2020 AP Maths Prelim

Module B - Finance and Modelling

Question 1

$$(1 + 0{,}16)^5 = (1 + 0{,}13)^2 (1 + x)^3$$ ✓ ✓ ✓

$\therefore x = 0{,}18044\ldots$ ✓ (5)

$\therefore$ Depreciation rate = 18,04 % ✓

[5]

Question 2

a) $T_{n+1} = \left(1 + \frac{0{,}12}{12}\right) T_n - 6000$ ✓ ✓ ✓

$\therefore T_{n+1} = 1{,}01\ T_n - 6000$ (5)

where $T_0 = 70\,000$, $n \in \mathbb{N}_0$ ✓ ✓

b) $T_8 = 260\,85{,}94602$ ✓

$T_9 = 20\,346{,}80548$ ✓

$\therefore$ Interest $= 6000 - (T_8 - T_9)$ ✓

$= R\ 260{,}86$ ✓ (4)

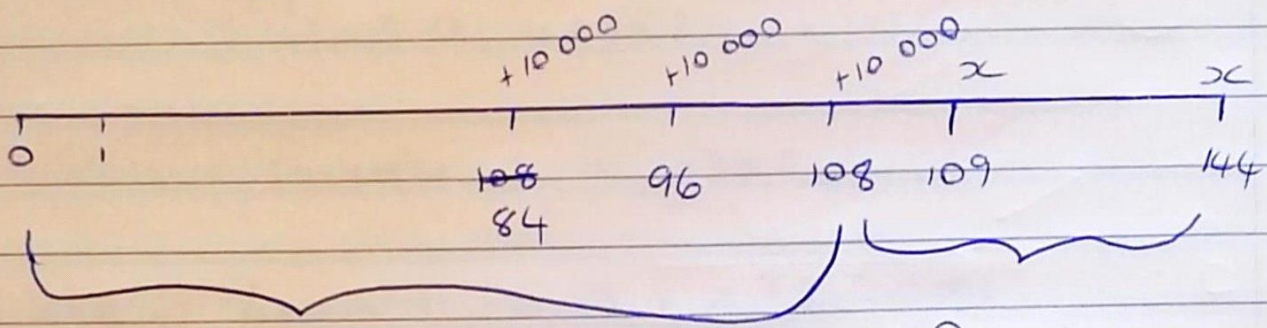
[9]

Question 3

a) $FV = \frac{1500\left[\left(1+\frac{0,126}{12}\right)^{144}-1\right]}{\frac{0,126}{12}}$ ✓ ✓ ✓

$= R\ 500\ 025,54$ ✓ ✓ (5)

b)



$500\,000 = \frac{1500\left[\left(1+\frac{0,126}{12}\right)^{108}-1\right]}{\frac{0,126}{12}}\left(1+\frac{0,126}{12}\right)^{36}$ ✓ ✓ ✓

$+ 10\,000\left[\left(1+\frac{0,126}{12}\right)^{60}+\left(1+\frac{0,126}{12}\right)^{48}+\left(1+\frac{0,126}{12}\right)^{36}\right.$ ✓ ✓ ✓

$+ x\left[\frac{\left(1+\frac{0,126}{12}\right)^{36}-1}{\frac{0,126}{12}}\right]$ ✓

$\therefore x = R\ 354,18$ ✓ ✓ ✓ (10)

[15]

Question 4

a) $\left(1+\frac{i^{(4)}}{4}\right)^4 = \left(1+\frac{i^{(12)}}{12}\right)^{12}$ ✓ ✓

$\therefore \left(1+\frac{0,142}{4}\right)^4 = \left(1+\frac{i^{(12)}}{12}\right)^{12}$

$\therefore i^{(12)} = 0,140352...$ ✓ (4)

$\therefore$ Interest = 14,0352% ✓

b) Value of Loan = 0,9 × 285 9009

= R 257 310,00 ✓

$$257\,310 = \frac{6000\left[1-\left(1+\frac{0,140352}{12}\right)^{-n}\right]}{\frac{0,140352}{12}}$$ ✓ ✓

$\therefore n = 59,8822...$ ✓ ✓

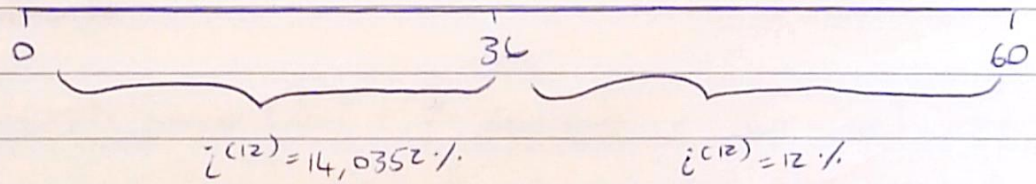
It will take 60 months. ✓ (6)

c) Last payment = $257\,310\left(1+\frac{0,140352}{12}\right)^{60}$ ✓ ✓

$$- \frac{6000\left[\left(1+\frac{0,140352}{12}\right)^{59}-1\right]}{\frac{0,140352}{12}}\left(1+\frac{0,140352}{12}\right)$$ ✓ ✓ ✓

= R 5296,86 ✓ (6)

d)



$$OB@36 = 257\,310\left(1 + \frac{0,140352}{12}\right)^{36} - \frac{6000\left[\left(1 + \frac{0,140352}{12}\right)^{36} - 1\right]}{\frac{0,140352}{12}}$$

$$= R\ 124\,391,2783$$

$$\therefore 124\,391,2783 = \frac{x\left[1 - \left(1 + \frac{0,12}{12}\right)^{-24}\right]}{\frac{0,12}{12}}$$

$$\therefore x = R\ 5855,53$$

∴ New monthly payments are R 5855,53. (6)

[22]

Question 5

a) birth rate (b) = % female × survival rate
× no. litter per cycle × litter size
$= (0.6) \times (0,86) \times \left(\frac{1}{3}\right) \times (1)$ ✓✓
$= 0,172$ ✓

death rate (d) = $\frac{1}{56}$ ✓

Growth rate (r) = b − d ✓
$= 0,172 - \frac{1}{56}$
$= 0,1541...$ (A) ✓ (6)

b) $P_{n+1} = (1 + 0,1541...) P_n - 2500$ ✓ ✓

$P_0 = 11500$ ✓, $n \in \mathbb{N}_0$ ✓ (4)

* Accept rounding as no d.p. mentioned.

c) $P_7 = 3346,86...$
≈ 3346 ✓✓✓ (3)

d) steady state when: $P_{n+1} = P_n$ ✓

$\therefore 11500 = (1,1541...) 11500 + C$ ✓

$\therefore C = -1772,64...$

$\therefore$ cull 1772 elephants. ✓ (3)

[16]

Or : $3346 = 3346(1 + 0,1541\ldots) + C$

$\therefore C = -515,762$

$\therefore$ cull 515 elephants.

→ Ambiguity in question.

Question 6

a) The owl population increases suddenly whilst the ~~mouse~~ mice population decreases. This is due to the introduction of owls in the area. (4)

b) i. A
ii. F
iii. B (6)

c) $b \cdot M_0 \cdot O_0$ = Mice killed by owls in the first month.

$\therefore b(13750)(6) = 83$

$\therefore b = 0,001$ (4)

d) The natural death rate of the owls. (1)

e) Owls will decrease due to natural death whilst mice will increase due to fewer predators. (2)

f) $K = 15000$ and is the carrying capacity. (2)

g) $M_{n+1} = M_n$ when there is equilibrium (6)

$\therefore$ ~~$1445 = 1445 + 0,2\left(1 - \frac{1445}{15000}\right) - 0,001(1445)O_n$~~

$\therefore$ ~~O_n~~ $= 1445 = 1445 + (0,2)(1445)\left(1 - \frac{1445}{15000}\right) - 0,001(1445)O_n$

$\therefore O_n = 180,73...$

≈ 180 owls. or ≈ 181 owls. [25]

Question 7

a) The number of sunflowers in any given season depends on the number of sunflowers the two seasons before and not just the previous season. ✓✓ (2)

b) $T_{n+1} = 3T_n + 5T_{n-1} + 5$ ✓ ✓ ✓

where $T_1 = 5$, $T_2 = 20$ and $n \in \mathbb{N}$ ✓ (4)

c) $T_6 = 3T_5 + 5T_4 + 5$

$= 3(1580) + 5(375) + 5$ ✓

$= 6620$

Yes she will. ✓ (2)

[8]