QUESTION 1

1.1 Consider: $f(x) = 2\ln(x-4)$

(a) Solve for x if $f(x) = 2$, giving your answer to two decimal places. (4)

(b) Determine the domain and range of $f(x)$ and hence sketch the graph of the function on the Answer Sheet provided. (6)

1.2 A student learns to type y words per minute after t days of practise. The relationship between y and t is given by:

$$y(t) = 120(1 - e^{-0,15t})$$

(a) How many words can the student type after 5 days? (2)

(b) How many days will it take the student to type 100 words per minute? (4)

(c) Sketch the graph of $y(t) = 120(1 - e^{-0.15t})$ on the Answer Sheet**,** showing its horizontal asymptote. (4)

(d) State the maximum number of words the student will be able to type per minute. Explain how the graph shows this. (2)

[22]

QUESTION 2

Use mathematical induction to prove that

$$\sum_{p=1}^{n} 2\left(\frac{1}{2}\right)^{p} = 2 - 2\left(\frac{1}{2}\right)^{n}$$

for all $n \in N$. **[13]**

QUESTION 3

3.1 (a) Given that $2 + 3i$ is a root of $x^4 - 4x^3 + 17x^2 - 16x + 52 = 0$, determine a quadratic factor of $f(x) = x^4 - 4x^3 + 17x^2 - 16x + 52$ (4)

(b) Hence solve the equation $x^4 - 4x^3 + 17x^2 - 16x + 52 = 0$, completely. (5)

3.2 Calculate the modulus and argument of $\frac{7-i}{3-4i}$. **Show all working details**. (7)

[16]

QUESTION 4

Solve the equation: $|x|^2 - 5|x| = 6$ and hence sketch the graph of

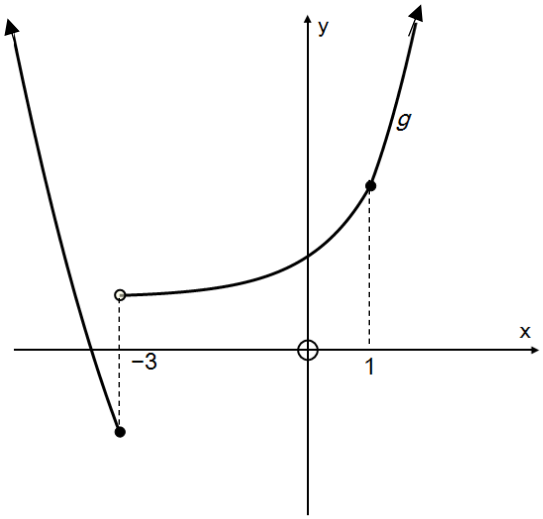
$f(x) = |x|^2 - 5|x| - 6$ on the Answer Sheet, showing all salient points. (9)

[9]

QUESTION 5

The figure below shows the graph of the piece-wise function:

$$g(x) = \begin{cases} (x+1)^2 - 6 & if \quad x \leq -3 \\ e^x - e + 4 & if \quad -3 < x \leq 1 \\ 2x^2 + 2 & if \quad x > 1 \end{cases}$$



5.1 State whether the following statements are true or false, explaining how you came to the conclusion.

(a) $\lim_{x \to -3} g(x)$ exists (3)

(b) $\lim_{x \to 1} g(x)$ exists (3)

5.2 Determine whether the function is differentiable at $x = 1$. Motivate your answer fully. (6)

[12]

QUESTION 6

It is given that: $f(x) = \begin{cases} 8 & if \quad x \leq -1 \\ 9 - x^2 & if \quad -1 < x \leq 2 \\ ax + b & if \quad x > 2 \end{cases}$

Determine the values of a and b such that f is differentiable at $x = 2$. **[8]**

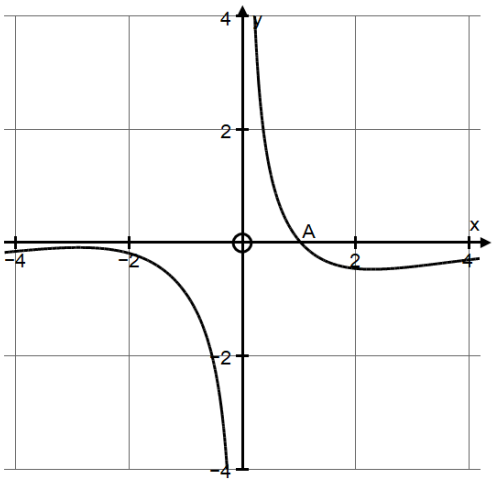
QUESTION 7

7.1 Determine $\frac{dy}{dx}$ if:

(a) $y = \ln\frac{\sin 2x}{2-x}$ (8)

(b) $y = \tan 3x.\, e^{2x+1}$ (6)

7.2 The graph of $y\sin x + \cos x = 3xy + \frac{1}{2}x$ is given below.



(a) By making use of Newton's method, determine the co-ordinates of A, rounded to four decimal digits. (9)

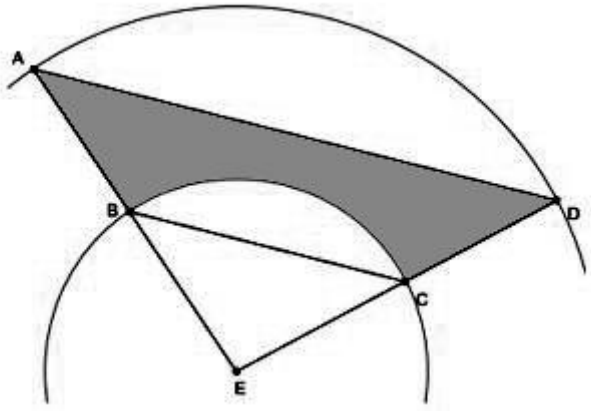
(b) Find an expression for $\frac{dy}{dx}$ in terms of x and y. (10)

(c) Hence, determine the equation of the tangent to the curve at A. (Round answers off to four decimal digits) (4)

[37]

QUESTION 8

E is the centre of two concentric circles. $AB = BE = CD = CE = r$ and $A\hat{E}D = \theta$ where $\theta \in (0;\pi)$.



8.1 Show that the perimeter of the shaded region is given by: (8)

$$p = 2r + r\theta + 4r\sin\frac{\theta}{2}.$$

8.2 Determine the area of the shaded region in terms of r and θ. (3)

8.3 If $r = 2\theta$, determine the value of θ so that the area of the shaded region is a maximum. Give answer correct to 3 decimal places. (9)

8.4 Show that this value of θ does maximise the area. (7)

[27]

QUESTION 9

Consider the function: $g(x) = \frac{2x^2+7x-15}{x+2}$

9.1 Determine the equations of all asymptotes. (6)

9.2 Prove that the graph has no stationary points. (6)

9.3 Determine the intercepts with the axes. (4)

[16]

QUESTION 10

10.1 Determine the following:

(a) $\int \frac{\cos x}{\sin x - 2}\,dx$ (4)

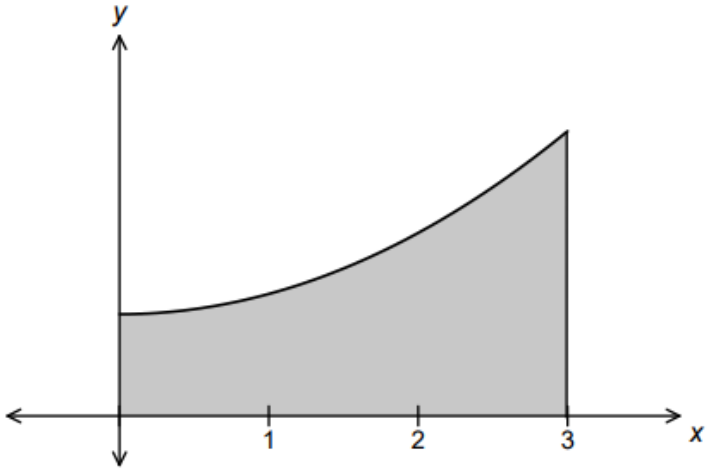
(b) $\int x\sqrt[3]{2+5x}\,dx$ (7)

10.2 Given: $f(x) = \frac{5x^2+20x+6}{x^3+2x^2+x}$

(a) Resolve $f(x)$ into its partial fractions. (8)

(b) Hence, determine $\int f(x)dx$. (5)

10.3 Kate determines the area enclosed by the curve $y = kx^2 + 2$ and the $x-$ axis between $x = 0$ and $x = 3$, using the Riemann sum.



Using n rectangles, she found the following expression for the Riemann sum.

$$A = \frac{48}{5} + \frac{27}{5n} + \frac{9}{5n^2}$$

(a) Determine the exact area. (2)

(b) Determine the value of k. (6)

(c) If the volume of the solid generated by rotating the shaded area about the $x-$ axis between 0 and p is $\frac{4984}{375}\pi$, determine the value of p. (8)

[40]

Total: 200 marks