ST. DAVID'S MARIST INANDA

ADVANCED PROGRAMME MATHEMATICS

PRELIMINARY EXAMINATION
PAPER 2: OPTIONAL MODULE
STATISTICS

GRADE 12

3 SEPTEMBER 2019

EXAMINER: MRS S RICHARD	**MARKS: 100**
MODERATOR: MR L VICENTE	**TIME: 1 hour**

NAME: Memo

INSTRUCTIONS:

- ✓ This paper consists of 16 pages and a separate 4-page formula sheet. Please check that your paper is complete. The last 3 pages are blank for working out.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ Round answers to 2 decimal places, unless stated otherwise.
- ✓ It is in your interest to show all your working details.
- ✓ Work neatly. Do NOT answer in pencil.
- ✓ Diagrams are not drawn to scale.

QUESTION	Q1 [14]	Q2 [11]	Q3 [15]	Q4 [14]	Q5 [10]	Q6 [7]	Q7 [15]	Q8 [14]	TOTAL [100]
LEARNER'S MARKS									

QUESTION 1

The two events A and B are such that $P(A) = 0{,}4$; $P(B) = 0{,}24$ and $P(A/B) = 0{,}25$

a) Prove that the probability that both events occur is 0,06. (2)

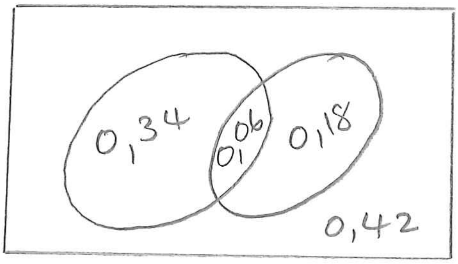
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$0{,}25 = \frac{P(A \cap B)}{0{,}24}$$

$$0{,}06 = P(A \cap B)$$

b) Calculate the probability that:

i) at least one of the events occur. (2)



P(at least one event occurs)

$= 1 - 0{,}42$

$= 0{,}58$

ii) exactly one event occurs. (3)

P(exactly one event occurs)

$= 0{,}34 + 0{,}18$

$= 0{,}52$

iii) B occurs given A has occurred. (3)

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} \checkmark$$

$$= \frac{0{,}06}{0{,}4} \checkmark$$

$$= 0{,}15 \checkmark$$

c) Are events A and B independent? Support your answer using calculations. (4)

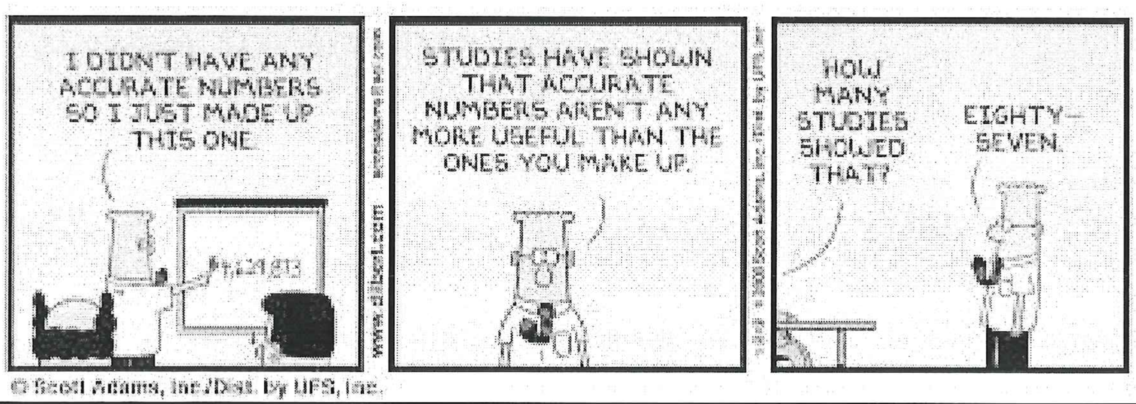
$$P(A) \times P(B) = 0{,}4 \times 0{,}24$$

$$= 0{,}096 \checkmark$$

$$P(A \cap B) = 0{,}06 \checkmark$$

$$P(A) \times P(B) \neq^{\checkmark} P(A \cap B)$$

$\therefore$ events NOT ✓ independent



[14]

QUESTION 2

a) In an Inter High school hockey tournament, with 7 teams competing, each team must play against every other team. How many games will be played? (2)

$$\binom{7}{2} = {}^{7}C_{2} = 21$$

b) One of the teams, consisting of 13 players, is to be transported in 3 different vehicles belonging to 3 of the players' parents. The vehicles can carry 3, 4 and 6 players respectively. In how many ways can the players be transported? (4)

$$\binom{13}{3}\binom{10}{4}\binom{6}{6} = 60060$$

$$\frac{13!}{3!\cdot 4!\cdot 6!} = 60060$$

c) The 3 players whose parents are helping with the transport would naturally like to travel with their parents. What is the probability that each of the 3 players ends up going with his own parents, if the other players are all allocated randomly? (Round answer to 5 decimal places) (5)

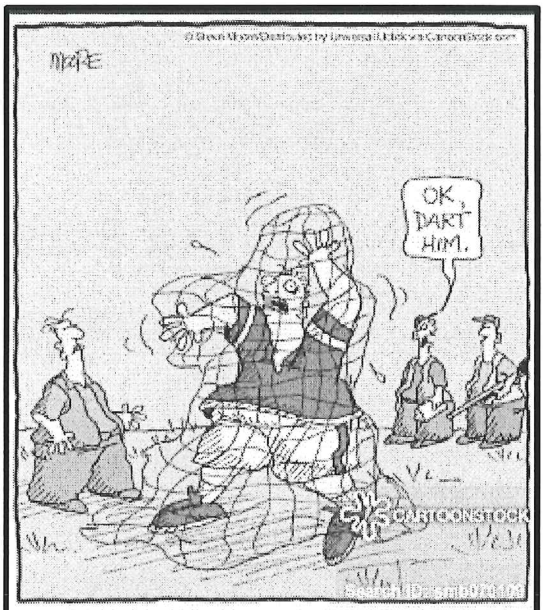
$$\binom{10}{2}\binom{8}{3}\binom{5}{5}$$

$$45 \times 56 \times 1$$

$$= 2520$$

$$P(\text{own parents}) = \frac{2520}{60060} \quad \text{use b)}$$

$$= 0{,}04196$$



After biologists draw blood samples and record vital statistics, the rugby player is tagged and released.

[11]

QUESTION 3 (Round answers to 4 decimal places, where applicable)

a) A factory makes a large number of coloured sweets and it is known on average that 20% of the sweets are green. A packet contains 25 sweets (a random sample from the factory).

i) Determine the expected value for the number of green sweets in a packet of 25. (1)

$20\% \times 25 = 5$ sweets ✓

ii) Calculate the probability that 9 of the sweets in the packet are green. (6)

$\binom{25}{9}(0,2)^9(0,8)^{16}$ ✓ ✓ ✓ ✓ ✓

$= 0,0294$ ✓

iii) Determine the standard deviation for the number of green sweets in a packet of 25. (2)

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{25(0,2)(0,8)}$ ✓

$= 2$ ✓

$Var[X] = npq$

$25 \times 0,2 \times 0,8$

$= 4$ ✓

$\therefore \sigma = 2$ ✓

You know how many ∴ hypergeometric.

b) A batch of 20 integrated circuit chips contain 20% that are defective. A sample of 10 is drawn at random. What is the probability that at least one of the chips will be defective? (6)

*

$P(\text{at least one})$

$= 1 - P(\text{none})$ ✓M

$= 1 - \dfrac{\binom{10}{0}\binom{10}{4}}{\binom{20}{4}}$ ✓ ✓ ✓

$= 0{,}9567$ ✓✓

OR

$P(X=0)$

$= \dfrac{{}^{4}C_{0} \times {}^{16}C_{10}}{{}^{20}C_{10}}$ ✓ ✓ ✓

$= \dfrac{14}{323}$

$= 0{,}0433\ldots$

[15]

$1 - \dfrac{14}{323}$ ✓

$= \dfrac{309}{323}$

$= 0{,}9567$ ✓✓

If Bin $2/6$ $1 - P(X=0)$ ✓ $= 1 - \binom{10}{0}$ ✓ …

Read carefully!

* A batch of 20 of which 20% defective ∴ 4 defective and 16 not defective

∴ hypergeometric

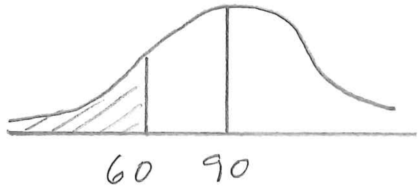
If "A batch of integ. circuit chips contain 20% that are defective" then would be binomial.

QUESTION 4

The length of life (in months) of a certain hair dryer is approximately normally distributed with mean of 90 months and standard deviation 15 months.

$5 \times 12 = 60$ months.

a) Each hair dryer is sold with a 5-year guarantee. What percentage of hair dryers fail before the guarantee expires? (8)



$\sigma = 15$ months

$\overline{X} = 90$ months

$z = \frac{60 - 90}{15} = -2$

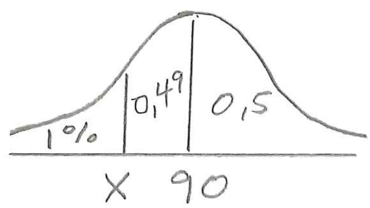
$P(z \leq -2) = 0{,}5 - 0{,}4772$

$= 0{,}0228$

$= 2{,}28\%$

b) The manufacturer decides to change the length of the guarantee so that no more than 1% of hair dryers fail during the guarantee period. How long should he make the guarantee? (6)

$z = \frac{X - \overline{X}}{\sigma}$



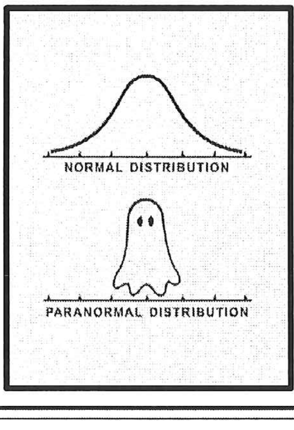
$X = \overline{X} + z\sigma$

$= 90 - 2{,}33(15)$

$= 55{,}05$ months

He should make the guarantee 55 months (4,58 years)

[14]



QUESTION 5

The pH value of water measures the degree of its acidity. The water in a particular dam is known to have pH values with variance 0,25. Environmentalists obtain ten samples of water from the dam and test them. The mean pH of the samples is 8,2.

a) Obtain a 96% confidence interval for the true population mean pH for the dam. Give your answer correct to 2 decimal places. (8)

variance $0{,}25$

$\therefore \sigma = 0{,}5$

$n = 10$

$\bar{x} = 8{,}2$

96% CI : $\bar{x} \pm 2{,}05 \frac{\sigma}{\sqrt{n}}$

maybe 2,06

$= 8{,}2 \pm 2{,}05 \frac{(0{,}5)}{\sqrt{10}}$

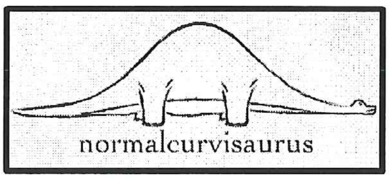
$= [7{,}88 ; 8{,}52]$

$[7{,}87 ; 8{,}53]$

b) Explain the meaning of your answer in a). (2)

If a large number of samples are tested, the mean pH will lie in the above interval in 96% of the cases

[10]



QUESTION 6

A discrete random variable has a probability mass function as defined below:

$$P(X = x) = \frac{2x+3}{k} \text{ for } x \in \{1,2,3\}$$

a) Determine the value of the constant k. (3)

x	1	2	3
$P(x)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$

✓

$$\frac{5}{k} + \frac{7}{k} + \frac{9}{k} = 1 \checkmark$$

$$21 = k \checkmark$$

b) Determine the expected value of the probability mass function. (4)

$$P(X=1) = \frac{5}{21}$$

$$P(X=2) = \frac{7}{21}$$

$$P(X=3) = \frac{9}{21}$$

$$E(X) = 1 \times \frac{5}{21} \checkmark + 2 \times \frac{7}{21} \checkmark + 3 \times \frac{9}{21} \checkmark$$

$$= \frac{46}{21}$$

[7]

$$= 2{,}19 \checkmark$$

QUESTION 7

A random variable has a probability density function given by:

$$f(x) = \begin{cases} k(1-\frac{1}{3}x), & \text{if } 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

a) Show that $k = \frac{3}{4}$. (5)

$$\int_0^2 k\left(1-\tfrac{1}{3}x\right) dx = 1$$

$$k\left[x - \tfrac{1}{3}\cdot\tfrac{x^2}{2}\right]_0^2 = 1$$

$$k\left[\left[2 - \tfrac{1}{3}\tfrac{(2)^2}{2}\right] - [0]\right] = 1$$

$$k\left(2 - \tfrac{2}{3}\right) = 1$$

$$k\left(\tfrac{4}{3}\right) = 1 \quad \therefore \quad k = \tfrac{3}{4}$$

On calc:

$$\int_0^2 1 - \tfrac{1}{3}x \, dx$$

$$= \tfrac{4}{3}$$

$$\therefore k\left(\tfrac{4}{3}\right) = 1$$

$$k = \tfrac{3}{4}$$

b) Determine the median. (10)

$$\int_0^m \tfrac{3}{4}\left(1 - \tfrac{1}{3}x\right) dx = 0{,}5$$

$$\tfrac{3}{4}\left[x - \tfrac{x^2}{6}\right]_0^m = 0{,}5$$

$$\left[x - \tfrac{x^2}{6}\right]_0^m = \tfrac{2}{3}$$

$$m - \tfrac{m^2}{6} - 0 = \tfrac{2}{3}$$

$$6m - m^2 = 4$$

$$m^2 - 6m + 4 = 0$$

$$m \neq 5{,}24 \text{ or } 0{,}76$$

[15]

QUESTION 8

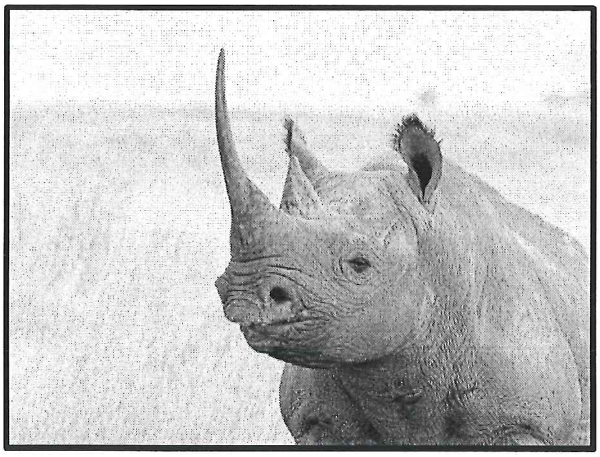
The following are the latest statistics on the number of rhinos poached in each Province in 2017 and 2018:

	2017	**2018**
Gauteng	8	15
Limpopo	114	110
Mpumalanga	92	83
North West	87	65
Eastern Cape	5	15
Free State	4	4
Western Cape	85	70
Northern Cape	0	5
KZN	0	99

a) Calculate the mean and the standard deviation of rhinos poached per province for the years 2017 and 2018. (4)

$\bar{x}_{2017} = 43,88\ldots$ ✓ $\bar{x}_{2018} = 51,77\ldots$ ✓

$\sigma_{2017} = 45,97\ldots$ ✓ $\sigma_{2018} = 39,829\ldots$ ✓



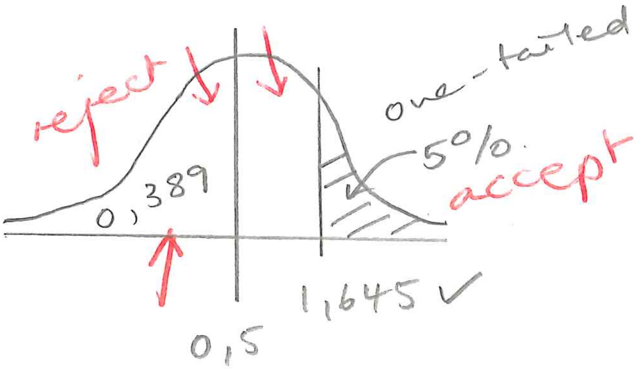
b) The Minister of Agriculture claimed the following:

"On average, 2018 showed an increase from 2017 in the number of rhinos poached across all provinces".

Hence, set up a formal hypothesis test at the 5% level of significance, to test his claim (assume the sample comes from a normal distribution) (10)

$H_0 : \mu_{2018} = \mu_{2017}$ ✓

$H_1 : \mu_{2018} > \mu_{2017}$ ✓



label and shade regions.

use a)

$$z = \frac{51,77\ldots - 43,88}{\sqrt{\frac{(39,83)^2}{9} + \frac{(45,97)^2}{9}}}$$ ✓ formula

$= 0,389$ ✓

lies in reject ✓ region

∴ there is not enough evidence to reject H_0 at 5% significance level

∴ we cannot say there is an ✓ increase.

[14]

[TOTAL: 100 MARKS]