MODULE 2 STATISTICS

QUESTION 1

The following number contains nine digits: **223 677 888**

If all nine digits are rearranged, determine the probability that the number made starts with a 3, given that it is odd. (12)

[12]

QUESTION 2

2.1 The sales staff at an insurance company make house calls to prospective clients. Past records show that 30% of the people visited will take out a new policy with the company. On a particular day, one salesperson visits 8 people.

(a) Calculate the probability that more than two people take out a new policy. (12)

(b) The company awards a bonus to any salesperson who sells more than 50 policies in a month. Angelique wants to calculate the probability that she gets a bonus in a month in which she visits 150 prospective clients.

(1) Show that a normal approximation is a suitable method to determine this probability. (2)

(2) Determine this probability using the normal approximation. (7)

2.2 A box contains 80 grapes of which 24 have seeds in them.

How many grapes should Caitlin take out of the box to be **at least** 85% sure that she will have **at least** one grape that is seedless? (hint: first determine the inequality in terms of n and then use trial and error to solve for n.) (8)

[29]

QUESTION 3

A random sample of 15 bags of chocolate pretzels had the following net weights, in grams:

242 238 245 249 237 220 223 230

224 245 221 246 242 234 239

3.1 Determine, to 2 decimals, the mean, $\bar{x}$ and standard deviation, s of this sample. (2)

3.2 Find a 90% confidence interval, to 1 decimal, for the true mean net weight in grams. (8)

[10]

QUESTION 4

The length of time, in tens of minutes, that patients spend waiting at a doctor's surgery is modelled by the following probability density function:

$$f(t) = \begin{cases} m(9 + 3t - 2t^2), & 0 \leq t \leq 3 \\ 0, & otherwise \end{cases}$$

4.1 Determine the value of m, showing all working. (9)

4.2 In minutes, determine:

(a) the expected waiting time $(E(t))$ of a patient at a doctor's surgery if

$$E(t) = \int_a^b t\,.\,f(t)dt$$ (10)

(b) the waiting time that occurs most frequently? (6)

[25]

QUESTION 5

A telephone company believes that, for young people, the average length of a telephone call on a land line is longer than on a mobile, due to the difference in call costs.

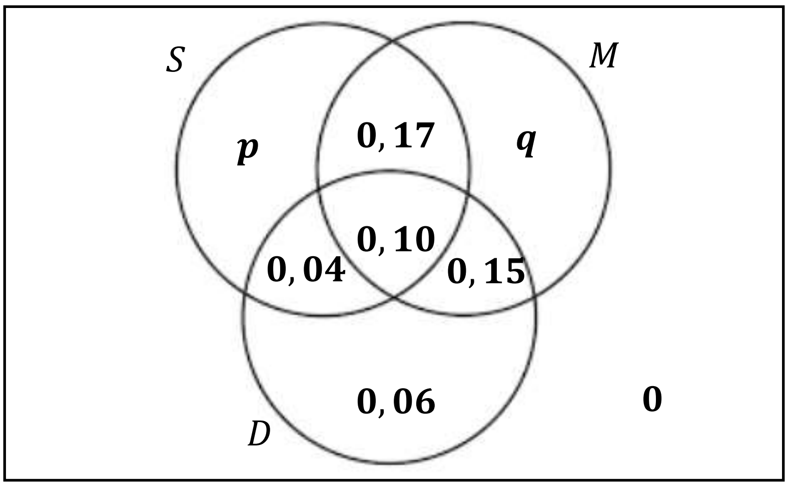
The company collected data on the time, t minutes, of 500 calls made by young people on **mobiles** and found $\mu_M = 14{,}7$ and $\sigma_M = 11{,}4$.
For 200 calls made on **land lines** by the same young people, showed $\mu_L = 15{,}9$ and $\sigma_L = 10{,}4$.
Stating your hypotheses clearly, test at the 5% level whether or not there is evidence that longer calls are made on land lines than on mobiles. (9)

[9]

QUESTION 6

The Venn diagram below shows the probabilities of customers having various combinations of a starter, main course or dessert at Polly's restaurant.

$S = the\ event\ a\ customer\ has\ a\ starter$
$M = the\ event\ a\ customer\ has\ a\ main\ course$
$D = the\ event\ a\ customer\ has\ a\ dessert$



6.1 If it is given that S and D are **statistically independent**, find the value of p. (4)

6.2 Hence, find the value of q. (2)

6.3 Find, to 3 decimals:

(a) $P(D \mid M \cap S)$ (3)

(b) $P(D \mid M \cap S')$ (3)

6.4 One evening 63 customers are booked into Polly's restaurant for an office party. Polly has asked for their starter and main course orders before they arrive. Of these 63 customers:

- 27 ordered a main course **and** a starter
- 36 ordered a main course **without** a starter

Using the values calculated in 6.3, estimate **how many** of these 63 customers will order a dessert. (3)

[15]

GRAND TOTAL: 100 marks

Memo

MODULE 2 STATISTICS

QUESTION 1

$n(first\ is\ 3\ \cap\ number\ is\ odd) = \checkmark\frac{7!}{\checkmark 2!3!}$

$= 420$

$n(number\ is\ odd) = \checkmark\frac{8!}{\checkmark 2!\,2!\,3!} + \frac{8!}{2!\,3!}\checkmark\checkmark$

$= 1680 + 3360$

$= 5040$

$P(first\ is\ 3\ |number\ is\ odd) \checkmark = \frac{n(first\ is\ 3 \cap number\ is\ odd)}{n(number\ is\ odd)}\checkmark\checkmark$

$= \frac{420}{5040}\checkmark\checkmark$

$= \frac{1}{12}$ or $0{,}0833\checkmark\ldots$ (12)

[12]

QUESTION 2

2.1

(a) $P(X > 2) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$ ✓✓✓✓✓

$= 1 - \binom{8}{0}(0{,}3)^0(0{,}7)^8 - \binom{8}{1}(0{,}3)^1(0{,}7)^7 - \binom{8}{2}(0{,}3)^2(0{,}7)^6$ ✓✓✓✓✓✓✓

$= 0{,}45$ ✓ (12)

(b)

(1) $np = 150 \times 0{,}3 = 45 > 5$ ✓

$nq = 150 \times 0{,}7 = 105 > 5$ ✓ (2)

(2) $\mu = 45$

$\sigma = \sqrt{npq} = \sqrt{150.0,3.0,7} = \frac{3\sqrt{14}}{2}$

$P(x > 50,5)$ ✓

$= P\left(z > \frac{50,5-45}{\sqrt{31,5}}\right)$ ✓ ✓ ✓

$= P(z > 0,98)$ ✓

$= 0,5 - 0,3365$ ✓

$= 0,1635$ ✓ (7)

2.2 $P(X \geq 1) \geq 0,85$ ✓

$1 - P(X = 0) \geq 0,85$

$\therefore P(X = 0) \leq 0,15$ ✓

$\frac{\binom{56}{0}\binom{24}{n}}{\binom{80}{n}} \leq 0,15$ ✓ ✓ ✓ NO marks for binomial from here

$\frac{\binom{24}{n}}{\binom{80}{n}} \leq 0,15$ ✓

if: $n = 1$ $P = 0,3$ ✓ trial and error

$n = 2$ $P = 0,087$

$\therefore n = 2$ ✓ (8)

[29]

QUESTION 3

3.1

$\bar{x} = 235{,}67g$ ✓

$s = 9{,}82g$ ✓ (2)

3.2 $\mu \in \left(235{,}67 - 1{,}645\left(\frac{9{,}82}{\sqrt{15}}\right);\ 235{,}67 + 1{,}645\left(\frac{9{,}82}{\sqrt{15}}\right)\right)$ ✓✓✓✓✓✓

$\therefore \mu \in (231{,}5\ ;\ 239{,}8)$ ✓✓ (8)

[10]

QUESTION 4

4.1 $f(t) = \begin{cases} m(9 + 3t - 2t^2), & 0 \le t \le 3 \\ 0, & otherwise \end{cases}$

$\int_0^3 m(9 + 3t - 2t^2)\, dt = 1$ ✓✓

$m\int_0^3 (9 + 3t - 2t^2)\, dt = 1$ ✓

$m\left[9t + \frac{3}{2}t^2 - \frac{2}{3}t^3\right]_0^3 = 1$ ✓✓✓✓

$\therefore m\left[\left(27 + \frac{27}{2} - 18\right)\right] = 1$ ✓

$\therefore \frac{45}{2}m = 1$

$\therefore m = \frac{2}{45}$ ✓ (9)

4.2 (a) if $E(t) = \int_a^b t\,.\,f(t)dt$

$= \frac{2}{45}\int_0^3 t.(9 + 3t - 2t^2)dt$ ✓✓

$= \frac{2}{45}\int_0^3 (9t + 3t^2 - 2t^3)dt$ ✓

$= \frac{2}{45}\left[\frac{9}{2}t^2 + t^3 - \frac{1}{2}t^4\right]_0^3$ ✓✓✓

$= \frac{2}{45}\left[\frac{9}{2}(3)^2 + (3)^3 - \frac{1}{2}(3)^4\right]$ ✓

$= \frac{6}{5}$ ✓

$\therefore E(t) = \frac{6}{5} \times 10 = 12\ minutes$ ✓ ✓ (10)

(b) $f(t) = \frac{2}{45}(9 + 3t - 2t^2)$

$= \frac{2}{5} + \frac{2}{15}t - \frac{4}{45}t^2$ ✓

$f'(t) = \frac{2}{15} - \frac{8}{45}t = 0$ ✓ ✓ ✓

$\therefore t = \frac{3}{4}$ ✓

$\therefore t = \frac{3}{4} \times 10 = 7{,}5\ minutes$ ✓ (6)

[25]

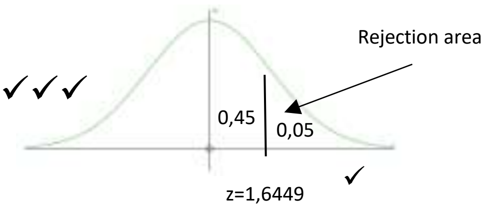
QUESTION 5

$H_0: \mu_L = \mu_M$ ✓

$H_1: \mu_L > \mu_M$ ✓

$z = \frac{15{,}9 - 14{,}7}{\sqrt{\frac{10{,}4^2}{200} + \frac{11{,}4^2}{500}}}$ ✓✓✓✓

$= 1{,}34$ ✓



There is not enough evidence to reject the null hypothesis at a 5% level of significance. There is no significant difference in the length of calls. ✓

(9)

[9]

QUESTION 6

6.1

$P(S \cap D) = P(S).P(D)$

✓ $0{,}14 = (0{,}31 + p)\,.\,0{,}35$ ✓ ✓

$0{,}14 = 0{,}1085 + 0{,}35p$

$0{,}35p = 0{,}0315$

$p = 0{,}09$ ✓ (4)

6.2 $q = 1 - 0{,}35 - 0{,}17 - 0{,}09$ ✓ca

$= 0{,}39$ ✓ (2)

6.3

(a) $P(D \mid M \cap S) = \frac{P(D \cap (M \cap S))}{P(M \cap S)}$

$= \frac{0{,}1}{0{,}27}$ ✓ ✓

$= 0{,}370$ ✓ (3)

(b) $P(D \mid M \cap S') = \frac{P(D \cap (M \cap S'))}{P(M \cap S')}$

$= \frac{0{,}15}{0{,}54}$ ✓ ✓ca

$= 0{,}278$ ✓ca (3)

6.4 $(27 \times 0{,}370) + (36 \times 0{,}278)$ ✓ ✓

$= 9{,}99 + 10{,}008$

$= 19{,}998$

$= 20$ people ✓

(3)

[15]

TOTAL: 100 marks