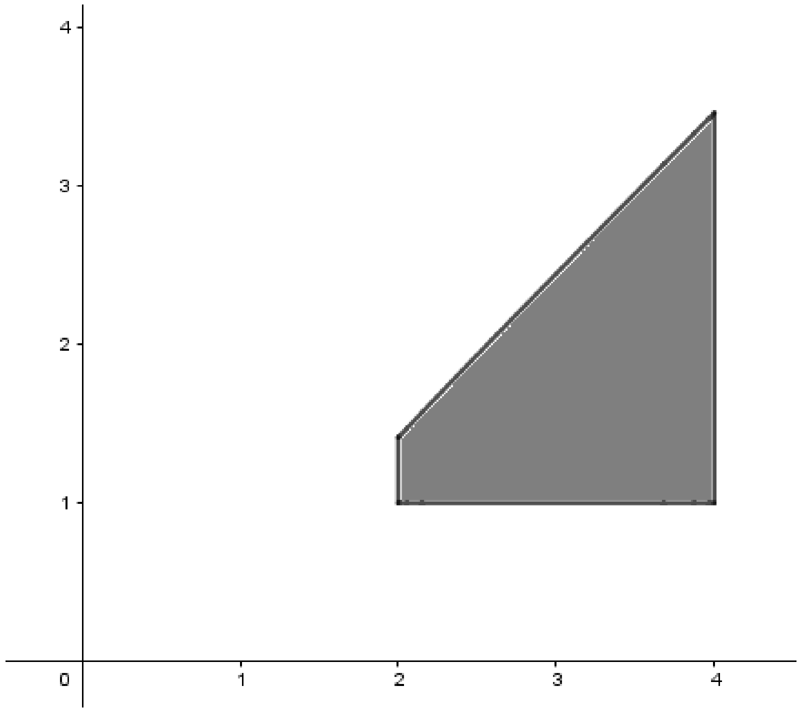
$k^2 + 5 = 7{,}655177822$ ✓

$k^2 = 2{,}655177822$

$k = 1{,}62947$✓

QUESTION 10 **10 MARKS**

Determine the volume of a solid obtained by rotating the region bounded by $y = \sqrt{x^2 - x}$, $x = 2$, $x = 4$, and $y = 1$ about the $x - axis$. (10)



$$V = \pi \int_a^b \left(f(x)\right)^2 dx \checkmark$$

$$= \pi \int_2^4 \left(\left(\sqrt{x^2 - x}\right)^2 - (1)^2\right) dx \checkmark\checkmark$$

$$= \pi \int_2^4 (x^2 - x - 1) dx \checkmark$$

$$= \pi \left[\frac{x^3}{3} \checkmark - \frac{x^2}{2} \checkmark - x \checkmark\right]_2^4$$

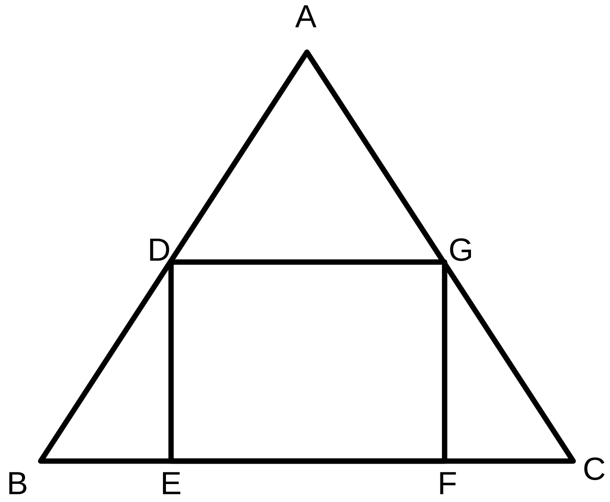
$$= \pi \left[\left(\frac{(4)^3}{3} - \frac{(4)^2}{2} - (4)\right) \checkmark - \left(\frac{(2)^3}{3} - \frac{(2)^2}{2} - (2)\right)\right] \checkmark$$

$$= \frac{32}{3}\pi \checkmark$$

QUESTION 11 **10 MARKS**

In the sketch ΔABC is an equilateral triangle with each side equal to a units.
DEFG is a rectangle with BE = FC = x units.

.



a) Prove that the area of the rectangle is $A(x) = x\sqrt{3}(a - 2x)$ (5)

$\hat{B} = \frac{1}{3}\pi$ ✓✓

$BE = x$

$\therefore DE = x\sqrt{3}$ ✓

$EF = a - 2x$ ✓

$A(x) = x\sqrt{3}(a - 2x)$ ✓

b) If $a = 2$, determine the maximum area of the rectangle. (5)

$A(x) = x\sqrt{3}(2 - 2x)$

$A(x) = 2x\sqrt{3} - 2x^2\sqrt{3}$ ✓

$A'(x) = 2\sqrt{3} - 4\sqrt{3}x$ ✓

$0 = 2\sqrt{3} - 4\sqrt{3}x$ ✓

$4\sqrt{3}x = 2\sqrt{3}$

$x = \frac{1}{2}$ ✓

$A\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)\sqrt{3}\left(2 - 2\left(\frac{1}{2}\right)\right)$

$A\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2}\ units^2$ is the maximum area ✓