ST BENEDICT'S

SUBJECT	**AP Mathematics**	**PAPER**	**Paper 2**
GRADE	**12**	**DATE**	11 July 2019
EXAMINER	Mr Benecke	**MARKS**	100
NAME	**Memo**	**MODERATOR**	Mrs Povall + Mrs Serafino
TEACHER		**DURATION**	1 hours

QUESTION NO	MAXIMUM MARK	ACTUAL MARK
1	13	
2	4	
3	8	
4	8	
5	20	
6	19	
7	12	
8	8	
9	8	
TOTAL	100	

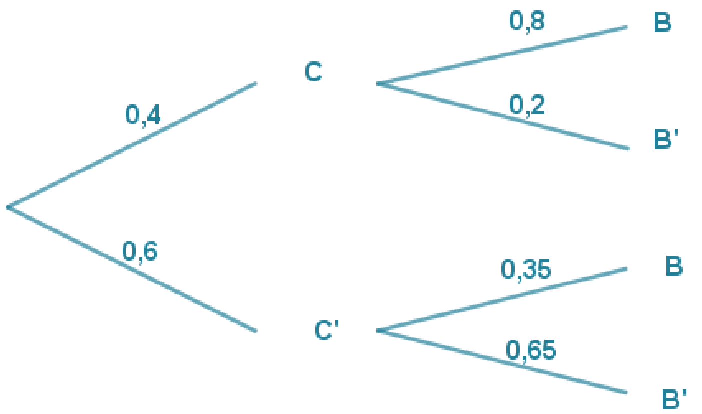
INSTRUCTIONS:

1. This paper consists of 9 questions and 10 pages.
2. Read the questions carefully.
3. Answer all questions.
4. Number your answers clearly and use the same numbering as in the question paper.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to **four** decimal digits where necessary.
7. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks. Equations may not be solved solely with a calculator.
8. It is in your interest to write legibly and present your work neatly.

QUESTION 1 **13 MARKS**

The probability that Dale plays cricket on any day is $0{,}4$. On a day when he plays cricket, the probability that he goes to bed early is $0{,}8$. On a day when he does not play cricket, the probability that he goes to bed early is $0{,}35$.

a) Determine the probability that he goes to bed early. (4)



$P(B) = (0{,}4 \times 0{,}8) + (0{,}6 \times 0{,}35) = 0{,}53$ ✓✓

b) Given that he played cricket, determine the probability that he will not go to bed early. (2)

$P(B'|C) = 0{,}2$ ✓✓

c) Given that he went to bed early, determine the probability that he played cricket. (4)

$P(C|B) = \frac{P(C\cap B)}{P(B)}$ ✓

$= \frac{0{,}4\times 0{,}8 ✓}{0{,}53 ✓}$

$= 0{,}6038$ or $\frac{32}{53}$ ✓

d) Are the events playing cricket and going to bed early independent.
Substantiate your answer mathematically. (3)

$P(C) \times P(B) = 0{,}4 \times 0{,}53 = 0{,}21$ ✓
$P(C \cap B) = 0{,}4 \times 0{,}8 = 0{,}32$ ✓

Not independent ✓

QUESTION 2 **4 MARKS**

Determine the probability of rolling a 6-sided dice three times, and getting a different number each time. (4)

$$P(X) = \frac{6\times5\times4\checkmark\checkmark}{6\times6\times6\checkmark\checkmark} = \frac{5}{9}$$

QUESTION 3 **8 MARKS**

On a shelf there are 4 **different** Mathematics textbooks and 8 **identical** English set books.

a) In how many ways can the books be arranged so that the Mathematics books are together? (4)

$$\frac{9!\checkmark\times4!\checkmark}{8!\checkmark} = 216\checkmark$$

b) In how many ways can the books be arranged so that all the Mathematics books are separated from each other? (4)

$$9\checkmark P\checkmark 4\checkmark = 3024\checkmark$$

QUESTION 4 **8 MARKS**

Let x be the amount of weight Johnny will gain/lose during the August holidays. The probability distribution is shown below.

x	-2	-1	0	1	2
$P(X = x)$	k	$0{,}2$	$5k$	$0{,}3$	$4k$

a) Prove that the value of $k = 0{,}05$ (4)

$k + 0{,}2 + 5k + 0{,}3 + 4k = 1$ ✓✓

$10k = 0{,}5$ ✓

$k = 0{,}05$✓

b) Determine the amount of weight (in kg's) that Johnny can expect to gain/lose during the August holidays. (4)

$(-2 \times 0{,}05) + (-1 \times 0{,}2) + (0 \times 5(0{,}05)) + (1 \times 0{,}3) + (2 \times 4(0{,}05))$ ✓

$= -0{,}1 - 0{,}2 + 0 + 0{,}3 + 0{,}4$✓

$= 0{,}4$✓

He can expect to gain $0{,}4\ kg$ during the August holidays. ✓

QUESTION 5 **20 MARKS**

The South African cricket team has registered 40 players to play in the 2019 Cricket World Cup. The probability of a cricket player picking up an injury during the course of the World Cup is $0{,}15$.

a) Determine the number of SA players that are expected to pick up an injury during the course of the competition. (2)

$40 \times 0{,}15 = 6\ players$ ✓✓

b) Determine the probability that at least 3 SA players will pick up an injury during the course of the World Cup? (8)

1✓ $- \binom{40}{0}(0{,}15)^0(0{,}85)^{40}$✓✓ $- \binom{40}{1}(0{,}15)^1(0{,}85)^{39}$✓✓ $- \binom{40}{2}(0{,}15)^2(0{,}85)^{38}$✓✓

$= 0{,}9514$ ✓

c) There are a total 600 cricket players taking part in the World Cup. Determine the probability that at least 100 players will pick up an injury during the course of the World Cup. Use a suitable approximation to determine the probability. (10)

$np = (600)(0{,}15) = 90$✓ $n(1-p) = (40)(0.85) = 510$✓

∴ a normal approximation can be used. ✓

$\sigma^2 = np(1-p) = 600(0{,}15)(0{,}85) = 76{,}5$ ✓✓

$P(X \geq 100) = P\left(Z \geq \frac{99{,}5✓ - 90}{\sqrt{76{,}5}}\right)$ ✓

$P(X \geq 100) = P(Z \geq 1{,}09)$✓

$P(X \geq 100) = 0{,}5 - 0{,}3621$✓

$P(X \geq 100) = 0{,}1379$✓

QUESTION 6 **19 MARKS**

Emily is planning her 18th birthday party at a restaurant. She has 20 female friends and 5 male friends. She can only invite 10 people to her party, so she decides to invite 10 people randomly.

a) Determine the probability that she invites only female friends. (4)

$$\frac{\binom{20}{10}\checkmark\binom{5}{0}\checkmark}{\binom{25}{10}\checkmark} = 0{,}0565\checkmark$$

b) Determine the probability that she will invite less than two male friends. (5)

$$\frac{\binom{20}{9}\checkmark\binom{5}{1}\checkmark}{\binom{25}{10}\checkmark} + \frac{\binom{20}{10}\binom{5}{0}}{\binom{25}{10}}\checkmark$$

$$= 0{,}2569 + 0{,}0565$$

$$= 0{,}3134 \quad or \quad \frac{793}{2530}\checkmark$$

c) Of Emily's 25 friends, there are 5 couples, and 15 other friends who are not part of a couple. In how many ways can Emily invite exactly 3 couples and 4 other friends, who may or may not part of a couple, to her party. (5)

$$\binom{5}{3}\checkmark\checkmark\binom{19}{4}\checkmark\checkmark = 38760\ ways\checkmark$$

d) Emily takes a random photograph of the friends invited in (c) standing in a line. What is the probability that all the couples are standing next to their partner in the photograph? (5)

$$\frac{2!\times2!\times2!\checkmark\times7!\checkmark\checkmark}{10!\checkmark} = \frac{1}{90} \quad or \quad 0{,}0111\ \checkmark$$

QUESTION 7 **12 MARKS**

a) A survey of 300 Americans revealed that 120 of them supported Donald Trump. Assuming a normally distributed population, find a 98% confidence interval for the true proportion of Americans who support Donald Trump. (8)

$\hat{p} = \frac{120}{300} = 0,4$ ✓

$z = \pm 2,33$ ✓

$$p = \hat{p} \pm Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$ ✓

$$p = 0,4 \pm 2,33\sqrt{\frac{0,4(0,6)}{300}}$$ ✓✓

$$p = \left(0,4 - 2,33\sqrt{\frac{0,4(0,6)}{300}}; 0,4 + 2,33\sqrt{\frac{0,4(0,6)}{300}}\right)$$ ✓

$p = (0,3341$ ✓$; 0,4659$ ✓$)$

b) Describe, in words, what this confidence interval means (2)

We are 98% confident that between 33,4% and 46,59% of Americans support Donald Trump. ✓✓

c) Why do you think this interval is so large? State two reasons: (2)

1. The sample size is small ✓
2. The confidence level is large ✓

QUESTION 8 **8 MARKS**

a) A sample of 80 Gr 12 learners average driving speeds (in km/h) revealed a 96% confidence interval of (60;86) as an estimate of the true population mean of all South African Gr 12 learners average driving speeds. Determine the standard deviation: (8)

$\bar{x} = \frac{60+86}{2} = 73$ ✓✓

$\mu = \bar{x} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right)$ ✓

$86 = 73 + 2{,}05$✓ $\left(\frac{\sigma}{\sqrt{80}}\right)$✓✓

$13 = 2{,}05\left(\frac{\sigma}{\sqrt{80}}\right)$✓

$\sigma = 56{,}7198$✓

QUESTION 9 **8 MARKS**

A recent study found that the mean weight of a cricket player is $90kg$ with standard deviation of $10kg$. A random sample of 40 SA cricket players is selected.

a) What is the probability that the average weight of the SA cricket players will be less than $92kg$. (5)

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \checkmark$$

$$z = \frac{92-90}{\frac{10}{\sqrt{40}}} \checkmark$$

$$z = 1{,}2649 \checkmark$$

$$P(X < 92) = P(Z < 1{,}2649)$$
$$= 0{,}5 + 0{,}3962\ (or\ 0{,}3980) \checkmark$$
$$= 0{,}8962\ (or\ 0{,}8980) \checkmark$$

b) Was it appropriate to use the central limit theorem (a)? Explain your answer. (3)

Yes✓, even though the distribution isn't normal✓, the sample size is larger than 30✓.