ST. DAVID'S MARIST INANDA

ADVANCED PROGRAMME MATHEMATICS

OPTIONAL MODULE: FINANCE AND MODELLING

GRADE 12

3 SEPTEMBER 2019

EXAMINER: MRS L NAGY	MARKS: 100 TIME: 1 HOUR

NAME: MEMO

INSTRUCTIONS:

- ✓ This paper consists of **15** pages. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ It is in your interest to show all your working details.
- ✓ Work neatly. Do NOT ANSWER IN PENCIL.

QUESTION	Q1 [18]	Q2 [31]	Q3 [11]	Q4 [9]	Q5 [31]	TOTAL [100]
LEARNER'S MARKS						

QUESTION 1 [18 marks]

(a) You open a savings account and immediately deposit R12 300. Six months after the initial deposit, you deposit another R3 000 in the same savings account followed by another 6 quarterly deposits of R3 000. The account earns interest at a rate of 5,6% per annum compounded quarterly for the first 2 years and then it changes to 6% p.a. compounded quarterly for the next four years. Calculate the value of your savings account at the end of this six-year period. (12)

$$F_V = 12300\left(1+\frac{0,056}{4}\right)^{8}\left(1+\frac{0,06}{4}\right)^{16}+3000\left[\frac{\left(1+\frac{0,056}{4}\right)^{7}-1}{\frac{0,056}{4}}\right]\left[\left(1+\frac{0,06}{4}\right)\right]^{16}$$

$$F_V = R45239,20$$

(b) However, you decided to withdraw R8 000 from the account mentioned above, one year after the savings account was opened. Hence, calculate the actual value of your savings account after 6 years. (6)

$A = 8000\left(1 + \frac{0,056}{4}\right)^{4}\left(1 + \frac{0,06}{4}\right)^{16}$ ✓✓✓

$A = R10732,44$ ✓✓

Decreased value

$$\begin{array}{r} R45239,20 \\ -\ 10732,44 \\ \hline R34506,76 \end{array}$$ ✓

QUESTION 2 [31 marks]

You start a restaurant with a loan of R1 000 000. You must repay this loan in equal monthly instalments, at the end of each month, over a 3-year period.
The interest rate is 7,2% per annum compounded monthly.

(a) Calculate your monthly instalments. (4)

$$1\,000\,000 = x\left[\frac{1-\left(1+\frac{0,072}{12}\right)^{-36}}{\frac{0,072}{12}}\right]$$

$$x = R\ 30\ 968,62$$

(b) Determine the amount outstanding immediately after you have paid your 12th instalment. (4)

$$BO = 30\,968,62\left[\frac{1-\left(1+\frac{0,072}{12}\right)^{-24}}{\frac{0,072}{12}}\right]$$

$$BO = R\ 690\ 288,46$$

Suppose your restaurant needs to be refurbished and you need a further loan of R350 000 immediately after the 12th instalment was made. Under the new agreement you must settle the second loan plus the balance outstanding of the first loan in equal monthly instalments of R40 000 each, plus one final payment less than R40 000. The new interest rate is 6,4% per annum compounded **quarterly**.

(c) How many **monthly** instalments of R40 000 will you have to make? (12)

$$\left(1+\frac{i}{12}\right)^{12} = \left(1+\frac{0,064}{4}\right)^{4} \checkmark$$

$$i_{NOM} = 12\left[\sqrt[12]{\left(1+\frac{0,064}{4}\right)^{4}} - 1\right] \checkmark$$

$$r = 6,366166875\% \text{ per annum} \checkmark\checkmark$$

$$104\,0288,46 = 40000\left[\frac{1-\left(1+\frac{0,063....}{12}\right)^{-n}}{\frac{0,063..}{12}}\right] \checkmark\checkmark$$

(+2) ✓✓ for working out

$$\therefore n = 28,05974559 \checkmark\checkmark$$

28 pmts of R40 000 each ✓✓

(d) What is the amount of your final payment? (7)

$$1040288,46 = 40000\left[\frac{1-\left(1+\frac{0,063\ldots}{12}\right)^{-28}}{\frac{0,063\ldots}{12}}\right] + y\left(1+\frac{0,063}{12}\right)^{-29}$$

$$y = R\,2394,82$$

(e) Calculate the total interest paid. (4)

$$30\,968,62 \times 12 + 40000 \times 28 + 2394,82$$

$$= R\,1494018,26$$

$$1494018,26 - 1350\,000 = R\,144\,018,26$$

QUESTION 3 [11 marks]

Tilapia is a small fresh-water fish that has made a huge splash in the seafood industry. You are starting a fish farm with 5 tilapia of which 60% is female. A female spawns every 4 weeks and will spawn approximately 600 eggs in the male's nest. The death rate of this tilapia is 25%. Approximately 70% of these eggs will survive to maturity.

(a) (1) Calculate the value of r and use this value to set up an equation using a Malthusian model. (5)

$r = b - d$

$r = 252 - 0,25$

$r = 251,75$ ✓

$b = 0,60 \times 600 \times 0,70$

$b = 252$ ✓✓

$P_{n+1} = (1 + 251,75)P_n \quad ; \quad P_0 = 5$ ✓

$= 252,75 . P_n$ ✓

(2) Calculate the population size of these tilapia at the end of each month for a period of 3 months. (3)

$P_1 = 1263,75$ ✓

$P_2 = 319412,81$ ✓

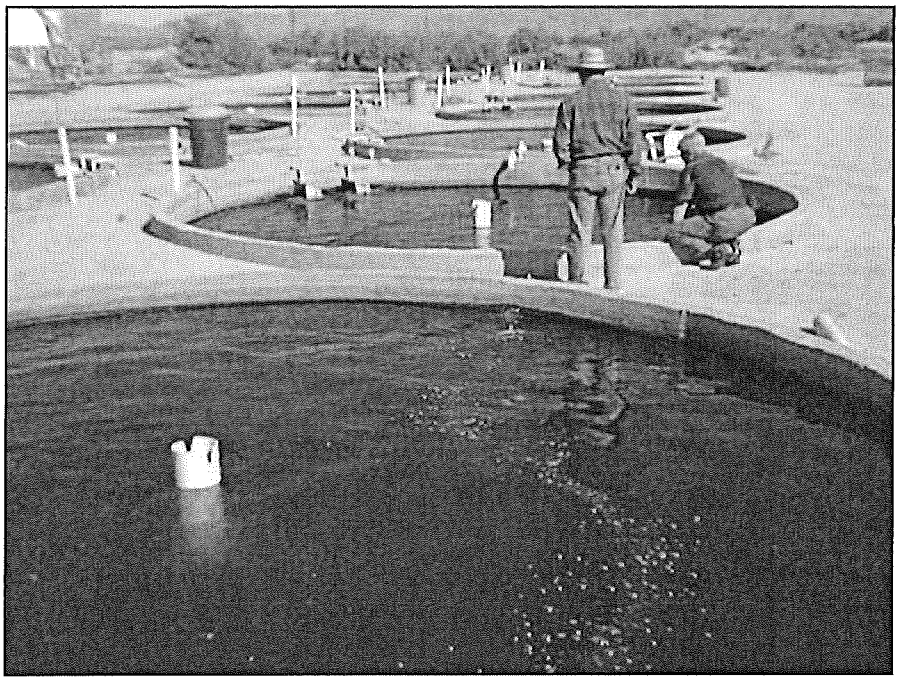
$P_3 = 807\,31588,36$ ✓

(b) How many tilapias would you need to sell every month to keep the tilapia population in a stable state from the third month onwards? (3) ✓✓

$807\ 31588{,}36 = 807315588{,}36\ (252{,}75) - x$

$x = 2{,}032417737 \times 10^{10}$ ✓

A tilapia farm may use tanks or ponds for breeding.



QUESTION 4 [9 marks]

A population of 34 giraffes are kept in a protected reserve. Their population increases before levelling off to a stable population.

The table below records the giraffe population at yearly intervals.

n	P_n	ΔP	$\frac{\Delta P}{P}$
0	34		
1	42	8,5	**A** = 0,20238 ✓✓
2	51	**B** = 10 ✓	0,19078
3	62	12	0,19355
4	75	14,5	0,19333
5	**C** = 91 ✓		

(a) Complete the table by calculating the values of A, B and C giving the answers to A, B and C where necessary correct to 5 decimal places. (4)

(b) Determine the equation of the line of best fit that represents the relationship between the growth rate and the population. ROUND off all values to 5 decimal places. (3)

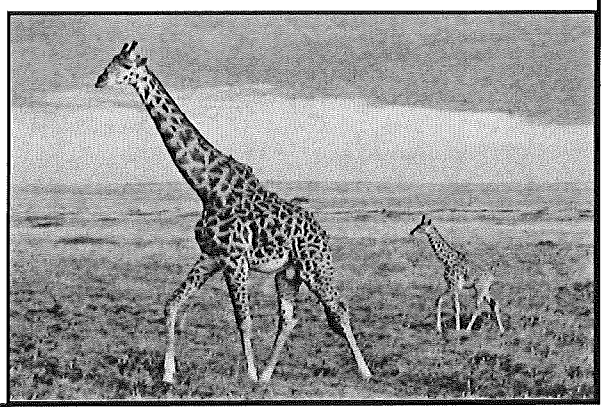
$y = A + Bx$

$y = -0,00106x + 0,20697$ ✓ ✓ ✓

(c) Calculate, as a percentage correct to 2 decimal places, the annual intrinsic growth rate for this model. (2)

$r = Y\text{-int} = 0,20697$ ✓

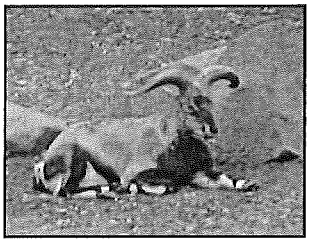
$\therefore 20,70\ \%$ ✓



QUESTION 5 [31 marks]

In China there is an area where 16 snow leopards live in the wild of which 50% are female. A female snow leopard gives birth once a year and on average there will be 2 cubs per litter. The life span of a snow leopard in this area is approximately 10 years. The favourite food of a snow leopard is the Himalayan blue sheep. The prey consumption of a snow leopard is on average 3 sheep per month.

Himalayan blue sheep are neither blue nor sheep as you can see below! They behave more like goats. Their coats have a blue sheen to them. They are on the endangered species list.



Currently, there are 8000 Himalayan blue sheep in an area which can sustain 20 000.
A female blue sheep gives birth to one baby calf a year.
65% of the blue sheep are females.
80% of the blue sheep calves survive to maturity.

Use the predator-prey model to answer the following questions. Let the snow leopard population be S and the Himalayan blue sheep population be B.

Hence, the models are:

$$S_{n+1} = S_n + b.f.S_n.B_n - c.S_n$$

$$B_{n+1} = B_n + a.B_n\left(1 - \frac{B_n}{K}\right) - b.S_n.B_n$$

The population cycles are in months.

(a) Use the information above to describe the purpose of the following parameters:

.

(1) c (2)

Per capita natural ✓ death rate of a snow leopard per cycle / month. ✓

(2) b (2)

Per capita attack rate of snow leopard on bharal

(b) Write down the value of the parameter K and describe the role of K in this model. (2)

K = 20000 ✓
Carrying capacity - the maximum number of Himalayan blue sheep which can be sustained by ✓ the environment.

(c) (1) Describe the role of the term $b.S_n.B_n$ in the prey-model. (2)

The number of Himalayan Blue Sheep killed and eaten by the snow leopard in a monthly ✓ cycle.

(2) Calculate the value of b, if $B_0 = 8000$. (4)

$b.S_n.B_n = 3 \times 16$

$b = \frac{48}{\sqrt{16 \times 8000}}$ ✓ ✓

$b = 0,000375$ ✓

(d) The population of the two species tends to an equilibrium or stable state.

(1) Show that the stable state achieved by the two species is represented by $\frac{c}{bf}$ and $\frac{a}{b}\left(1-\frac{B_n}{K}\right)$ (5)

$S_{n+1} = S_n$ ✓

$c.\ S_n = b.f.S_n.B_n$ ✓

$c = b.f.B_n$

$\frac{c}{bf} = B_n$

$B_{n+1} = B_n$ ✓

$b.S_n.B_n = a.B_n\left(1-\frac{B_n}{K}\right)$ ✓

$b.S_n = a\left(1-\frac{B_n}{K}\right)$ ✓

$S_n = \frac{a}{b}\left(1-\frac{B_n}{K}\right)$

(2) Calculate, correct to 5 decimal places, the values of the following parameters:

i) a (2)

$a = 1 \times \frac{1}{12} \times 0{,}65 \times 0{,}80$ ✓

$a = \frac{13}{300}$ ✓ or $0{,}04333$

ii) b (2)

Asked twice ; 0,000375 ✓✓

iii) c (1)

$\frac{1}{10}$ ✓

(3) If the parameter f = 0,05482, calculate the number of snow leopards and blue sheep representing the state of equilibrium for this area in China. (5)

$B_n = \frac{c}{bf} = \frac{\frac{1}{10}}{0,000375 \times 0,05482}$ ✓ $= 486,44$

$\therefore 486$ ✓

$S_n = \frac{a}{b}\left(1 - \frac{B_n}{K}\right)$

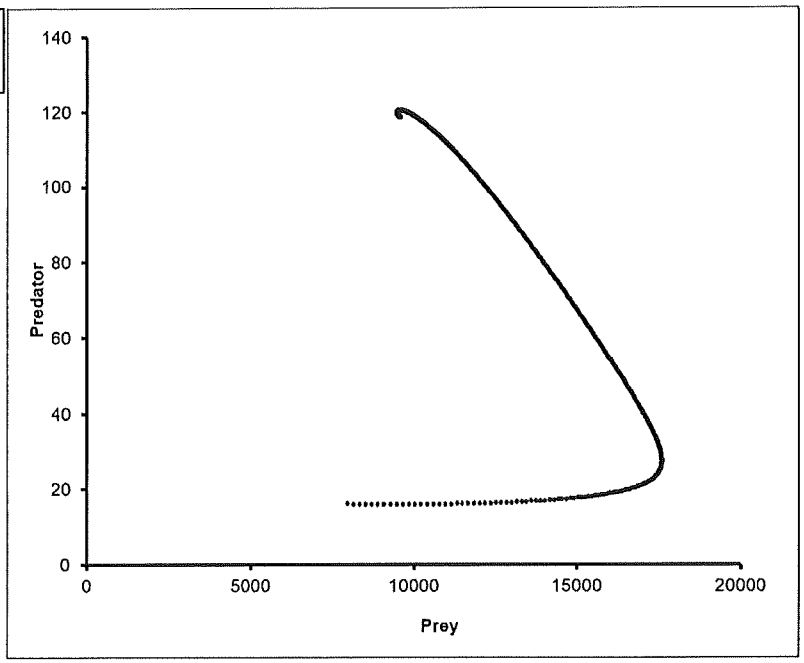
$= \frac{0,04333}{0,000375}\left(1 - \frac{486}{20000}\right)$ ✓

$= 112,7$

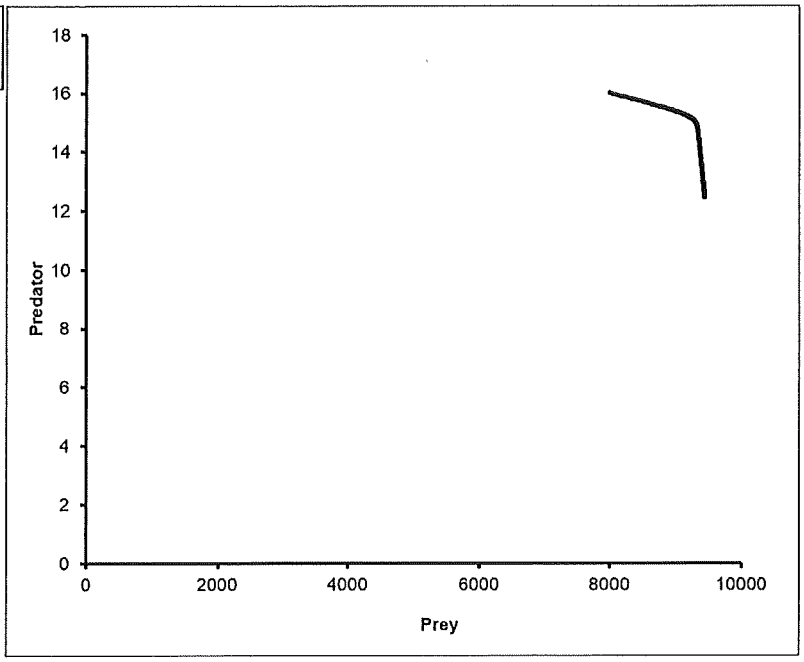
$= 113$ ✓ Rounding ✓

(4) What might happen to the equilibrium number for both species should the parameter K be halved? Use the given graphs below to help you formulate your answer. (4)

Graph 1
K=20 000



Graph 2
K=10 000

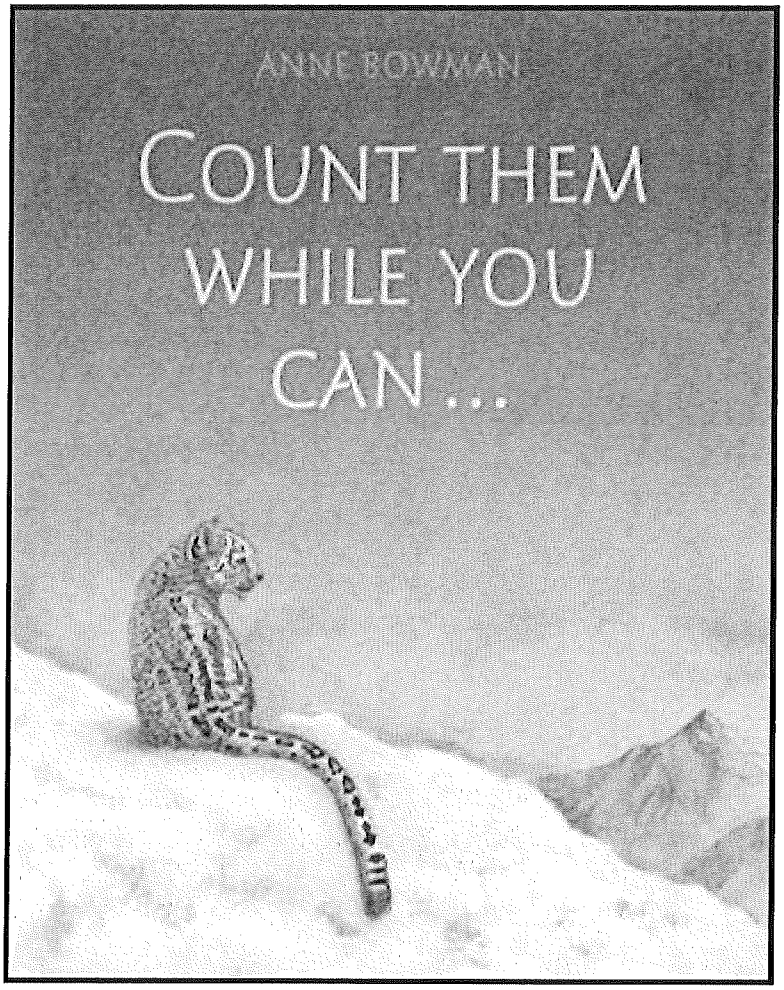


A state of equilibrium will be achieved in less time/ fewer cycles ✓

The Himalyan Blue sheep will decrease in number and remain below 10 000, the new carrying capacity. ✓

Initially the snow leopards will increase because of an abundance of food but then the snow leopards' population will decline because of less food as the carrying capacity is much smaller. ✓

Both populations became smaller at the point of equilibrium. ✓


ANNE BOWMAN
COUNT THEM WHILE YOU CAN...