## ST BENEDICT'S

| SUBJECT | **AP Mathematics** | PAPER | **Paper 1** |
|---|---|---|---|
| GRADE | **12** | DATE | 11 July 2019 |
| EXAMINER | Mr Benecke | MARKS | 200 |
| NAME | | MODERATOR | Mrs Povall + Mrs Serafino |
| TEACHER | | DURATION | 2 hours |

| QUESTION NO | DESCRIPTION | MAXIMUM MARK | ACTUAL MARK |
|---|---|---|---|
| 1 | Solving for $x$ | 27 | |
| 2 | Limits | 12 | |
| 3 | Inverse Functions | 12 | |
| 4 | Differentiation | 39 | |
| 5 | Continuity and Differentiability | 13 | |
| 6 | Trigonometry | 8 | |
| 7 | Drawing Functions | 15 | |
| 8 | Interpretation and Newton-Raphson | 22 | |
| 9 | Integration | 32 | |
| 10 | Solids of Revolution | 10 | |
| 11 | Min/Max Problem | 10 | |
| **TOTAL** | | 200 | |

**INSTRUCTIONS:**

1. This paper consists of 11 questions and 8 pages.

2. Read the questions carefully.

3. Answer all questions.

4. Number your answers clearly and use the same numbering as in the question paper.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Round off your answers to **four** decimal digits where necessary.

7. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks. Equations may not be solved solely with a calculator.

8. It is in your interest to write legibly and present your work neatly.

## QUESTION 1                                                                    27 MARKS

Solve for $x$ in each of the following:

a)   $|x|^2 - 5|x| = 14$                                                         (6)

b)   $\dfrac{|x^2 - 3x|(x+3)}{|x-3|} \leq 0$                                      (5)

c)   $\ln(e^{2x} - 12) - x = 0$                                                  (7)

d)   Given $f(x) = x^4 + 4x^3 + 3x^2 + 4x + 2$ and $f(i) = 0$, solve for $x$ if $f(x) = 0$   (9)


## QUESTION 2                                                                    12 MARKS

Evaluate:

a)   $\lim\limits_{x \to 4} \dfrac{x^2 - 2x - 8}{x - 4}$                          (3)

b)   $\lim\limits_{x \to 0} \dfrac{\sin 4x}{2\sin 2x}$                            (4)

c)   $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2 - 1}}{2x - 3}$                 (5)


## QUESTION 3                                                                    12 MARKS

Given $f(x) = \ln(x + 4)$

a)   State the domain and range of $f(x)$.                                       (2)

b)   Determine $f^{-1}(x)$, the inverse of $f(x)$ in the form $f^{-1}(x) = \ldots$   (3)

c)   Sketch the graphs of $f^{-1}(x)$ and $f(x)$ on the same axes, clearly labelling intercepts
     with the axes and asymptotes.                                              (7)

**QUESTION 4**                                                                                         **39 MARKS**

Determine:

a)   $f'(x)$ if $f(x) = \sqrt{5x}$ **by first principles.**                                            (8)

b)   $\frac{dy}{dx}$ if $y = (4x^2 + 2)^9$                                                              (3)

c)   $D_x\left[e^{x^3}\right]$                                                                          (3)

d)   $f'(x)$ if $f(x) = \ln\left(\frac{x}{x^2-1}\right)$                                                (7)

e)   $D_x[2x^4.\cos(x^3 - 1)]$                                                                          (6)

f)   $\frac{dy}{dx}$ if $x^2 + xy + y^2 = 9$                                                            (6)

g)   $f^n(x)$ if $f(x) = \frac{1}{x}$                                                                   (6)

**QUESTION 5**                                                                                         **13 MARKS**

Given:
$$f(x) = \begin{cases} x^2 & if \ x < 2 \\ |x - 6| & if \ x \geq 2 \end{cases}$$

a)   Write $f(x)$ as a split function without the absolute value notation.                             (4)
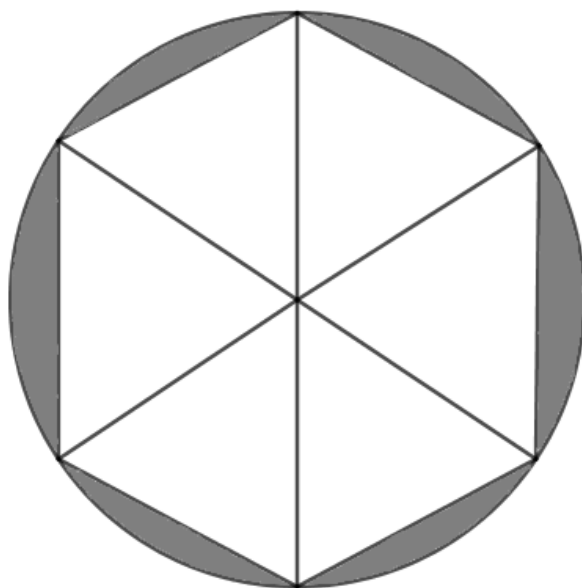
b)   Determine the value of $f'(1)$ and $f'(6)$.                                                        (3)

c)   Determine if $f(x)$ is continuous and differentiable at $x = 2$. Justify your answer fully.       (6)

## QUESTION 6                                                                8 MARKS

Six identical triangles are inscribed in a circle. Area of the total shaded region is $10\ units^2$.
Determine the radius of the circle.                                              (8)
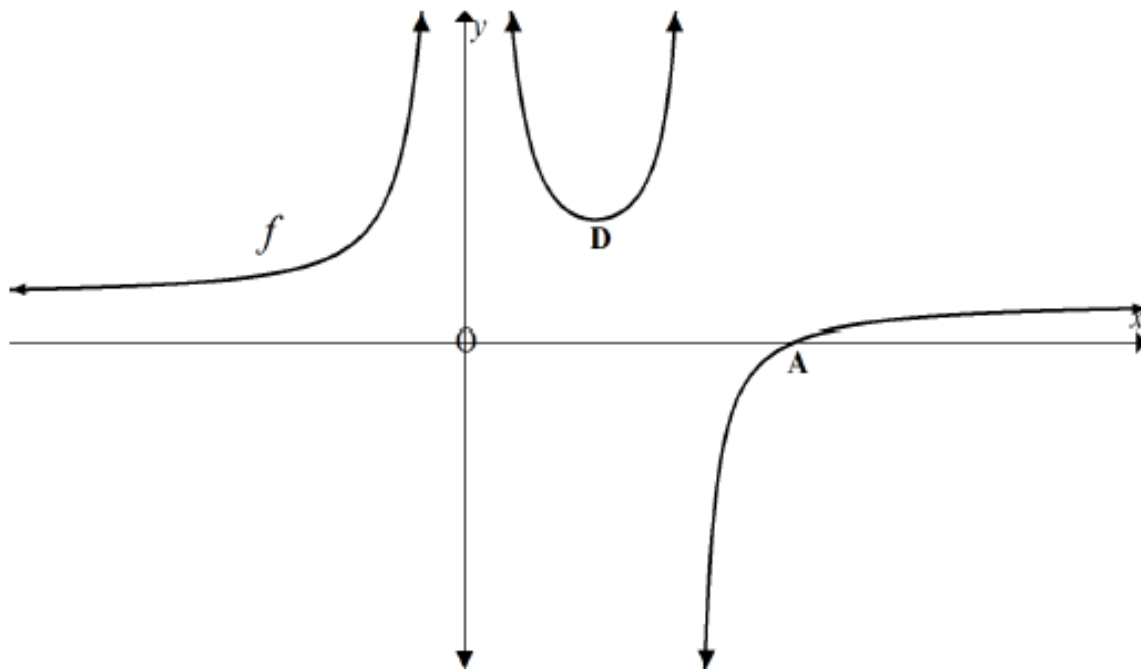


## QUESTION 7                                                               15 MARKS

Consider the function $f(x) = \frac{x^2+x-12}{x+3}$

a)   Determine the equation of the vertical asymptote.                           (2)

b)   Determine the equation of the oblique asymptote.                            (4)

c)   Sketch the graph of $f$.                                                    (9)

The graph of $f(x) = \frac{1}{x^2} - \frac{1}{x-2} + 1$ is shown, with a turning point at D(1,14; 2,93) and an x-intercept at the point indicated by A



a)     Give the equations of all the vertical and horizontal asymptotes of the graph of $f$.     (4)

b)     Use Newton's method to determine the coordinates of A, the x-intercept, correct to 4 decimal places. Use $x = 3$ as your initial value. (You must find $x_1$ manually, the rest can be done on the calculator, do not use the "solve function").     (8)

c)     For which values of $x$ is the function $f$ strictly increasing?     (4)

d)     Draw a rough sketch of $f'$ on the same system of axes above. Show clearly where $f'$ has asymptotes and intercepts with the axes, if any.     (6)

**QUESTION 9**                                                               **32 MARKS**

a)      Determine the following integrals:

1)      $\int e^{2x+3}\ dx$                                                        (3)
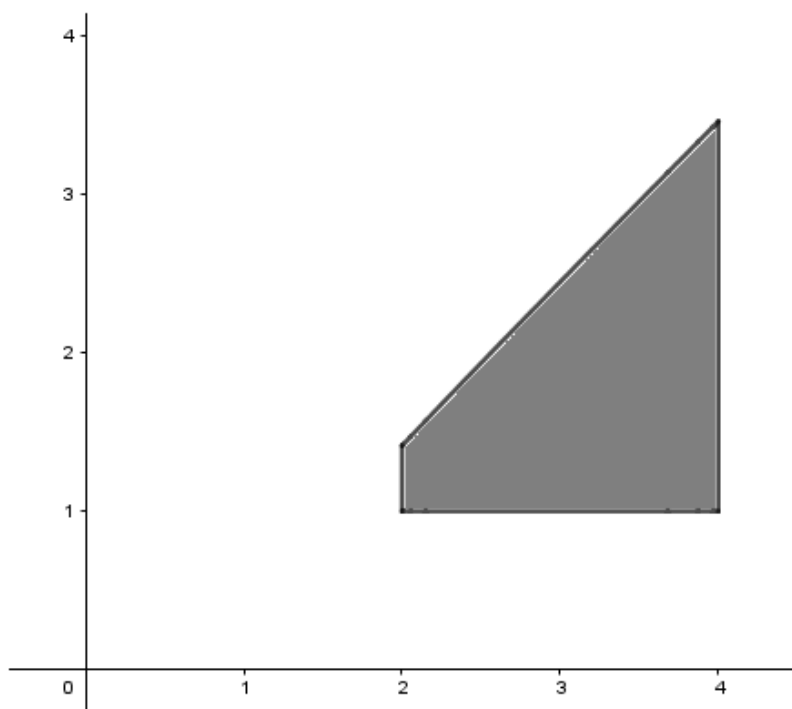
2)      $\int \frac{2x^7 - x^3}{x^8 - x^4}\ dx$                                        (4)

3)      $\int 3x(x + 5)^8\ dx$                                                    (7)

4)      $\int (2x + 3)\sin 4x\ dx$                                                (9)

b)      Find $k$ if $\int_0^k 3x\sqrt{x^2 + 5}\ dx = 10$                          (9)
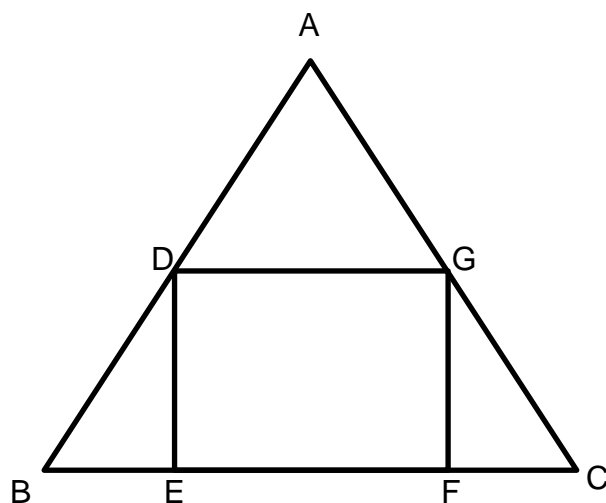
**QUESTION 10**                                                              **10 MARKS**

Determine the volume of a solid obtained by rotating the region bounded by $y = \sqrt{x^2 - x}$, $x = 2$, $x = 4$, and $y = 1$ about the $x - axis$.                                (10)

## QUESTION 11                        **10 MARKS**

In the sketch ∆ABC is an equilateral triangle with each side equal to $a$ units.
DEFG is a rectangle with BE = FC = $x$ units.

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a)    Prove that the area of the rectangle is $A(x) = x\sqrt{3}(a - 2x)$                 (5)

b)    If $a = 2$, determine the maximum area of the rectangle.               (5)