WESTERFORD HIGH SCHOOL
Advanced Programme Mathematics
Grade 12 Paper 2 – Matrices and Graph Theory

September 2019	Time: 1 hour
Examiner: K Court	Marks: 100

INSTRUCTIONS

1. This question paper consists of 4 questions over 7 pages.
2. The diagrams are reprinted on a Diagram Sheet on the last 2 pages. **Detach and hand in the Diagram Sheet as you need to use it to show your working**.
3. Answer all the questions and show necessary working clearly and legibly, guided by mark allocation.
4. Calculators may be used unless stated otherwise.
5. A **formula sheet** will be provided.
6. Diagrams are not necessarily drawn to scale.
7. Give answers rounded correct to **2 decimal places,** unless otherwise specified.

QUESTION 1

1.1. Given the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Write down A^{-1} in terms of a, b, c and d. (3)

1.2. Use an inverse matrix, and show some working, to solve for C if

$$BC = \begin{pmatrix} -3 & -1 \\ 11 & -5 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & -5 \\ 2 & 1 \end{pmatrix}.$$

(5)

1.3. Give the value of the following determinant:

$$\begin{vmatrix} 2 & 0 & 0 \\ 1 & b & 0 \\ -1 & 2 & d \end{vmatrix}$$

(2)

1.4. Given $M = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ x+4 & 3x+1 & 2x+3 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 0 & -2 \\ 5 & 8 & -1 \\ -3 & 1 & 10 \end{pmatrix}$

1.4.1. Determine ONLY the values of p, q and r if $MP = \begin{pmatrix} p & \dots & \dots \\ \dots & q & \dots \\ \dots & \dots & r \end{pmatrix}$. (6)

1.4.2. Determine $|M|$, the determinant of M. (6)

1.4.3. An inverse matrix can be found by determining a matrix of minors, then co-factors etc. If the matrix of co-factors for M is $\begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ a & b & c \end{pmatrix}$ determine the **co-factors** a, b and c. (5)

1.4.4. Could you have predicted the answer to 1.4.2? Explain. (1)

1.5. Another way of finding an inverse matrix is by transforming the larger matrix $[M: I_{n\times n}]$ to $[I_{n\times n}: M^{-1}]$, using a sequence of row transformations. The process for the matrix M has been started below.

$$M = \begin{pmatrix} 1 & 2 & -1 \\ 5 & 7 & 1 \\ -3 & 0 & -4 \end{pmatrix}$$

$$\begin{matrix} r1 \\ r2 \\ r3 \end{matrix}\left(\begin{array}{cccc|cc} 1 & 2 & -1 & 1 & 0 & 0 \\ 5 & 7 & 1 & 0 & 1 & 0 \\ -3 & 0 & -4 & 0 & 0 & 1 \end{array}\right)$$

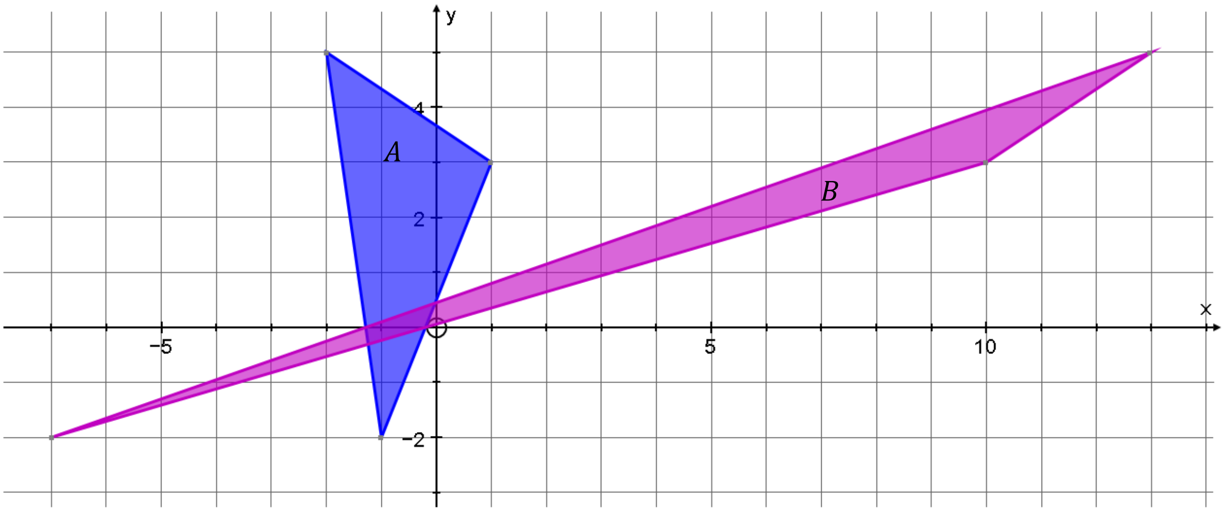
$$\begin{matrix} r1 \\ r2 - 5r1 \\ r3 + 3r2 \end{matrix}\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -3 & 6 & -5 & 1 & 0 \\ 0 & 6 & -7 & 3 & 0 & 1 \end{array}\right)$$

1.5.1. Complete the process to find M^{-1}. (10)

1.5.2. Hence show that the determinant is -15, by writing the matrix with integer values. (2)

[40]

QUESTION 2



2.1. The triangle A is defined by the matrix $\begin{pmatrix} -1 & -2 & 1 \\ -2 & 5 & 3 \end{pmatrix}$. The triangle has been transformed to create triangle B. Write down the transformation matrix used. (2)

2.2. In another transformation, triangle A is first reflected in the y −axis and then rotated by 90° anti-clockwise to create triangle E.

2.2.1. Give the transformation matrix that would achieve the above transformation in one step. (4)

2.2.2. Draw the resultant triangle, E, onto the grid provided on the answer sheet. (4)

2.2.3. Describe in words a different **single transformation** of A that would have achieved the same result. (2)

2.3. Triangle A is transformed by the matrix $\begin{pmatrix} 0{,}342 & 0{,}940 \\ 0{,}940 & -0{,}342 \end{pmatrix}$.

Describe the transformation fully in words (work to ONE decimal place). (6)

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QUESTION 3

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$Matrix\ X$ $\quad$ $Matrix\ Y$ $\quad$ $Matrix\ Z$

3.1. Draw a graph to represent each of the adjacency matrices above.
Use vertex labels A, B, C and D. (6)

3.2. Give 2 terms used in graph theory to describe all three graphs. (2)

3.3. Give an additional term used to describe the third graph. (1)

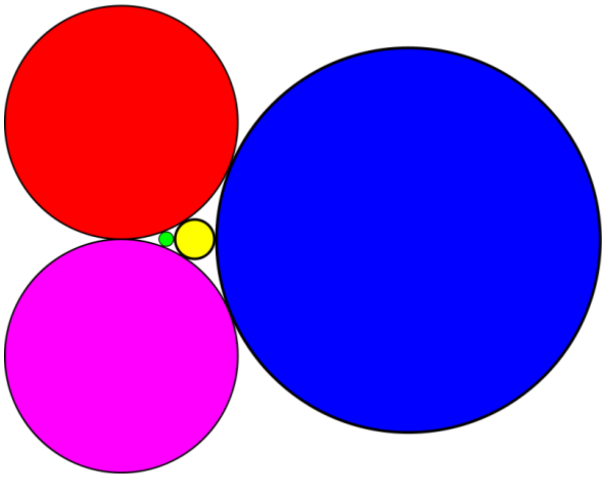
3.4. Draw the complement of the graph in the first matrix. (4)

3.5. Give the name of the particular type of circuit found in graph 3. (1)

[14]

QUESTION 4

Two circles drawn in a plane are said to "kiss" whenever they intersect in exactly one point.
A "coin graph" is a graph based on a set of circles, none of which overlap. Each circle results in a vertex and each "kiss" results in an edge.



4.1. Draw a coin graph for the diagram given above. (4)

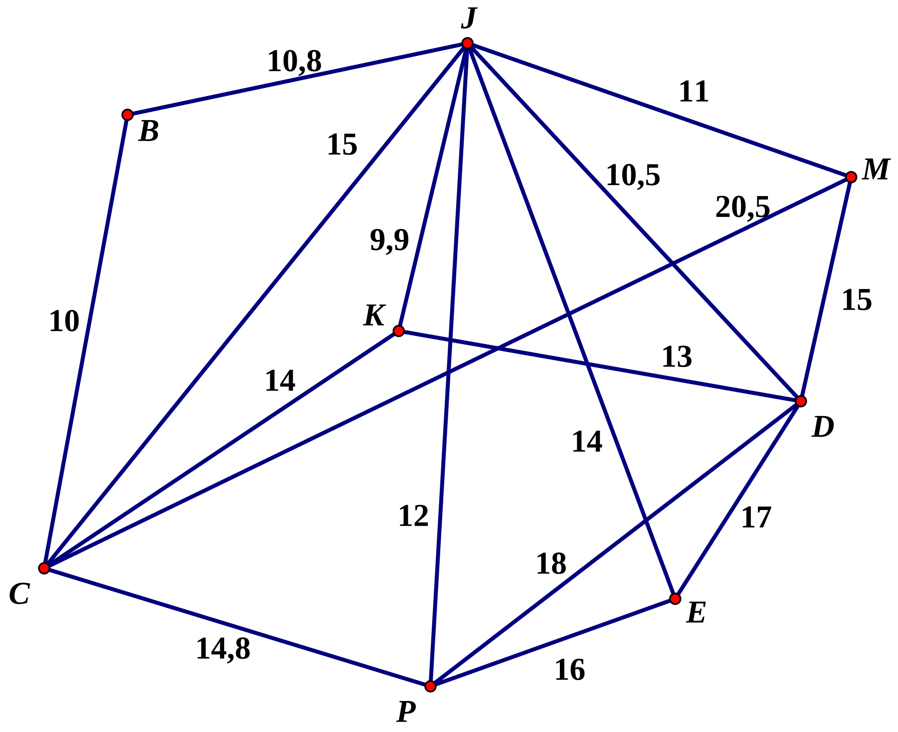
4.2. Is this a planar graph? Explain. (2)

[6]

QUESTION 5

The diagram below shows the cost of airfares between some major centres in South Africa in 100's of rands. The centres are Johannesburg (J), Kimberley (K), Bloemfontein (B), Cape Town (C), Port Elizabeth (P), East London (E), Durban (D) and Mbombela (M).

This diagram is repeated for you on the Diagram Sheet.



A director based in Johannesburg is required to visit each of the centres once every six months.

5.1. Determine a lower bound for the cost of the airfares, by removing Johannesburg and using Kruskal's algorithm. Clearly state the order in which you choose each vertex. (6)

5.2. In what way, if any, would the lower bound found be different if Prim's algorithm was used instead of Kruskals algorithm. (2)

5.3. Find an upper bound for the cost of the director's airfares using the Nearest-Neighbour algorithm and Johannesburg as the starting point. (7)

5.4. Knowing this upper bound, determine a better route for the director to use to visit all his centres. (7)

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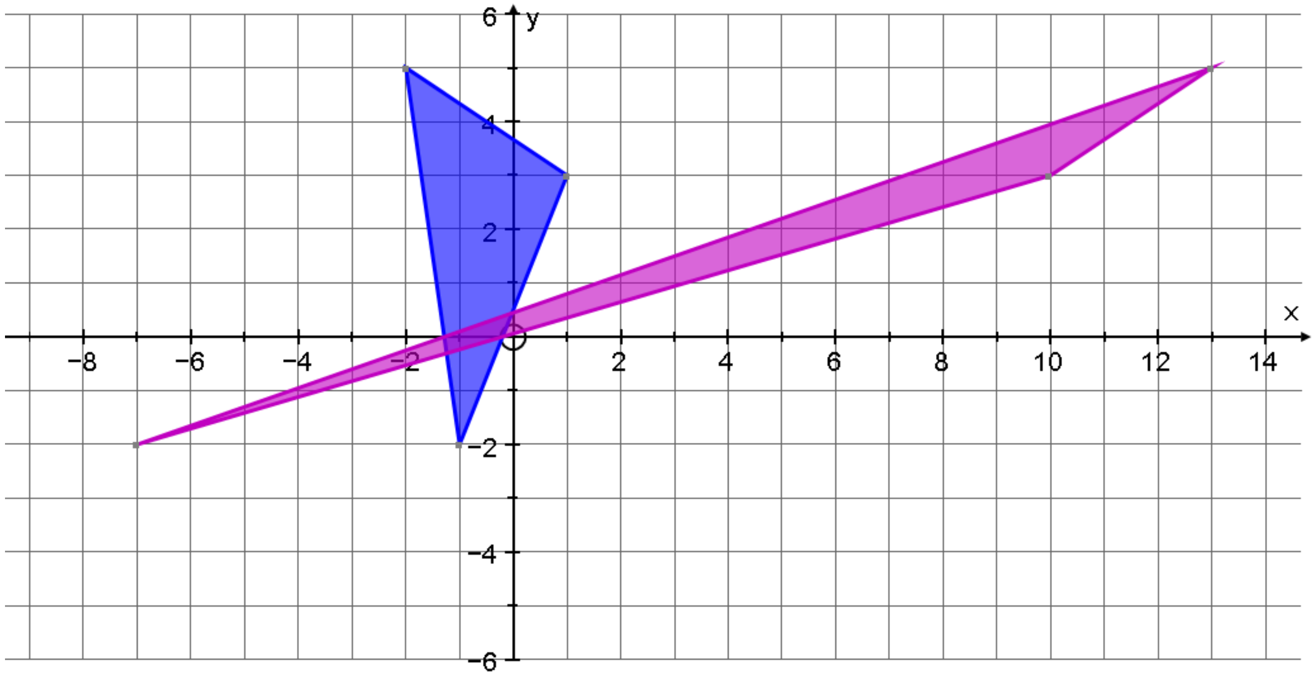
DIAGRAM SHEET

Attach this sheet to your answers. **Name: __________________________**

Question 2



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Question 4