



GRADE 12
ADVANCED PROGRAMME MATHEMATICS
Preliminary Examination Paper 1
ALGEBRA & CALCULUS

Time:	2 Hours	200 marks
Date:	23 September 2020	
Examiner:	Ms A Smith	Moderator: Mr J Ruiz-Mesa

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 8 pages and an Information Booklet of 2 pages (i-ii). Please check that your question paper is complete.
 2. Answer all the questions in the ANSWER BOOKLET.
 3. Approved, non-programmable, non-graphical calculators may be used, unless otherwise indicated.
 4. Work neatly and show all the necessary steps in your calculations.
 5. Diagrams have not been drawn to scale.
 6. Trigonometric calculations should be done using RADIANS and answers should be given in RADIANS.
 7. Round off your answers to TWO decimal digits, unless otherwise indicated.
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QUESTION 1

1.1 Solve for $x \in \mathbb{R}$, without the use of a calculator and showing all working:

(a) $|x^2 - 15| = 2x$ (6)

(b) $2\ln x + 5 = 12\log_x e$ (8)

1.2 Given: $f(x) = x^2 - 2x$ and $g(x) = |x|$

(a) Determine $h(x)$, if $h(x) = f(g(x))$. (2)

(b) Sketch the graph of $h(x)$. Clearly indicate all intercepts with the axes as well as the coordinates of the turning point(s). (6)

(c) Use the graph you sketched in (b) to solve for $x \in \mathbb{R}$ where $h(x) < x + 2$. (8)

[30]

QUESTION 2

2.1 The cubic equation $2x^3 - 5x^2 + px - 5 = 0$ has a solution $x = 1 - 2i$.

Determine the value of p and the other two solutions. (8)

2.2 If $w = a + bi$ and $w^2 = 5 - 12i$, determine all possible real values of a and b . (10)

[18]

QUESTION 3

Use mathematical induction to prove that

$$\sum_{i=1}^n n \times 2^n = (n-1) \cdot 2^{n+1} + 2 \quad (12)$$

[12]

QUESTION 4

4.1 A function is defined as follows:

$$f(x) = \begin{cases} ax - 21 & \text{if } x < 5 \\ x^2 - ax + b & \text{if } x \geq 5 \end{cases}$$

Determine the values of a and b if it is given that f is differentiable at $x = 5$.

(Take special care with your notation). (8)

4.2 Given: $g(x) = \frac{2x^2 + 3x - 2}{x - 3}$

(a) Determine the x -intercepts of g . (4)

(b) Determine all possible asymptotes of g . (6)

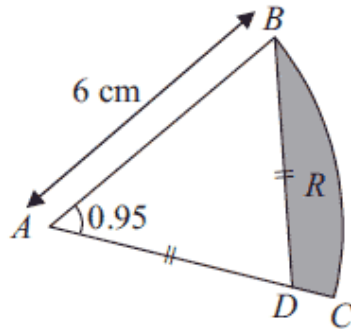
(c) Determine the coordinates of the stationary points of g . (8)

(d) Determine the nature of the stationary points of g . (6)

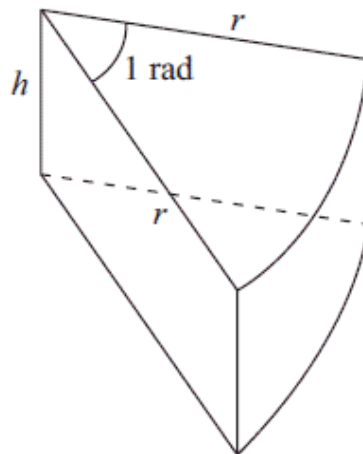
[32]

QUESTION 5

- 5.1 The diagram shows ABC, a sector of a circle with radius 6 cm and centre A. The region R, shown shaded below, is bounded by lines CD, DB and the arc BC. $\hat{BAC} = 0,95$ radians and $AD = BD$.



- (a) Determine the lengths of line BD and arc BC. (6)
- (b) Determine the area of region R. (4)
- 5.2 The diagram below shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian. The volume of the box is 300 cm^3 .



Determine the value of r for which the surface area of the box will be at a minimum.

(9)

[19]

QUESTION 6

6.1 Determine $\frac{dy}{dx}$ if $y = \frac{e^{2x}}{\sin 3x + 2}$ (6)

6.2 Determine the gradient of the tangent to the curve $2y^3 + 2x^3y = y + 4$ at the point $(-2; 3)$. (8)

6.3 The function $f(x) = \cos^3 x - x \ln x$ has a root on the interval $x \in [1; 2]$.

Use Newton-Raphson iteration to determine this root. You should:

- use an initial guess of $x = 1$
- show the iterative formula you use
- show your first two approximations
- give your answer to 5 decimal places

(8)

[22]

QUESTION 7

7.1 Given: $\int_0^5 h(x) dx = -3$

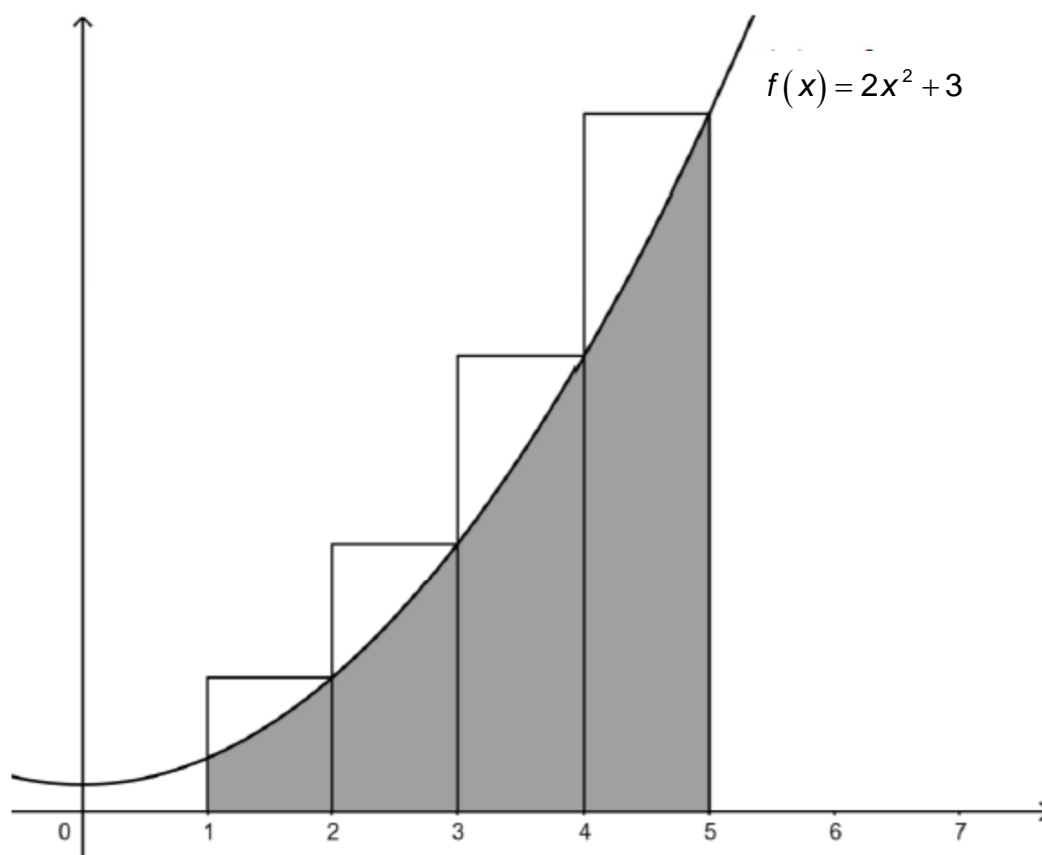
Determine $\int_{-5}^5 h(x) dx$ if:

(a) $h(x) = h(-x)$ (2)

(b) $2h(x) = 5h(-x)$ (4)

(c) $h(x) = -2h(-x)$ (4)

7.2 Use the rectangles to estimate the shaded area in the diagram below. (4)



[14]

QUESTION 8

8.1 Determine the following integrals:

(a) $\int \frac{e^{5x}}{3} dx$ (3)

(b) $\int \tan^3 2x \cdot \sec^2 2x dx$ (6)

(c) $\int \cot^2 4x dx$ (6)

(d) $\int x(5x-2)^{\frac{2}{3}} dx$ (8)

8.2 Given: $g(x) = \frac{9x^2 - 15x - 6}{x^3 - 3x^2 - 9x - 5}$

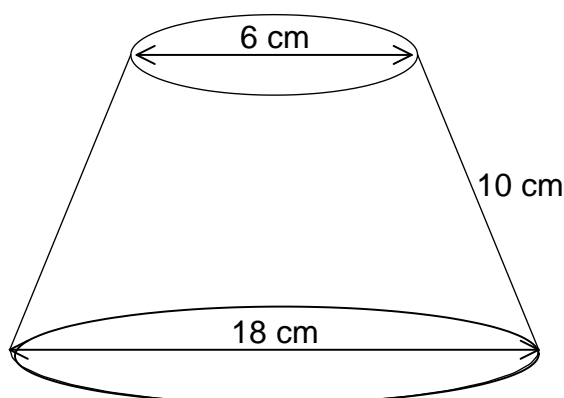
(a) Resolve $g(x)$ into partial fractions. (10)

(b) Hence, or otherwise, determine $\int g(x) dx$. (6)

[39]

QUESTION 9

The diagram shows a salt container in the shape of a frustum of a cone. The frustum has a base diameter of 18 cm, top diameter of 6 cm and slant height of 10 cm. This has been created by rotating a certain function about the x -axis.



9.1 Using a suitable sketch on a Cartesian plane, determine the integral that would represent the volume of the frustum. (8)

9.2 Hence or otherwise, determine to what height the container must be filled to contain 856 cm^3 of salt. (6)

[14]

Total: [200]

EXAMINATION NUMBER:

2	0	1	0	1	2	0	2	0			
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MARKING GRID

Question	Algebra	Calculus
1	/30	
2	/18	
3	/12	
4		/32
5		/19
6		/22
7		/14
8		/39
9		/14
TOTAL PER TOPIC	/60	/140
TOTAL MARK		/200