AP MATHEMATICS
GRADE 12
PRELIMINARY EXAMINATION
PAPER 1
23 SEPT. 2020

QUESTION 1

1.1 (a) $|x^2 - 15| = 2x$

$x^2 - 15 \geq 0$ or $x^2 - 15 < 0$

$x^2 - 15 = 2x$ ✓ $\quad -x^2 + 15 = 2x$ ✓

$x^2 - 2x - 15 = 0 \quad x^2 + 2x - 15 = 0$

$(x-5)(x+3) = 0 \quad (x+5)(x-3) = 0$

$x = 5$ ✓ or $x \neq -3$ ✓ $\quad x \neq -5$ ✓ or $x = 3$ ✓

$\therefore \quad x = 5 \text{ or } x = 3$

(6)

(b) $2\ln x + 5 = 12 \log_x e$

$2\ln x + 5 = \frac{12}{\ln x}$ ✓

$2(\ln x)^2 + 5\ln x - 12 = 0$ ✓

$(2\ln x - 3)(\ln x + 4) = 0$ ✓

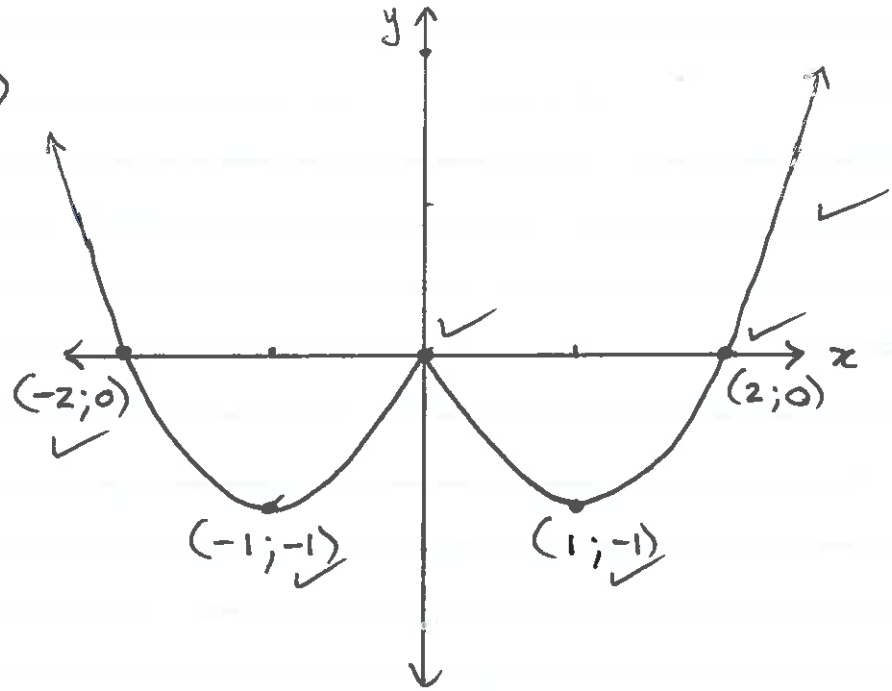
$\ln x = \frac{3}{2}$ ✓ or $\ln x = -4$ ✓

$x = e^{3/2}$ ✓ $\quad x = e^{-4}$ ✓✓

(8)

1.2 (a) $h(x) = (|x|)^2 - 2|x|$ ✓✓ (2)

(b)



(6)

(c) $x^2 - 2x = x + 2$ ✓ or $x^2 + 2x = x + 2$ ✓

$x^2 - 3x - 2 = 0$ $\quad$ $x^2 + x - 2 = 0$

$x = 3,56$ ✓ or $x \neq -0,56$ ✓ $\quad$ $x \neq 1$ ✓ or $x = -2$ ✓

$\therefore x = 3,56$ $\quad$ $\therefore x = -2$

$x \in (-2 ; 3,56)$ ✓ ✓

(8)

[30]

QUESTION 2

2.1 $x = 1 - 2i$ and $x = 1 + 2i$ ✓ are solutions

$-b = (1 - 2i) + (1 + 2i)$ $\quad$ $c = (1 - 2i)(1 + 2i)$

$-b = 2$ $\quad$ $= 1 - 4i^2$

$b = -2$ ✓ $\quad$ $= 5$ ✓

$\therefore x^2 - 2x + 5$ ✓ is a factor

$$2x^3 - 5x^2 + px - 5 = 0$$
$$(x^2 - 2x + 5)(2x - 1) = 0 \quad \checkmark$$
$$2x^3 - 5x^2 + 12x - 5 = 0 \quad \checkmark$$
$$\therefore \quad p = 12 \quad \checkmark$$

$$x = 1 - 2i \quad \text{or} \quad x = 1 + 2i \quad \text{or} \quad x = \frac{1}{2} \quad \checkmark$$

(8)

2.2 $$(a + bi)^2 = 5 - 12i \quad \checkmark$$
$$a^2 + 2abi + b^2 i^2 = 5 - 12i$$
$$(a^2 - b^2) + 2abi = 5 - 12i$$

$$a^2 - b^2 = 5 \quad \checkmark \qquad 2ab = -12 \quad \checkmark$$
$$b = \frac{-6}{a} \quad \checkmark$$

$$a^2 - \left(\frac{-6}{a}\right)^2 = 5 \quad \checkmark$$
$$a^2 - \frac{36}{a^2} = 5$$

$$a^4 - 5a^2 - 36 = 0 \quad \checkmark$$
$$(a^2 - 9)(a^2 + 4) = 0 \quad \checkmark$$
$$a = \pm 3 \quad \checkmark$$

$$b = \frac{-6}{3} = -2 \quad \checkmark$$

$$\text{or} \quad b = \frac{-6}{-3} = 2 \quad \checkmark$$

$$\therefore \quad w = 3 - 2i$$
$$\text{or} \quad w = -3 + 2i$$

(10)
[18]

QUESTION 3

$$1 \times 2^1 + 2 \times 2^2 + \ldots + n \times 2^n = (n-1).2^{n+1} + 2$$

For $n = 1$: $\quad LHS = 2$ ✓

$$RHS = (1-1).2^{1+1} + 2$$
$$= 2 \quad ✓$$

$\therefore$ True for $n = 1$

For $n=k$: $\quad 1 \times 2 + 2 \times 2^2 + \ldots + k \times 2^k = (k-1).2^{k+1} + 2$ ✓✓

For $n=k+1$: $\quad 1 \times 2 + 2 \times 2^2 + \ldots + k \times 2^k + (k+1) \times 2^{k+1} =$ ✓

$$(k-1).2^{k+1} + 2 + (k+1) \times 2^{k+1}$$ ✓

$$RHS = 2^{k+1}\left[(k-1) + (k+1)\right] + 2 \quad ✓$$

$$= 2^{k+1}(2k) + 2 \quad ✓$$

$$= 2^{(k+1)+1}(k) + 2$$

$$= \left[(k+1) - 1\right].2^{(k+1)+1} + 2 \quad ✓$$

$\therefore$ True for $n = k+1$, if true for $n = k$. ✓✓

$\therefore$ True for $n = 1$, $n = 2$, $n = 3$, ...

[12]

QUESTION 4

4.1 $$f'(x)=\begin{cases} a & \text{if } x<5 \\ 2x-a & \text{if } x\geq 5 \end{cases}$$ ✓ ✓

$$\lim_{x\to 5^-} f'(x) = \lim_{x\to 5^+} f'(x)$$ ✓

$$a = 2(5)-a$$ ✓

$$2a = 10$$

$$a = 5$$ ✓

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$$ ✓

$$5(5)-21 = 5^2-5(5)+b$$ ✓

$$4 = b$$ ✓

(8)

4.2 (a) $$2x^2+3x-2=0$$ ✓

$$(2x-1)(x+2)=0$$ ✓

$$x=\tfrac{1}{2}$$ ✓ or $$x=-2$$ ✓

(4)

(b) $$x=3$$ ✓✓

$$g(x) = \frac{2x(x-3)+9x-2}{x-3}$$ ✓

$$= 2x + \frac{9(x-3)+25}{x-3}$$ ✓

$$= 2x+9+\frac{25}{x-3}$$ ✓

$$\therefore\ y = 2x+9$$ ✓

(6)

(c) $g(x) = 2x + 9 + 25(x-3)^{-1}$

$g'(x) = 2 - 25(x-3)^{-2}$ ✓ ✓

$0 = 2 - \frac{25}{(x-3)^2}$ ✓

$0 = 2(x-3)^2 - 25$ ✓

$(x-3)^2 = \frac{25}{2}$

$x = 3 \pm \frac{5}{\sqrt{2}}$

$x = \frac{6 \pm 5\sqrt{2}}{2}$ ✓ ✓

$x \approx 6{,}54$ or $x \approx -0{,}54$

$y = 15 + 10\sqrt{2}$ ✓ $y = 15 - 10\sqrt{2}$ ✓

$y \approx 29{,}14$ $y \approx 0{,}86$

$(6{,}54 ; 29{,}14)$ $(-0{,}54 ; 0{,}86)$ (8)

(d)

x	-1	$-0{,}54$	0	6	$6{,}54$	7
$g'(x$	$+$	0	$-$	$-$	0	$+$

✓✓ ✓ ✓

Maximum turning point at $(-0{,}54 ; 0{,}86)$ ✓

Minimum turning point at $(6{,}54 ; 29{,}14)$ ✓

(6)

[32]

QUESTION 5

5.1 (a) $A\hat{B}D = 0,95$ radians

$$A\hat{D}B = \pi - 2(0,95) \checkmark$$
$$= 1,24159\ldots \text{ radians} \checkmark$$

$$\frac{BD}{\sin 0,95} = \frac{6}{\sin 1,24159\ldots} \checkmark$$

$$BD = 5,16 \text{ cm} \checkmark$$

$$\text{arc } BC = 6(0,95) \checkmark$$
$$= 5,7 \text{ cm} \checkmark$$

(6)

(b) $A = \frac{1}{2}r^2\theta - \frac{1}{2}r \times AD \times \sin\theta$

$$= \frac{1}{2}(6)^2(0,95) - \frac{1}{2}(6)(5,16)\sin 0,95 \checkmark \checkmark$$

$$= 17,1 - 12,5916\ldots \text{ or } 12,5854\ldots \checkmark$$

$$= 4,51 \text{ cm}^2 \checkmark$$

(4)

5.2 $V = 300$

$\frac{1}{2} r^2 (1) h = 300$ ✓

$h = \frac{600}{r^2}$ ✓

$A = (r + r + r(1))h + 2\left(\frac{1}{2} r^2 (1)\right)$ ✓

$= 3rh + r^2$

$= 3r\left(\frac{600}{r^2}\right) + r^2$ ✓

$= \frac{1800}{r} + r^2$ ✓

$A' = -1800r^{-2} + 2r$ ✓

$0 = \frac{-1800}{r^2} + 2r$ ✓

$1800 = 2r^3$ ✓

$r^3 = 900$

$r = 9{,}65 \text{ cm}$ ✓

(9)
[19]

QUESTION 6

6.1 $$y = \frac{e^{2x}}{\sin 3x + 2}$$

$$\frac{dy}{dx} = \frac{2.e^{2x}(\sin 3x + 2) - (3\cos 3x).e^{2x}}{(\sin 3x + 2)^2}$$ ✓

(6)

6.2 $$6y^2 \cdot \frac{dy}{dx} + 6x^2y + 2x^3 \cdot \frac{dy}{dx} = \frac{dy}{dx}$$ ✓ ✓

$$\frac{dy}{dx}(6y^2 + 2x^3 - 1) = -6x^2y$$

$$\frac{dy}{dx} = \frac{-6x^2y}{6y^2 + 2x^3 - 1}$$ ✓

$$m_{tan} = \frac{-6(-2)^2(3)}{6(3)^2 + 2(-2)^3 - 1}$$ ✓

$$= -\frac{72}{37}$$ ✓ or $-1,95$

(8)

6.3 $$f(x) = \cos^3 x - x \ln x$$

$$f'(x) = 3\cos^2 x(-\sin x) - 1 \ln x - x\left(\frac{1}{x}\right)$$

$$= -3\cos^2 x.\sin x - \ln x - 1$$

Set $x_0 = 1$

$$x_1 = x_0 - \frac{\cos^3 x_0 - x_0 \ln x_0}{-3\cos^2 x_0 \sin x_0 - \ln x_0 - 1}$$ ✓

$$= 1,090808\ldots$$ ✓

$$x_2 = 1,093015\ldots$$ ✓

$$\therefore \quad x \approx 1,09302$$ ✓

(8)

[22]

QUESTION 7

7.1 (a) -6 ✓✓ (2)

(b) $h(-x) = \frac{2}{5}h(x)$ ✓

$-3 + \frac{2}{5}(-3)$ ✓✓

$= -\frac{21}{5}$ ✓ OR $-4,2$ (4)

(c) $h(-x) = \frac{h(x)}{-2}$ ✓

$-3 + \frac{-3}{-2}$ ✓✓

$= -\frac{3}{2}$ ✓ or $-1,5$ (4)

7.2 $(2(2)^2+3)(1) + (2(3)^2+3)(1) + (2(4)^2+3)(1) +$

$(2(5)^2+3)(1)$ ✓

$= 11 + 21 + 35 + 53$ ✓✓

$= 120$ units^2 ✓ (4)

[14]

QUESTION 8

8.1 (a) $\int \frac{e^{5x}}{3}\,dx$

$= \frac{1}{3} \times \frac{1}{5} \int 5.e^{5x}\,dx$

$= \frac{e^{5x}}{15} + c$ ✓✓✓ (3)

(b) $\int \tan^3 2x \cdot \sec^2 2x \, dx$

$= \frac{1}{2}\int (\tan 2x)^3 (2\sec^2 2x)\, dx$ ✓

$= \frac{1}{2} \times \frac{\tan^4 2x}{4} + c$

$= \frac{\tan^4 2x}{8} + c$ ✓

(6)

(c) $\int \cot^2 4x \, dx$

$= \int (\operatorname{cosec}^2 4x - 1)\, dx$ ✓✓

$= \frac{-\cot 4x}{4} - x + c$ ✓

(6)

(d) $\int x (5x-2)^{2/3} dx$

Set $f(x) = x$ ✓ and $g'(x) = (5x-2)^{2/3}$ ✓

$f'(x) = 1$ ✓ $\quad g(x) = \frac{(5x-2)^{5/3}}{5/3 \times 5}$ ✓

$= \frac{3}{25}(5x-2)^{5/3}$

$\int x (5x-2)^{2/3} dx$

$= \frac{3}{25} x (5x-2)^{5/3} - \int \frac{3}{25}(5x-2)^{5/3}(1)\, dx + c$ ✓

$= \frac{3}{25} x (5x-2)^{5/3} - \frac{3}{25}\frac{(5x-2)^{8/3}}{8/3 \times 5} + c$ ✓

$= \frac{3}{25} x (5x-2)^{5/3} - \frac{9}{1000}(5x-2)^{8/3} + c$

(8)

8.2 (a) $\dfrac{9x^2-15x-6}{(x-5)(x+1)^2} = \dfrac{A}{x-5} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$ ✓ ✓

$9x^2-15x-6 = A(x+1)^2 + B(x-5)(x+1) + C(x-5)$ ✓

Set $x=-1$: $18 = C(-6)$ ✓

$C = -3$ ✓

Set $x=5$: $144 = A(6)^2$ ✓

$A = 4$ ✓

$9x^2-15x-6 = 4(x^2+2x+1) + B(x^2-4x-5) - 3(x-5)$ ✓

$9 = 4 + B$ ✓

$B = 5$ ✓

$g(x) = \dfrac{4}{x-5} + \dfrac{5}{x+1} - \dfrac{3}{(x+1)^2}$

(b) $\int \frac{4}{x-5}\,dx + \int \frac{5}{x+1}\,dx - \int \frac{3}{(x+1)^2}\,dx$ ✓ (10)

$= 4\ln|x-5| + 5\ln|x+1| - \dfrac{3(x+1)^{-1}}{-1} + C$ ✓ ✓ ✓

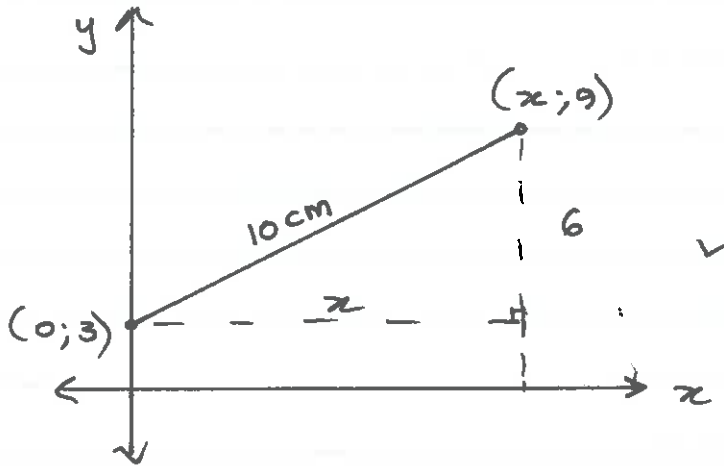
$= 4\ln|x-5| + 5\ln|x+1| + \dfrac{3}{(x+1)} + C$ ✓

(6)

[39]

QUESTION 9

9.1



$$x^2 = 10^2 - 6^2 \checkmark$$

$$x = 8 \checkmark$$

$$m = \frac{9-3}{8-0} \checkmark$$

$$= \frac{3}{4} \checkmark$$

$$y = \frac{3}{4}x + 3 \checkmark$$

$$V = \pi \int_0^8 \checkmark \left(\frac{3}{4}x + 3\right)^2 dx \checkmark$$

(8)

9.2

$$856 = \pi \int_k^8 \left(\frac{9}{16}x^2 + \frac{9}{2}x + 9\right) dx \checkmark$$

$$\frac{856}{\pi} = \left[\frac{3}{16}x^3 + \frac{9}{4}x^2 + 9x\right]_k^8 \checkmark$$

$$\frac{856}{\pi} = \frac{3}{16}(8)^3 + \frac{9}{4}(8)^2 + 9(8) - \left(\frac{3}{16}k^3 + \frac{9}{4}k^2 + 9k\right) \checkmark$$

$$\frac{3}{16}k^3 + \frac{9}{4}k^2 + 9k + \frac{856}{\pi} - 312 = 0 \checkmark$$

$$k = 2{,}5 \checkmark$$

$$\therefore \quad 8 - 2{,}5 = 5{,}5 \text{ cm} \checkmark$$

(6)

[14]