AP prelim: Statistics 2020

Question 1: [30]

1.1a) $P(\text{Male} \mid \text{Angry}) = \dfrac{P(\text{Male} \cap \text{Angry})}{P(\text{Angry})}$

$= \dfrac{15 ✓✓}{38 ✓✓}$ or $0{,}3947$

(4)

b) $P(\text{Furious} \mid \text{Female}) = \dfrac{P(\text{Furious} \cap \text{Female})}{P(\text{Female})}$

$= \dfrac{6 ✓✓}{39}$

(4)

$= \dfrac{2}{13} ✓✓$ or $0{,}1538$

1.2

65 sweets $\rightarrow$ 18 toffees

65 sweets $\rightarrow$ 20 jelly babies

65 sweets $\rightarrow$ 27 chocolates

20 jelly babies + 27 chocolates $\rightarrow$ 47 non-toffees

$P(\text{no toffee}) = \dfrac{\binom{47}{3}\binom{18}{0}}{\binom{65}{3}}$ ✓✓ ✓✓

(6)

$= \dfrac{1081}{2912}$ ✓✓ or $0{,}3712$

1.3 $p = 0,65$
$1-p = 0,35$

(a) $P(X=x) = \binom{3}{x}(0,65)^{x}(0,35)^{3-x}$
If X = number of children who want the meal.
$X = \{0, 1, 2, 3\}$

$P(X \geq 2) = P(X=2) + P(X=3)$ ✓✓
$= \binom{3}{2}(0,65)^2(0,35)^1$ ✓✓ $+ \binom{3}{3}(0,65)^3(0,35)^0$ ✓✓
$\frac{2873}{4000}$ ✓ or $0,7183$

(7)

(b) To use the normal approximation
$np > 5$ and $n(1-p) > 5$

$np = 100 \times 0,65$ and $n(1-p) = 100 \times 0,35$
$= 65$ ✓ $= 35$ ✓

$\therefore$ both conditions are met

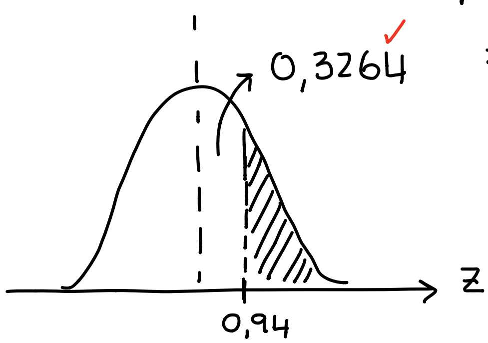
$\mu = 65$ (from above)
$\sigma^2 = np(1-p)$
$\sigma^2 = \frac{91}{4}$
$\sigma = \frac{\sqrt{91}}{2}$ ✓ $(4,7697)$

Continuity correction:

$$P(X \geq 70) \rightarrow P(X > 69{,}5)$$

$$P(X > 69{,}5) = P\left(Z > \frac{69{,}5 - 65}{\frac{\sqrt{91}}{2}}\right)$$

$$= P(Z > 0{,}94)$$



$$= 0{,}5 - 0{,}3264$$

$$= 0{,}1736$$

(9)

Question 2

2.1 $P(X = x) = \frac{1}{K}(2x+1) \quad x \in \{0, 1, 2, 3\}$

(a) $\frac{1}{K}(2(0)+1) + \frac{1}{K}(2(1)+1) + \frac{1}{K}(2(2)+1) + \frac{1}{K}(2(3)+1) = 1$

$$\frac{1}{K} + \frac{3}{K} + \frac{5}{K} + \frac{7}{K} = 1$$

$$\frac{16}{K} = 1$$

(4)

$$\therefore 16 = K$$

(b)

x	0	1	2	3
$P(x)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

$$E(X) = \left(0 \times \tfrac{1}{16}\right) + \left(1 \times \tfrac{3}{16}\right) + \left(2 \times \tfrac{5}{16}\right) + \left(3 \times \tfrac{7}{16}\right) \checkmark\checkmark$$
$$= \frac{17}{8} \checkmark \text{ or } 2{,}125$$

$$Var(X) = \left(0^2 \times \tfrac{1}{16}\right) + \left(1^2 \times \tfrac{3}{16}\right) + \left(2^2 \times \tfrac{5}{16}\right) + \left(3^2 \times \tfrac{7}{16}\right) - \left(\tfrac{17}{8}\right)^2 \checkmark$$
$$= \frac{55}{64} \checkmark$$

(7)

$$\therefore \sigma(X) = \sqrt{\frac{55}{64}} \checkmark$$
$$= \frac{\sqrt{55}}{8} \checkmark \text{ or } 0{,}927$$

2.2 $f(x) = \begin{cases} \frac{1}{x^2} ; & \frac{1}{2} \leq x \leq a \\ 0 ; & \text{elsewhere} \end{cases}$

(a) $\displaystyle\int_{\frac{1}{2}}^{a} \frac{1}{x^2}\, dx = 1 \checkmark\checkmark$

$$\left[-\frac{1}{x}\right]_{\frac{1}{2}}^{a} \checkmark = 1$$
$$-\frac{1}{a} - \left(-\frac{1}{\frac{1}{2}}\right) \checkmark = 1$$
$$-\frac{1}{a} + 2 = 1 \checkmark$$
$$\therefore a = 1$$

(5)

(b) $f(x) = \frac{1}{x^2} \qquad \frac{1}{2} \leq x \leq 1$

$$\int_{\frac{1}{2}}^{m} \frac{1}{x^2}\,dx = \frac{1}{2}$$

$$\left[-\frac{1}{x}\right]_{\frac{1}{2}}^{m} = \frac{1}{2}$$

$$-\frac{1}{m} - \left(-\frac{1}{\frac{1}{2}}\right) = \frac{1}{2}$$

$$-\frac{1}{m} + 2 = \frac{1}{2}$$

$$-\frac{1}{m} = -\frac{3}{2}$$

$$\therefore m = \frac{2}{3}$$

(5)

Question 3

3.1 $X \sim N(6, 25)$

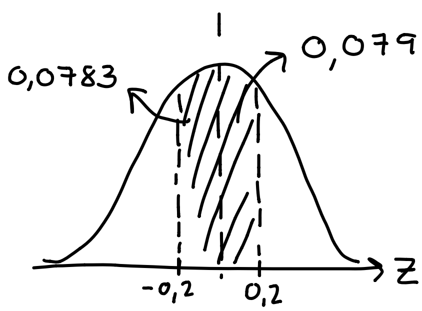
$\mu = 6$ and $\sigma^2 = 25$

$\sigma = 5$

(a) $P(5 \leq x \leq 7)$

$= P\left(\frac{5-6}{5} \leq z \leq \frac{7-6}{5}\right)$

$= P(-0{,}2 \leq z \leq 0{,}2)$



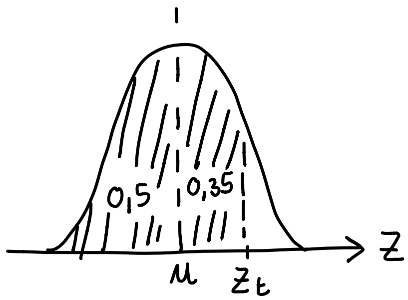
$\therefore P(-0{,}2 \leq z \leq 0{,}2) = 2 \times 0{,}0793$

$= 0{,}1586$

(4)

(b) $P(X < t) = 0{,}85$

$P\left(Z < \frac{t-\mu}{\sigma}\right) = 0{,}85$ ✓



$z_t = 1{,}04$ ✓

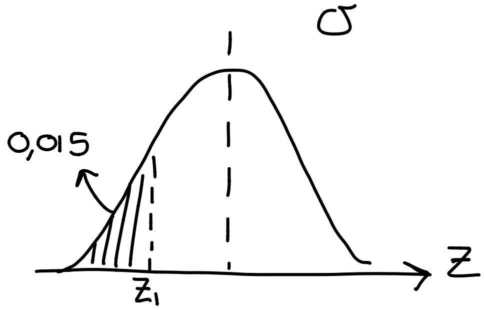
(4)

$\therefore \quad \frac{t-6}{5} = 1{,}04$ ✓

$t = 11{,}2$ ✓

3.2 $\quad 3\sigma^2 = 2\mu \quad$ and $\quad P\left(X < \frac{\mu}{3}\right) = 0{,}015$

$P\left(Z < \frac{\frac{\mu}{3} - \mu}{\sigma}\right) = 0{,}015$



$\therefore \; z_1 = -2{,}17$ ✓✓

$\frac{\frac{\mu}{3} - \mu}{\sigma} = -2{,}17$ ✓

$\frac{\mu}{3} - \mu = -2{,}17\sigma$

$-\frac{2}{3}\mu = -2{,}17\sigma$

$$2\mu = 6{,}51\sigma \quad \ldots (1)$$
$$2\mu = 3\sigma^2 \quad \ldots (2)$$

$$6{,}51\sigma = 3\sigma^2$$
$$0 = \sigma(3\sigma - 6{,}51)$$
$$\sigma = 0 \text{ or } 3\sigma = 6{,}51$$
$$\sigma = 2{,}17$$

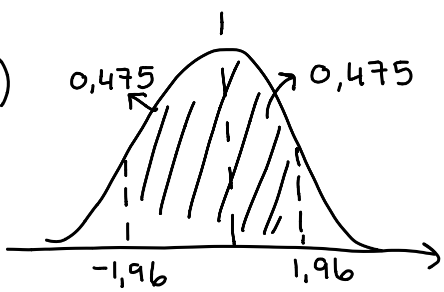
(8)

$$2\mu = 3(2{,}17)^2$$
$$\mu = 7{,}0634$$

Question 4

4.1 $p = 0{,}081$

(a)



$z = \pm 1{,}96$

$$p \pm z\sqrt{\frac{p(1-p)}{n}}$$

(8)

$$0{,}081 \pm 1{,}96\sqrt{\frac{0{,}081(0{,}919)}{209}}$$

$(0{,}044 \,;\, 0{,}1180)$

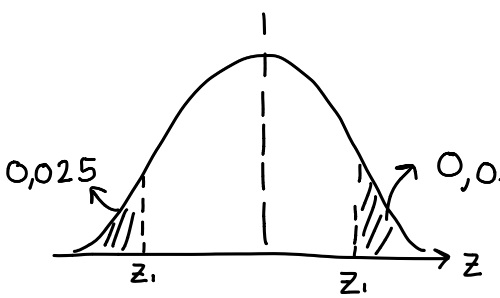
$(4{,}4\% \,;\, 11{,}80\%)$

(b) Increase the sample size ✓✓ (2)
Decrease confidence level (any one)

4.2 $\mu = 9$ hours
$\sigma = 3$ hours

(a) $\bar{x} = 11{,}96$ hours ✓✓

$H_0: \mu = 9$
$H_1: \mu \neq 9$ ✓✓



$z_1 = \pm 1{,}96$ ✓

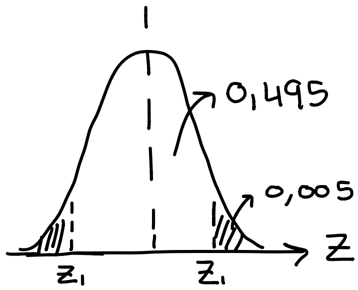
$$Z_{test} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{11{,}96 - 9}{\frac{3}{\sqrt{10}}}$$ ✓ ✓ (10)

$$= 3{,}12$$ ✓

$3{,}12 > 1{,}96$ $\therefore$ the test statistic ✓✓ lies in the rejection region. There is sufficient evidence to reject H_0 & therefore not fair to claim a 9hr mean time.

(b)



$Z_1 = \pm 2{,}58$ ✓

$\bar{x} - z\left(\frac{\sigma}{\sqrt{n}}\right)$

$11{,}96 - 2{,}58\left(\frac{3}{\sqrt{10}}\right)$ ✓✓ (5)

$(9{,}5124 \,;\, 14{,}4076)$ ✓✓

Question 5

(a) _ B _ B _ B _ B _ B _

$6P4$ ✓ $\times 5!$ ✓

$= 43\ 200$ ✓ (3)

(b) $P(E) = \dfrac{n(E)}{n(S)}$

$n(S)$: $9!$

$n(E)$: $\frac{4}{G}\ \underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{7!}\ \frac{5}{B}$

$4 \times 7! \times 5$

$\therefore P(E) = \dfrac{4 \times 7! \times 5}{9!}$ ✓✓✓ ✓ (5)

$= \dfrac{5}{18}$ ✓ or $0{,}2778$