AP Mathematics Elective MEMO.

1.1 a) $-x+10=2$

$x=8$ (2)

$\therefore P=\begin{pmatrix} 8 & 2 \\ -5 & -1 \end{pmatrix}$

b) $P^{-1}=\frac{1}{2}\begin{pmatrix} -1 & -2 \\ 5 & 8 \end{pmatrix}$ (3) $=\begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{5}{2} & 4 \end{pmatrix}$

1.2.

$\det A = p(-p+1)-2(-3)=0$

$-p^2+p+6=0$

$p^2-p-6=0$

$(p-3)(p+2)=0$

$\therefore p=3$ or $p=-2$ (4)

transpose = 1 mark.

2a) $2C^t-D = 2\begin{pmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 2 \\ k+3 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} -4 & 2 & 2 \\ 2 & -2 & 0 \\ 4 & 0 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 2 \\ k+3 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} -5 & 1 & 0 \\ -k-1 & -2 & -2 \\ 3 & -1 & -3 \end{pmatrix}$ (2)

2b. $\text{Det } D = 1(-2) - 1(k+3-2) + 2(k+3)$
$= -2 - k - 1 + 2k + 6$
$= k+3$

(2)

c $k+3 = 0$
$k = -3$

(2)

Q3 $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 3 & -5 & -2 & 1 \\ -7 & 11 & a & -11 \end{array}\right]$

Set up matrix

$R2 - 3R1$; $R3 + 7R1$

$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 0 & 1 & -5 & 13 \\ 0 & -3 & 7+a & -39 \end{array}\right]$

$R3 + 3R2$

$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 0 & 1 & -5 & 13 \\ 0 & 0 & -8+a & 0 \end{array}\right]$

for infinitely many solutions

$-8 + a = 0$
$\therefore a = 8.$

(4)

b) $y - 5z = 13$
$\therefore y = 13 + 5z.$

$\therefore x - 2(13 + 5z) + z = -4$
$x - 26 - 10z + z = -4$
$x - 9z = 22$

(4)

Q4

$$P = \begin{pmatrix} -1 & 3 & 0 & -2 \\ -3 & 1 & 0 & 4 \\ -2 & 1 & 4 & 1 \\ 2 & -1 & 3 & 0 \end{pmatrix}$$

$R2 - 3R1$; $R3 - 2R1$; $R4 + 2R1$

$$P = \begin{pmatrix} -1 & 3 & 0 & -2 \\ 0 & -8 & 0 & 10 \\ 0 & -5 & 4 & 5 \\ 0 & 5 & 3 & -4 \end{pmatrix}$$

$$\therefore \det P = -1\left[-8(-16-15) + 10(-15-20)\right] \times (-1)$$

$$= -102$$

or

$$2(-8+0-48) + 1(24-0+16) + 3(15-5-20)$$

Q4

$$P = \begin{pmatrix} 2 & -1 & 3 & 0 \\ -3 & 1 & 0 & 4 \\ -2 & 1 & 4 & 1 \\ -1 & 3 & 0 & -2 \end{pmatrix}$$

$R1 + R2$; $R1 + R3$; $R4 + 3R1$

$$= \begin{pmatrix} 2 & -1 & 3 & 0 \\ -1 & 0 & 3 & 4 \\ 0 & 0 & 7 & 1 \\ 5 & 0 & 9 & -2 \end{pmatrix}$$

$$\det P = +1 \begin{pmatrix} -1 & 3 & 4 \\ 0 & 7 & 1 \\ 5 & 9 & -2 \end{pmatrix}$$

$$= +1\left[-1(-23) - 3(-5) + 4(-35)\right]$$

$$= -102$$

Q5a) $\begin{pmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$ ✓ ✓

$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ ✓ ✓ (4)

b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ✓ ✓ (2)

c) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ ✓✓ order

$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ ✓ ✓ (4)

Q6a) $\Delta A \rightarrow \Delta B$

STRETCH ✓ ; parallel to x-axis ✓ / invariant y-axis

$k = 2$ ✓

$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ ✓ ✓ (5)

b) SHEAR ✓ ; invariant y-axis ✓ $k = \frac{1}{2}$ ✓

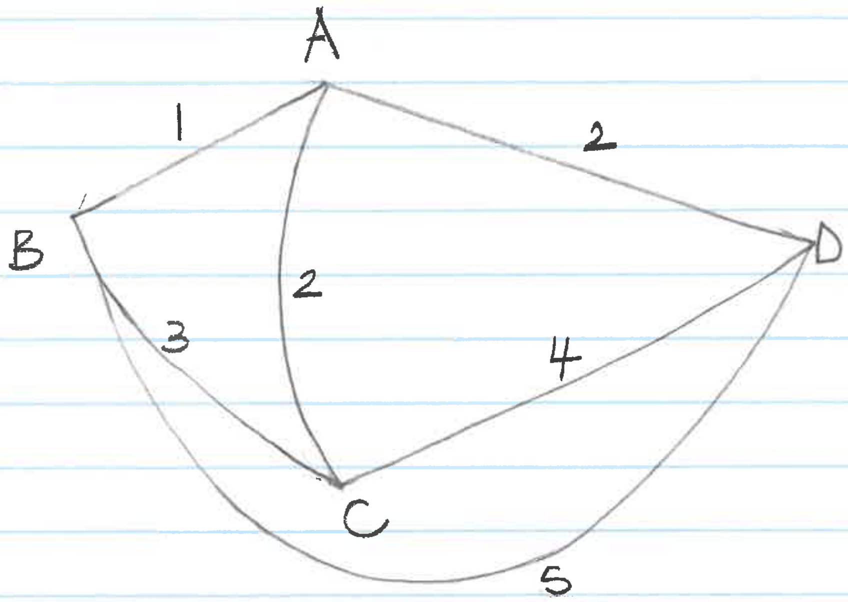
$\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}$ ✓ ✓ (5)

Q7a) construct an edge from P to U.

b)

	A	B	C	D
A	0	1	2	2
B	1	0	3	5
C	2	3	0	4
D	2	5	4	0

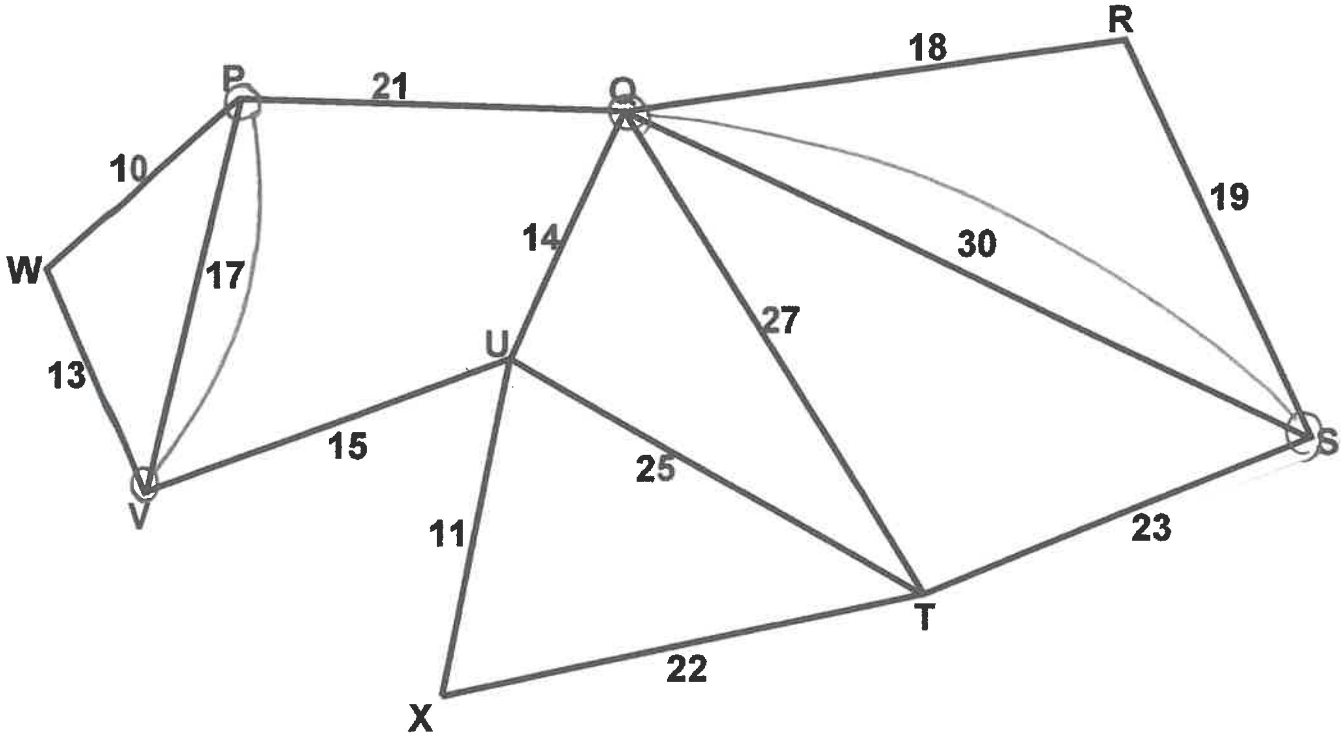
(4)



(4)

QUESTION 8 **[10]**

Determine an Eulerian circuit for the following graph, by using the Chinese Postman Algorithm. Clearly show the process, the final weight and a circuit starting and ending at vertex P.



odd Vertices P ; V ; Q ; S.

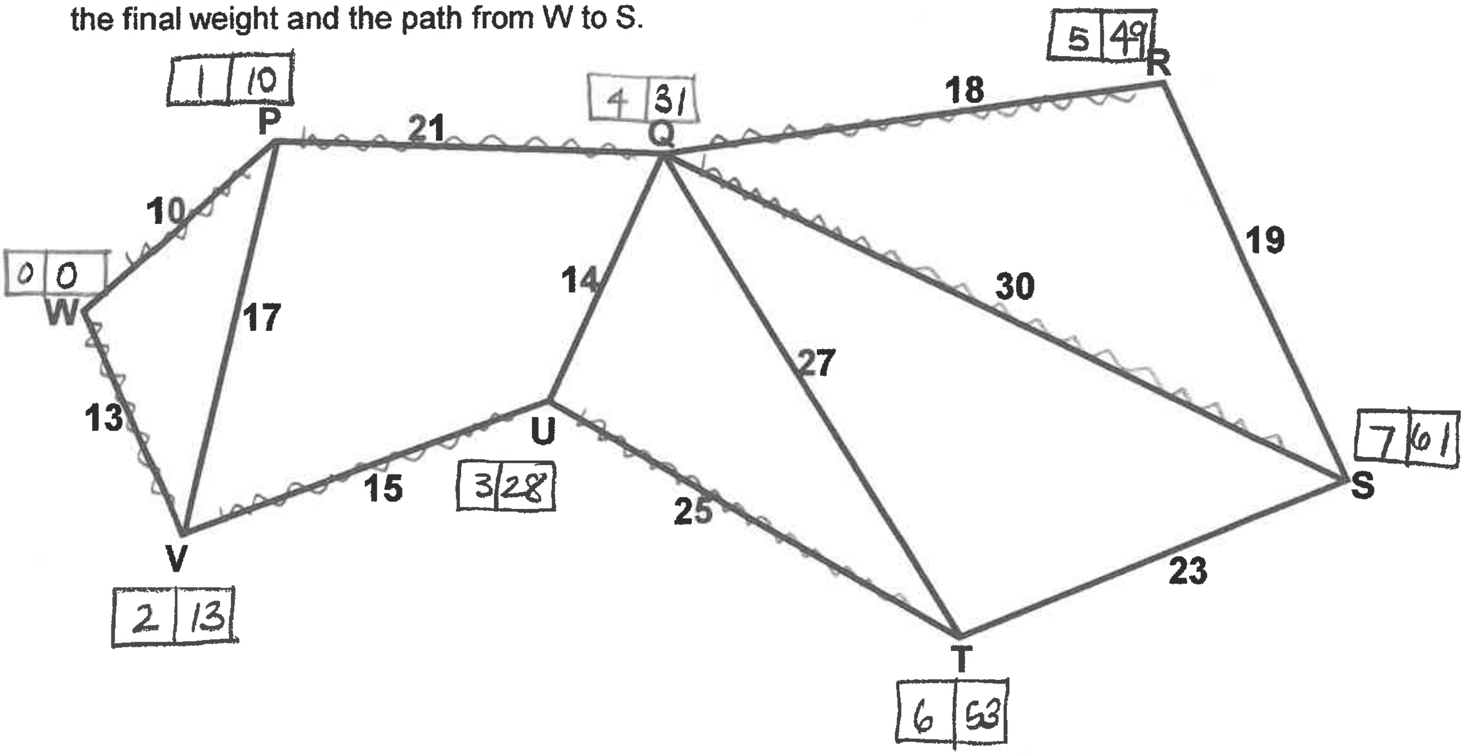
PV	17	QS	30.	=	47
PQ	21	VS	59	≠	80
PS	51	VQ	29	≠	80

∴ circuit weight = 265 + 47
= 312.

PW → WV → VP → PV → VU → UX → XT → TU → UQ → QT → TS → SQ → QS → SR → RQ → QP.

QUESTION 9 **[8]**

By applying Dijkstra's Algorithm, determine the shortest path from W to S. Clearly show the process, the final weight and the path from W to S.



W → P → Q → S

= 61.

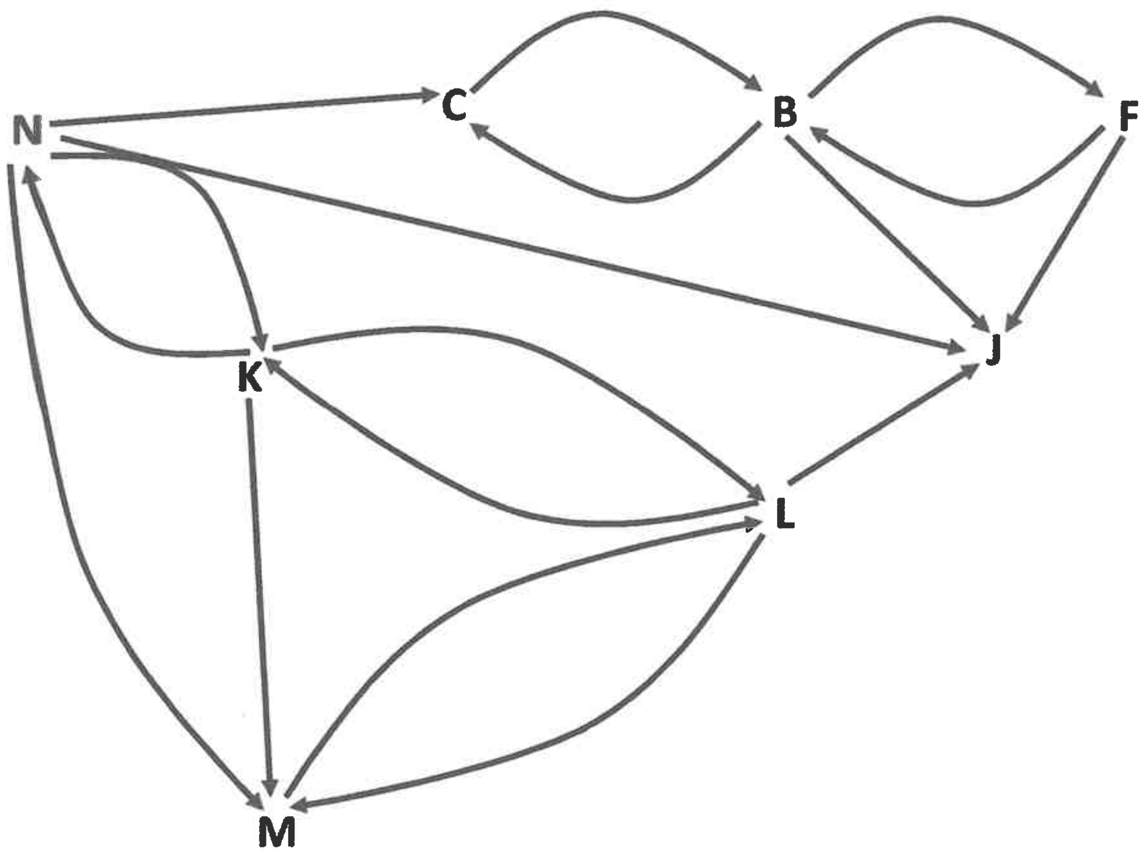
QUESTION 10 [6]

Natalie is having a kitchen tea for her friend, Emma, who is getting married. She wants to throw a surprize party for Emma's closest friends. The invitations are given telephonically. The vertices represent the friends and the edges represent the knowledge of the next friend's phone number.

a) What is the minimum number of calls that must be made to let every friend know about the tea party. 7 (2)

b) If each person can make a maximum of two calls, will the whole team be informed? Explain why. Yes. (+ reason) (2)

c) Who has most of the friends phone numbers? Natalie (2)

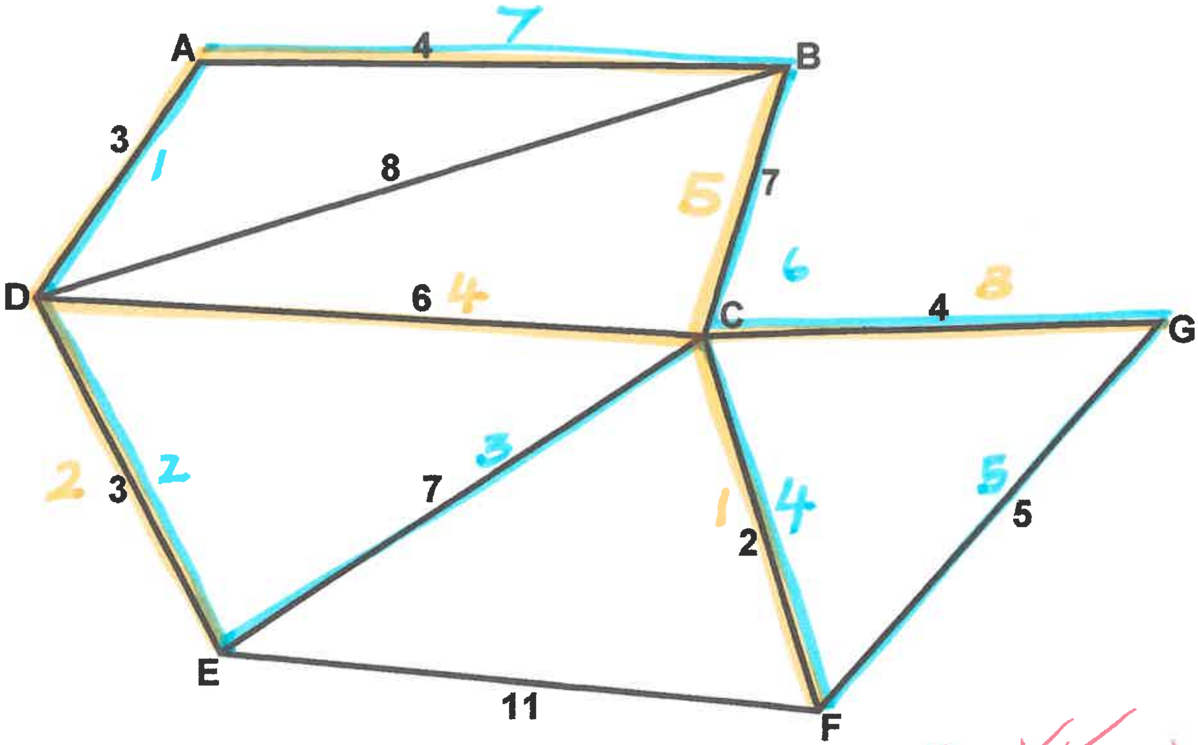


QUESTION 11 [12]

In the graph below, determine a range of weights between which an optimal circuit would lie.

a) Remove Vertex A and use Kruskal's, or Primm's, for the lower bound. (7)

b) Start at Vertex A for the upper bound. (5)



Lower Bound. $CF + DE + CG + DC + BC$ $(+ AD + AB$
$2 + 3 + 4 + 6 + 7$ $+ 3 + 4)$
$= 29$

upper Bound $AD \to DE \to EC \to CF \to FG \to GCB \to BA$
$= 3 + 3 + 7 + 2 + 5 + \underset{}{11}^{(4+7)} + 4 = 35$

$\therefore$ optimal circuit $\in [29; 35]$

QUESTION 12 (7)

Show that $\binom{n+2}{3} - \binom{n}{3} = n^2$; for all integers, n where $n \geq 3$; where $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

$*$ remember $n! \Rightarrow n(n-1)(n-2) \ldots\ldots (n-m)$

or $n! = n(n-1)!$

Q12. $\binom{n+2}{3} - \binom{n}{3}$ if $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!}$

$= \dfrac{(n+2)!}{(n-1)!\,3!} - \dfrac{n!}{(n-3)!\,3!}$

$= \dfrac{(n+2)(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}\,3!} - \dfrac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}\,3!}$

$= \dfrac{n(n+2)(n+1) - n(n-1)(n-2)}{6}$

$= \dfrac{n(n^2+3n+2) - n(n^2-3n+2)}{6}$

$= \dfrac{\cancel{n^3} + 3n^2 + \cancel{2n} - \cancel{n^3} + 3n^2 - \cancel{2n}}{6}$

$= \dfrac{6n^2}{6}$

$= n^2$

$=$ RHS