

Grade 12 AP Maths P2 2019 memo

1.1	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$ $A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$ (3)
1.2	$B^{-1}BC = B^{-1} \begin{pmatrix} -3 & -1 \\ 11 & -5 \end{pmatrix}$ $C = B^{-1} \begin{pmatrix} -3 & -1 \\ 11 & -5 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -5 \\ 2 & 1 \end{pmatrix}$ $B^{-1} = \frac{1}{3+10} \begin{pmatrix} 1 & 5 \\ -2 & 3 \end{pmatrix} \checkmark \checkmark \text{ inverse}$ $C = \frac{1}{13} \begin{pmatrix} 1 & 5 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 11 & -5 \end{pmatrix} \checkmark$ $C = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} \checkmark \checkmark \text{ -1 per error}$ (5)
1.3	$\begin{vmatrix} 2 & 0 & 0 \\ 1 & b & 0 \\ -1 & 2 & d \end{vmatrix}$ $\text{Det} = 2(bd - 2 \times 0) - 0 + 0 = 2bd$ (2)
1.4.1	$M = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ x+4 & 3x+1 & 2x+3 \end{pmatrix}$ $P = \begin{pmatrix} 1 & 0 & -2 \\ 5 & 8 & -1 \\ -3 & 1 & 0 \end{pmatrix}$ Row by column for diagonals $1 + 15 - 6 = 10$ $0 + 8 + 3 = 11$ $-2(x+4) - 1(3x+1) + 10(2x+3) =$ $-2x - 8 - 3x - 1 + 20x + 30 = 15x + 21$ MP $= \begin{pmatrix} p = 10 & & \\ & q = 11 & \\ & & r = 15x + 21 \end{pmatrix}$ (6)
1.4.2	$\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ x+4 & 3x+1 & 2x+3 \end{vmatrix}$ By regular method: $ M = 1[(2x+3) - 3(3x+1)]$ $-3[4(2x+3) - 3(x+4)]$ $+2[4(3x+1) - (x+4)] \checkmark \checkmark \checkmark$ $= [-7x] - 3[5x] + 2[11x] \checkmark$ $= -7x - 15x + 22x \checkmark$ $= 0 \checkmark$

	By diagonals method $(2x+3) + 3 \times 3(x+4) + 2 \times 4(3x+1)$ $-2(x+4) - 12(2x+3) - 3(3x+1)$ $= 2x + 3 + 9x + 36 + 24x + 8$ $-2x - 8 - 24x - 36 - 9x - 3 \checkmark$ $= 0 \checkmark$ (6)
1.4.3	$M = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ x+4 & 3x+1 & 2x+3 \end{pmatrix}$ Minors: $9 - 2 = 7 \checkmark$ $3 - 8 = -5 \checkmark$ $1 - 12 = -11 \checkmark$ $\therefore$ co-factors are $a = 7$ $b = 5$ $c = -11 \checkmark \checkmark \text{ signs changed}$ (5)
1.4.4	Could have known it would be zero as row three is $x \times \text{row 1} + \text{row 2}$ $\checkmark$ i.e. a multiple of 1 row added to another row (1)
1.5.1	$\begin{array}{l} r1 \\ r2 - 5r1 \\ r3 + 3r2 \end{array} \begin{pmatrix} 1 & 2 & -1 & & 1 & 0 & 0 \\ 0 & -3 & 6 & & -5 & 1 & 0 \\ 0 & 6 & -7 & & 3 & 0 & 1 \end{pmatrix}$ $\begin{array}{l} 3r1 - r3 \\ r3 + 2r2 \end{array} \begin{pmatrix} 3 & 0 & 4 & & 0 & 0 & -1 \\ 0 & -3 & 6 & & -5 & 1 & 0 \\ 0 & 0 & 5 & & -7 & 2 & 1 \end{pmatrix}$ $\checkmark \checkmark \quad \checkmark \checkmark$ $\begin{array}{l} 5r1 - 4r3 \\ 5r2 - 6r3 \end{array} \begin{pmatrix} 15 & 0 & 0 & & 28 & -8 & -9 \\ 0 & -15 & 0 & & 17 & -7 & -6 \\ 0 & 0 & 5 & & -7 & 2 & 1 \end{pmatrix}$ $\checkmark \checkmark \quad \checkmark \checkmark$ $\begin{array}{l} r1 \div 15 \\ r2 \div -15 \\ r3 \div 5 \end{array} \begin{pmatrix} 1 & 0 & 0 & & \frac{28}{15} & -\frac{8}{15} & -\frac{9}{15} \\ 0 & 1 & 0 & & -\frac{17}{15} & \frac{7}{15} & -\frac{6}{15} \\ 0 & 0 & 1 & & -\frac{7}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix}$ $\checkmark \checkmark$ (10)
1.5.2	$M^{-1} = \frac{1}{-15} \begin{pmatrix} -28 & 8 & 9 \\ 17 & -7 & -6 \\ 21 & -6 & -3 \end{pmatrix}$ $\checkmark \checkmark$

	(2)
2.1	Shear factor 3 // to x-axis $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ ✓✓ (2)
2.2.1	$A = \begin{pmatrix} -1 & -2 & 1 \\ 1 & 5 & 3 \end{pmatrix}$ Reflect in y $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓ Rotate 90 anticlockwise $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ✓ Combo : $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ✓✓ (4)
2.2.2	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & -2 & 1 \\ -2 & 5 & 3 \end{pmatrix}$ $\begin{pmatrix} 2 & -5 & -3 \\ 1 & 2 & -1 \end{pmatrix}$ ✓ On grid ✓✓✓ (4)
2.2.3	Reflection in the line $y = -x$ ✓ ✓ (2)
2.3	Pattern of matrix $\Rightarrow$ Reflection in a line ✓ 2θ in first quad since $\sin$ and $\cos > 0$ for three of matrix values. $\sin 2\theta = 0,940$ $\cos 2\theta = 0,342$ ✓ $2\theta = 70^\circ$ ✓ or $2\theta = 1,22175 \dots^c$ $\theta = 35^\circ$ ✓ or $\theta = 0,61087 \dots^c$ $\therefore$ line is $y = \tan 35^\circ x$ ✓ $\therefore$ line is $y = 0,7x$ ✓ (6)
3.1	 Matrix X ✓✓ Matrix Y ✓✓ Matrix Z ✓✓ (6)

3.2	Simple and connected ✓✓ (2)
3.3	Regular or Complete ✓ (1)
3.4	Graph of X Complement Graph X ✓✓✓✓ (4)
3.5	Hamiltonian Circuit ✓ (1)
4.1	 ✓ 5 vertices ✓✓✓ 9 edges (4)
4.2	Yes planar ✓ as it can be drawn without any edges having to cross each other . The edges only meet at vertices. ✓ (2)
5.1	Lower Bound : CB 10 KD 13 ✓ CK 14 ✓ CP 14,8 ✓

	DM 15 PE 16 ✓ = 82,8 Add back 10,5 and 9,9 ✓ TOTAL = 10 320 ✓ (6)
5.2.	No difference in end result. ✓ Would have had to choose edges in a different order e.g. CK before KD and PE before ✓ (2)
5.3	Upper Bound Nearest-Neighbour $J \rightarrow 9.9 \rightarrow K$ ✓ $\rightarrow 13 \rightarrow D$ $\rightarrow 15 \rightarrow M$ ✓ $\rightarrow 20.5 \rightarrow C$ $\rightarrow 10 \rightarrow B$ ✓ (via J) $\rightarrow 22.8 \rightarrow P$ ✓ $\rightarrow 16 \rightarrow E$ ✓ $\rightarrow 14 \rightarrow J$ ✓ Total = 121.2 in hundreds of Rands $\therefore \text{Total} = R12\,120$ ✓ (7)
5.4	Better Solution: $J \rightarrow 10.8 \rightarrow B$ $\rightarrow 10 \rightarrow C$ $\rightarrow 14 \rightarrow K$ $\rightarrow 13 \rightarrow D$ $\rightarrow 16 \rightarrow E$ $\rightarrow 17 \rightarrow P$ $\rightarrow 12 \rightarrow J$ $\rightarrow 11 \rightarrow M$ $\rightarrow 11 \rightarrow J$ ✓ ✓ ✓ ✓ ✓ ✓ Total = 114.8 in hundreds of Rands $\therefore \text{Total} = R11\,480$ ✓ (7) OR $J \rightarrow 11 \rightarrow M$ $\rightarrow 15 \rightarrow D$ $\rightarrow 17 \rightarrow E$ $\rightarrow 16 \rightarrow P$ $\rightarrow 14.8 \rightarrow C$ $\rightarrow 14 \rightarrow K$

	$\rightarrow 9.9 \rightarrow J$ $\rightarrow 10.8 \rightarrow B$ $\rightarrow 10.8 \rightarrow J$ ✓ ✓ ✓ ✓ ✓ ✓ Total = 119.30 in hundreds of Rands $\therefore \text{Total} = R11\,930$ ✓ OR $J \rightarrow 10.8 \rightarrow B$ $\rightarrow 10 \rightarrow C$ $\rightarrow 14.8 \rightarrow P$ $\rightarrow 16 \rightarrow E$ $\rightarrow 17 \rightarrow D$ $\rightarrow 15 \rightarrow M$ $\rightarrow 11 \rightarrow J$ $\rightarrow 9.9 \rightarrow K$ $\rightarrow 9.9 \rightarrow J$ Total = 114.40 in hundreds of Rands $\therefore \text{Total} = R11\,440$ And others! (7)