1

AP memo Paper 1. Aug 2020

1a i) $\dfrac{2x-23}{(2x-1)(x+5)} = \dfrac{A}{(2x-1)} + \dfrac{B}{(x+5)}$

$\therefore 2x-23 = A(x+5) + B(2x-1)$ ✓

if $x=-5$: $-33 = A(0) + B(-11)$ ✓

$B = 3$ ✓

if $x = \frac{1}{2}$: $-22 = A\left(\frac{11}{2}\right) + B(0)$ ✓

$A = -4$ ✓ (6)

$\therefore \dfrac{2x-23}{(2x-1)(x+5)} = \dfrac{-4}{(2x-1)} + \dfrac{3}{(x+5)}$

ii) $\displaystyle\int \frac{2x-23}{(2x-1)(x+5)} = -\int \frac{4\,dx}{(2x-1)} + \int \frac{3\,dx}{(x+5)}$ ✓

$= -4\cdot\frac{1}{2}\ln|2x-1| + 3\ln|x+5| + c$ ✓ (4)

$= -2\ln|2x-1| + 3\ln|x+5| + c$

16

b i) $(e-1)(2) - 2 = 2e-4$ ✓ (3)

ii) $\frac{1}{2}\left(2 + \frac{2}{e}\right)(e-1) - 2$ ✓

$= \frac{1}{2}\left(2e - 2 + 2 - \frac{2}{e}\right) - 2$ ✓

$= e - \frac{1}{2} - 2$ ✓ (3)

6

2a) $\overset{\frown}{AD} = r\theta$ ✓

$BC^2 = (2r)^2 + (2r)^2 - 2(2r)(2r)\cos\theta$ ✓

$= 4r^2(1-2\cos\theta)$ ✓

$BC = 2r\sqrt{1-2\cos\theta}$ ✓

Perim $= 2r + r\theta + 2r\sqrt{1-2\cos\theta}$ ✓✓ (6)

OR: $\dfrac{BC}{\sin\theta} = \dfrac{2r}{\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)} = \dfrac{2r}{\cos\frac{\theta}{2}}$ ✓ ✓ ✓

$BC = \dfrac{2r\sin\theta}{\cos\frac{\theta}{2}}$ ✓ (6)

Perim $= 2r + r\theta + \dfrac{2r\sin\theta}{\cos\frac{\theta}{2}}$ ✓ ✓

2b) Shaded area

$=$ area ΔBEC $-$ area sector AED ✓

$= \frac{1}{2}(2r)(2r)\sin\theta - \frac{1}{2}r^2\theta$ ✓ ✓

$= 2r^2\sin\theta - \frac{1}{2}r^2\theta$ ✓ (4)

2c) If $r=1$

$A(\theta) = 2\sin\theta - \frac{1}{2}\theta$ ✓

For max $A'(\theta) = 0$ ✓

$2\cos\theta - \frac{1}{2} = 0$ ✓

$\cos\theta = \frac{1}{4}$ ✓

$\theta = 1{,}3181$ ✓ (8)

$\therefore$ max area $= 1{,}28\ u^2$ ✓✓

18

3 For continuity ✓

$\lim\limits_{x\to a^-} g(x) = \lim\limits_{x\to a^+} g(x) = g(a)$

$\therefore a - 3 = a^3 - 2a - 1$ ✓ ✓

$\therefore 0 = a^3 - 3a + 2$ ✓ (6)

$\therefore a = -2$ ✓ or $a = 1$ ✓ (calc)

2

4a) $h(x) = \frac{\cancel{(x^2+2)}(x+2)}{\cancel{(x^2+2)}(x^2-2)}$ (2)

$= \frac{x+2}{x^2-2}$ $(x \neq \pm\sqrt{2})$

b) Vert asymptotes

$x = \pm\sqrt{2}$ (2)

Horiz Asymptotes:

$y = \lim_{x\to\pm\infty} \frac{x+2}{x^2-2}$

$y = \lim_{x\to\pm\infty} \frac{\cancel{x^2}(\frac{1}{x}+\frac{2}{x^2})}{\cancel{x^2}(1-\frac{2}{x^2})}$ (2)

$y = 0$

(No oblique asymptotes) (17)

c) At stat points, $h'(x) = 0$

$\therefore \frac{(x^2-2)-(x+2)(2x)}{(x^2-2)^2} = 0$

$\therefore x^2-2-2x^2-4x = 0$ $(D \neq 0)$

$\therefore -x^2-4x-2 = 0$

(15) $x^2+4x+2 = 0$

$(x+2)^2 = -2+(2)^2 = 2$

$x+2 = \pm\sqrt{2}$ (6)

$x = -2\pm\sqrt{2}$

d) x int $(-2;0)$ (3)

y int $(0;-1)$

5a) $\log_x e = \frac{\log_e e}{\log_e x}$ (change of base law)

$= \frac{1}{\ln x}$ (2)

5b) $\therefore \ln x - \frac{4}{\ln x} - 3 = 0$

(Let $\ln x = k$)

$k - \frac{4}{k} - 3 = 0$ (6)

$k^2 - 3k - 4 = 0$

$(k-4)(k+1) = 0$

$k = 4$ or $k = -1$

$\ln x = 4$ or $\ln x = -1$

$x = e^4$ or $x = e^{-1}$

(54,60 or 0,37)

c) $(g \circ f)(x) = g(\ln(x-1))$

$= e^{\ln(x-1)+2-2} + 1$

$= e^{\ln(x-1)} + 1$ (4)

$x-1 > 0$ $\therefore x > 1$

Domain $(1;\infty)$ (4)

Range $y > 1$ $(1;\infty)$

6a) $\sin x - \cos x = 0$

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4} + k\pi$ $k \in \mathbb{Z}$ (4)

$A(\frac{\pi}{4};0)$ $B(\frac{5\pi}{4};0)$

b) $A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx$

$= -\cos x - \sin x \Big]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ (6)

$= 2\sqrt{2}\ u^2$

6c) $\pi \int_0^k (\sin x - \cos x)^2\,dx = \pi(k - \frac{1}{4})$

$\int_0^k (\sin^2 x - 2\sin x\cos x + \cos^2 x)\,dx = k - \frac{1}{4}$

$\int_0^k (1 - \sin 2x) = k - \frac{1}{4}$

3

6c

$\therefore x + \frac{1}{2}\cos 2x \Big]_0^k = k - \frac{1}{4}$

$\therefore \cancel{k} + \frac{1}{2}\cos 2k - \frac{1}{2} = \cancel{k} - \frac{1}{4}$

$\frac{1}{2}\cos 2k = \frac{1}{4}$

$\cos 2k = \frac{1}{2}$

(19)

$2k = \frac{\pi}{3}$ (9)

$\underline{k = \frac{\pi}{6}}$

7. $F(2)=0$ $\therefore$ x int $(-2;0)$

$f'(x)>0 \Rightarrow$ decr if $x<1$

$f'(1)=0 \Rightarrow$ TP: $x=1$

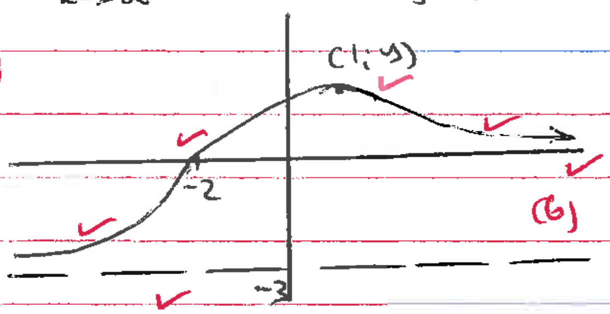
$f''(x)>0$ $\{x<-2$ or $x>2$ concave up

$f''(x)<0$ $-2<x<2$ concave down

$\lim_{x\to-\infty} f(x)=3 \Rightarrow$ H Asym: $y=-3$

$\lim_{x\to\infty} f(x)=0 \Rightarrow$ Hori Asymp $y=0$

(6)



8a) $f(g(x)) = \cos\left(\frac{2}{e^{2x}}\right)$

$\frac{d}{dx} f(g(x)) = -\sin\left(\frac{2}{e^{2x}}\right).(-2)(2)e^{-2x}$

$= 4\sin\left(\frac{2}{e^{2x}}\right).e^{-2x}$ (8)

(25)

8b) $2y + 2xy' + \pi \cos y\, y' = 0$

$y'(2x + \pi\cos y) = -2y$

$y' = \frac{-2y}{2x + \pi\cos y}$ (5) + (3)

at $(1;\frac{\pi}{2})$

$y' = \frac{-2\frac{\pi}{2}}{2+\pi\cos\pi} = \frac{-\pi}{2}$ (=8)

$y - \frac{\pi}{2} = \frac{-\pi}{2}(x-1)$

$\underline{y = \frac{-\pi}{2}x + \pi}$

c) $\frac{1}{\sec x + \tan x}(\sec x\tan x + \sec^2 x)$

$= \frac{1}{\cancel{(\sec x + \tan x)}} \sec\,\cancel{(\tan x + \sec x)}$ (4)

$= \sec x$

d) $-e^{-x}\sqrt{2}.x^{\frac{3}{2}} + e^{-x}.\frac{1}{2}(2x^3)^{-\frac{1}{2}}6x^2$ (5)

$= \frac{-\sqrt{2}e^{-x}x^{\frac{3}{2}} + 3x^2e^{-x}}{\sqrt{2}x^{\frac{3}{2}}}$

9a) $1-2\tan 0 = 1$ (>0)

$2-2\tan 1 = -1,11$ (<0)

$\therefore$ continuous & so there must be an x int between $x=0$ & $x=1$ (4)

b) $f'(x) = 1-2\sec^2 x$

$x_{r+1} = x_r - \frac{(1+x_r-2\tan x_r)}{1-2\sec^2 x_r}$

(10) Let $x_0 = \frac{1}{2}$ (6)

$x_1 = 0,755173...$

$x_2 = 0,709194776...$

$x_3 = 0,706337955...$

$\therefore x \approx 0,7063$

4

NB -1 No c ≥ 2 cases

10 a) i) RHS $= \sec^2 x + \frac{1}{2}(\cos 2x - 1)$

$= 1 + \tan^2 x + \frac{1}{2}(2\cos^2 x - 2)$

$= 1 + \tan^2 x + \cos^2 x - 1$

$= \tan^2 x + \cos^2 x$

$=$ LHS

(5)

or. LHS $= \frac{\sin^2 x}{\cos^2 x} + \cos^2 x$

$= \frac{\sin^2 x + \cos^4 x}{\cos^2 x}$

or RHS $= \frac{1}{\cos^2 x} + \frac{1}{2}(2\cos^2 x - 2)$

$= \frac{1}{\cos^2 x} + \cos^2 x - 1$

$= \frac{1 + \cos^4 x - \cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x + \cos^4 x}{\cos^2 x}$

ii) $\int_0^{\frac{\pi}{4}} (\tan x + \cos x)^2 \, dx$

(41)

$= \int_0^{\frac{\pi}{4}} (\tan^2 x + 2\tan x \cos x + \cos^2 x)\,dx$

$= \int_0^{\frac{\pi}{4}} (\tan^2 x + 2\sin x + \cos^2 x)\,dx$

$= \int_0^{\frac{\pi}{4}} [\sec^2 x + \frac{1}{2}(\cos 2x - 1) + 2\sin x]\,dx$

$= \int_0^{\frac{\pi}{4}} [\sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2} + 2\sin x]\,dx$

$= \tan x + \frac{1}{4}\sin 2x - \frac{1}{2}x - 2\cos x \Big]_0^{\frac{\pi}{4}}$

$= 1 + \frac{1}{4}(1) - \frac{\pi}{8} - \sqrt{2} - (-2)$

$= \frac{13}{4} - \frac{\pi}{8} - \sqrt{2}$ (1,44) Not nec.   (8)

10 b) i) $\int \cos 5x \sin 3x \, dx$

$= \frac{1}{2}\int (\sin 8x - \sin 2x)\,dx$

$= \frac{1}{2}\left(-\frac{1}{8}\cos 8x + \frac{1}{2}\cos 2x\right) + c$

(5)

$= -\frac{1}{16}\cos 8x + \frac{1}{4}\cos 2x + c$

ii) Let $u = x^3$; $du = 3x^2\,dx$

$\therefore dx = \frac{du}{3x^2}$

$\therefore \int x^2 \sin x^3 \, dx = \frac{1}{3}\int \sin u \, du$

$= -\frac{1}{3}\cos(x^3) + c$

iii) Let $u = x^3 + 3x^2 - 4$

$du = (3x^2 + 6x)\,dx$

$dx = \frac{du}{3(x^2 + 2x)}$

$\therefore$ Integral $= \frac{1}{3}\int u^{\frac{1}{2}}\,du$

(6)

$= \frac{1}{3} \cdot \frac{2}{3}(x^3 + 3x^2 - 4)^{\frac{3}{2}} + c$

$= \frac{2}{9}(x^3 + 3x^2 - 4) + c$

(iv) Let $u = x^2 - 1$ $\therefore du = 2x\,dx$

$dx = \frac{du}{2x}$

also when $x = \sqrt{2}$; $u = 1$

& when $x = 2$; $u = 3$

$\therefore$ Int $= \int_1^3 \frac{1}{u}\,du$

$= \frac{1}{2}\ln u \Big|_1^3$

$= \frac{1}{2}\ln 3$ (0,55)

v) $f(x) = \ln x \therefore f'(x) = \frac{1}{x}$

$g'(x) = x^3 \therefore g(x) = \frac{x^4}{4}$

$\therefore$ Integ. $= \frac{1}{4}x^4 \ln x \Big|_1^2 - \frac{1}{4}\int_1^2 x^3\,dx$

$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\left(\frac{1}{4}x^4\right)\Big]_1^2$

$= 1,84$

$\left(\frac{256}{4}\ln 2 - \frac{1}{16}(256) \right.$

$\left. = 64\ln - 16 \right)$

5

11a) i) $x+iy = a^2 + 2abiy + b^2i^2$

$= a^2 - b^2 + 2abiy$

Re: $x = a^2 - b^2$

Im: $y = 2ab$ (5)

ii) $a = -12$; $b = 5$

$x = (-12)^2 - (5)^2 = 119$ (4)

$y = 2(-12)(5) = -120$

b) RTP $6^{k+1} + 4$ div. by 5

From assumption

$6^k + 4 = 5p$

$\therefore 6^{k+1} + 4 = 6.6^k + 4$ (8)

$= 6^k + 4 + 5.6^k$

(17) $= 5p + 5.6^k$

$= 5(p + 6^k)$

which is div. by 5

12a) $f(x) = x^2 - 4x$ (1)

b) $\frac{b-1}{n} = \frac{4}{n}$ $\therefore b = 5$ (2)

c) Estimated area

$= \frac{64}{3} + \frac{32}{4} - \frac{32}{3 \times 64}$

$= \frac{175}{6}$ (3)

d) $\frac{64}{3}$ $(n \to \infty)$ (2)

(10)

e) Bigger

- all rectangles lie (2)

above graph