AP: MEMO-2020
PRELIM EXAM P1

QUESTION 1:

a) $\tan\left(\frac{\pi}{6}\right) = \frac{4}{PA}$ ✓

$PA = 4\sqrt{3}$ ✓

(4)

$PB = PO + OC$

$PO = \sqrt{(4\sqrt{3})^2 + 4^2}$

$= 8$ ✓

$\therefore PB = 12$ ✓

b) Area $= \frac{\pi}{3}\frac{(12)^2}{2} - \frac{1}{2}(4\sqrt{3}).4.2$ ✓✓ ✓✓

$= 47{,}7$ ✓

(5)

c) Perimeter $= AB + BD + ED + AO + OE$

$= 2(12 - 4\sqrt{3}) + 12 \cdot \frac{\pi}{3} + 8$ ✓ ✓ ✓

(4)

$= 30{,}7$ ✓

QUESTION 2:

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = \frac{2^n - 1}{2^n}$

Prove true for $n = 1$ ✓

LHS $= \frac{1}{2}$ RHS $= \frac{2^1 - 1}{2} = \frac{1}{2}$ ✓

$\therefore$ LHS = RHS ✓

Assume true for $n = k$ ✓

$\frac{1}{2} + \dots + \frac{1}{2^k} = \frac{2^k - 1}{2^k}$ ✓

(12)

Prove true for $n = k+1$ ✓

LHS $= \frac{1}{2} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$ ✓

$= \frac{2^k - 1}{2^k} + \frac{1}{2^{k+1}}$ ✓

Prove to RHS $= \frac{2^{k+1} - 1}{2^{k+1}}$ ✓

$= \frac{(2^k - 1)2^1 + 1}{2^{k+1}}$ ✓

$= \frac{2^{k+1} - 2 + 1}{2^{k+1}}$

$= \frac{2^{k+1} - 1}{2^{k+1}}$ ✓

✓

By PMI it is true for all positive integers.

QUESTION 3:

ai. $\frac{3w+7}{5} = \frac{p-4i}{3-i}$

$\frac{3w+7}{5} = \frac{p-4i}{3-i} \times \frac{3+i}{3+i}$ ✓

$= \frac{3p+pi-12i-4i^2}{9+1}$ ✓✓

$= \frac{3p+4+i(p-12)}{10}$ ✓ (8)

$\therefore 3w = \frac{3p+4+i(p-12)}{2} - 7$

$w = \frac{3p+4+i(p-12)-14}{6}$ ✓✓

$= \frac{3p-10}{6} + \frac{i(p-12)}{6}$ ✓✓

OR $(3-i)(3w+7) = 5p-20i$

$9w+21-3wi-7i = 5p-20i$ ✓

let $w = a+bi$ ✓

$9(a+bi)+21-3i(a+bi) = 5p-13i$ ✓

$9a+9bi+21-3ai-3bi^2 = 5p-13i$ ✓

Real: $9a+3b = 5p-21$ ✓

Im: $-3a+9b = -13$ ✓

$\therefore b = \frac{p-12}{6}$ ✓ $a = \frac{3p-10}{6}$ ✓

$\therefore w = \frac{3p-10}{6} + \frac{p-12}{6} \cdot i$

2. $\arg w = -\pi/2$

Real $= 0$

$\frac{3p-10}{6} = 0$ ✓

$3p = 10$ (2)

$p = \frac{10}{3}$ ✓

b. $2 + \ln\sqrt{1+x} + 3\ln\sqrt{1-x} = \ln\sqrt{1-x^2}$

$2 + \ln\sqrt{1+x} + 3\ln\sqrt{1-x} = \ln\sqrt{1+x} + \ln\sqrt{1-x}$ ✓

$2 + 2\ln\sqrt{1-x} = 0$ ✓✓

$\ln\sqrt{1-x} = -1$ ✓ (8)

$\sqrt{1-x} = e^{-1}$ ✓

$1-x = e^{-2}$ ✓

$1-e^{-2} = x$ ✓

$1-\frac{1}{e^2} = x$ ✓ (0.865)

QUESTION 4:

a. $\lim_{x\to 2} f(x) = 5$ $\lim_{x\to 2} g(x) = 2$

1. $\lim_{x\to 2} (f(x) \cdot g(2))$

$= 5 \cdot g(2)$

(3)

2. $\lim_{x\to 2} (2f(x) - g(x))$

$= 2 \cdot 5 - 2$

$= 8$

(3)

b. $g(x) = \begin{cases} -x^2 - x + 3 & x \le 0 \\ |x-3| & x > 0 \end{cases}$

$\lim_{x\to 0} (-x^2 - x + 3) = \lim_{x\to 0} (-x+3) = f(0)$

$3 = 3 = f(0)$

$\therefore$ continuous.

$\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^+} f'(x)$

$f(x) = -x^2 - x + 3$ $f(x) = -x + 3$

$f'(x) = -2x - 1$ $f'(x) = -1$

$f'(0) = -1$ $f'(0) = -1$

(9)

$\therefore$ Differentiable.

QUESTION 5:

$f(x) = 1 + 2\ln(4-x)$

a) y int: $x = 0$

$y = 1 + 2\ln 4 = 3{,}8$

x int: $y = 0$

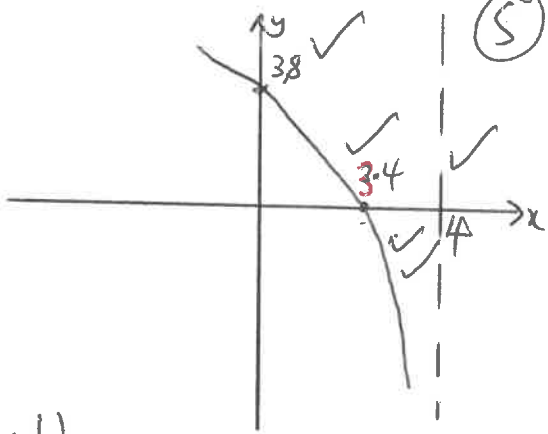
$-\frac{1}{2} = \ln(4-x)$

$e^{-1/2} = 4 - x$

$x = 4 - e^{-1/2} = 3{,}4$

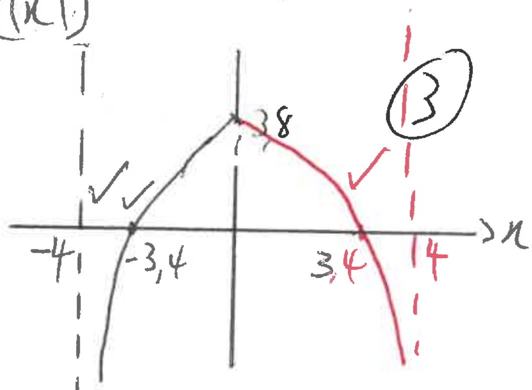
Asymptote: $4 - x > 0$

$x < 4$



(5)

b) $f(|x|)$



(3)

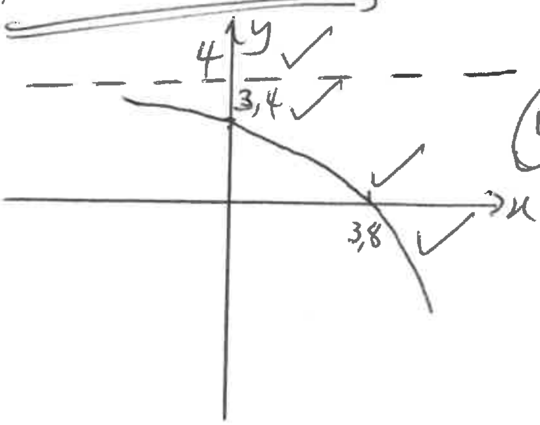
c) $x = 1 + 2\ln(4-y)$

$e^{\frac{x-1}{2}} = 4 - y$

$f^{-1}(x) = 4 - e^{\frac{x-1}{2}}$

(4)

d)



(4)

QUESTION 6

a. $y = \ln(x\sqrt{1-x^2} \cdot \cos x)$

$= \ln x + \ln(1-x^2)^{1/2} + \ln \cos x$

$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\left[\frac{1}{1-x^2} \cdot -2x\right] + \frac{1}{\cos x}(-\sin x)$

$= \frac{1}{x} - \frac{x}{1-x^2} - \frac{\sin x}{\cos x}$.

8

b.1. $y = \ln\left(\frac{x}{3-x}\right) = \ln x - \ln(3-x)$

$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{3-x} = k$

$3 - x + x = kx(3-x)$

6

$kx^2 - 3kx + 3 = 0$

2. $kx^2 - 3kx + 3 = 0$

$9k^2 - 4(k)3 < 0$

$9k^2 - 12k < 0$

$0 < k < \cancel{4}\,3$

3

c) $f(x) = e^{2x}$

$g(x) = x^2 + 2x - 5$

$h(x) = e^{2x} - x^2 - 2x + 5$

$h'(x) = 2e^{2x} - 2x - 2$

$x_r = -2{,}8 = \text{ans}$

8

$x_{r+1} = \text{ans} - \frac{e^{2\text{ans}} - \text{ans}^2 - 2\text{ans} + 5}{2e^{2\text{ans}} - 2\text{ans} - 2}$

$= -3{,}44970$

d. $x = y \cdot \sec(5/y)$

$1 = \sec(5/y)\frac{dy}{dx} + y \cdot \sec(5/y) \cdot \tan(5/y) \cdot \left(\frac{-5}{y^2}\right)\frac{dy}{dx}$

$\frac{1}{\sec(5/y) - 5/y \sec(5/y)\tan(5/y)} = \frac{dy}{dx}$

9

QUESTION 7:

$$f(x) = \frac{2x^2+3x-1}{2x+1}$$

a) VA : $x = -\frac{1}{2}$

$$\begin{array}{r} x+1 \\ 2x+1\,\overline{)\,2x^2+3x-1} \\ \underline{2x + x} \\ 2x-1 \end{array}$$

(6)

$y = x+1$

OR $\frac{1}{2}\left(x+\frac{1}{2}\right)(2x+2)+R = 2x^2+3x-1$

$\therefore$ OA : $y = \frac{1}{2}(2x+2) = x+1$

b) $y = -1$ $x = -1{,}8$ $x = 0{,}3$

$$f'(x) = \frac{(2x+1)(4x+3) - 2(2x^2+3x-1)}{(2x+1)^2} = 0$$

(8)

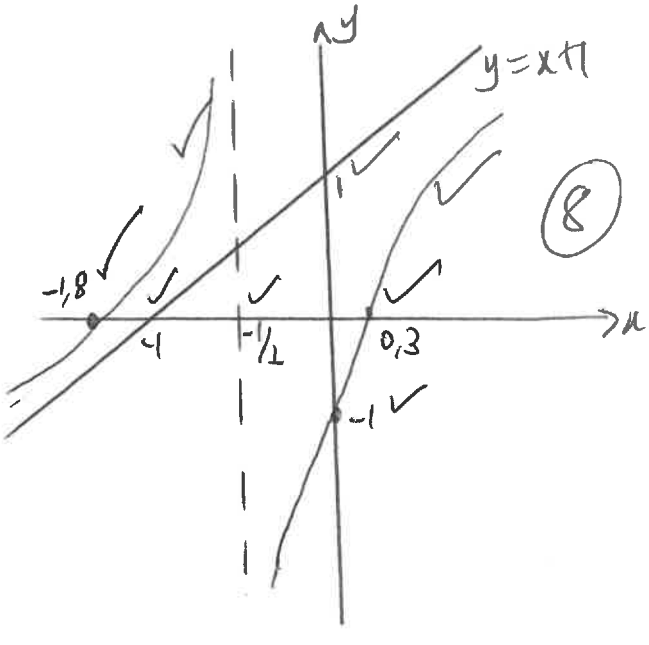
$8x^2+10x+3-4x^2-6x+2 = 0$

$4x^2+4x+5 = 0$

$\therefore x = -\frac{1}{2} \pm i$

no turning point.

c)



(8)

QUESTION 8:

a1. $$\int_1^{\ln 3} \frac{e^x - e^{3x}}{1+e^x}\,dx$$

$$= \int_1^{\ln 3} \frac{e^x(1-e^x)(1+e^x)}{1+e^x}\,dx$$

$$= \int_1^{\ln 3} e^x - e^{2x}\,dx$$

$$= e^x - \frac{e^{2x}}{2}\Big|_1^{\ln 3}$$

$$= -\frac{3}{2} - \left(e^1 - \frac{e^2}{2}\right)$$

$$= \left|\frac{e^2}{2} - e - \frac{3}{2}\right| = 0{,}52$$

(7)

2. $$\int \frac{\cos x + \sin x}{\sin 2x}\,dx$$

$$= \int \frac{\cos x + \sin x}{2\sin x \cos x}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sin x} + \frac{1}{\cos x}\,dx$$

$$= \frac{1}{2}\left[\frac{\ln|\sin x|}{\cos x} + \frac{\ln|\cos x|}{-\sin x}\right] + c$$

(7)

3. $\int x^3 \ln 2x \, dx$

$f(x) = \ln 2x \quad g'(x) = x^3$

$f'(x) = \frac{1}{x} \quad g(x) = \frac{x^4}{4}$

$= \frac{x^4}{4} \cdot \ln 2x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx$

7

$= \frac{x^4}{4} \ln 2x - \frac{x^4}{16} + c$

b) $\int_1^2 \frac{6x+1}{6x^2 - 7x + 2} \, dx$

$\frac{6x+1}{(3x-2)(2x-1)} = \frac{A}{(3x-2)} + \frac{B}{2x-1}$

$6x+1 = A(2x-1) + B(3x-2)$

Sub $x = \frac{1}{2}$. $4 = -\frac{1}{2}B$

$-8 = B$

11

Sub $x = \frac{2}{3}$: $5 = \frac{1}{3}A$

$15 = A$

$= \int_1^2 \frac{15}{3x-2} + \frac{-8}{2x-1} \, dx$

$= \frac{15 \ln|3x-2|}{3} - \frac{8 \ln|2x-1|}{2} \Big|_1^2$

$= (5\ln 4 - 4\ln 3)$

QUESTION 9

a) $A_1 = \int_1^e \ln\sqrt{x} \, dx$

$A_2 = \int_1^e (\ln x - \ln\sqrt{x}) \, dx$

6

$A_2 = \int_1^e (\ln x - \frac{1}{2}\ln x) \, dx$

$= \frac{1}{2}\int_1^e \ln x \, dx = A_1$ (Shown)

b) $A_3 = e \times 1 - \int_1^e (A_1 + A_2) \, dx$

$= e - \int_1^e 2 \cdot \frac{1}{2} \cdot \ln x \, dx$

8

$= e - \int_1^e \ln x \, dx$

$= e - [x\ln x - x]_1^e$

$= e - ((e\ln e - e) - (0 - 1))$

$= e - (e\ln e - e + 1)$

$= e - 1$

or $\int_0^1 x \, dy = \int_0^1 e^y \, dy$

$= e^y \big|_0^1 = e - 1$

c) $A_1 + A_2 + A_3 = e$ $\quad A_1 = A_2$

$2A_1 = e - A_3$

$2A_1 = 1$

4

$A_1 = \frac{1}{2}$

QUESTION 10.

a) $V = \pi \int_0^2 \left(\frac{x}{12-x^3}\right)^2 dx$

$= \pi \int_0^2 \frac{x^2}{(12-x^3)^2} dx$

8

let $u = 12 - x^3$

$du = -3x^2 dx$

$\int u^{-2} \cdot \frac{du}{-3} = \frac{u^{-1}}{3} + c$

$\therefore \pi \left[\frac{1}{3(12-x^3)} \right]_0^2$

$= \pi \left(\frac{1}{12} - \frac{1}{36} \right)$

$= \frac{\pi}{18}$

b) Width of rectangle $= 2x$

length $= 2y$

$A = 4xy$

$x^2 + y^2 = 16$

$A = 4x\sqrt{16-x^2}$

$\frac{dA}{dx} = 4\sqrt{16-x^2} + 4x \cdot \frac{1}{2}(16-x^2)^{-1/2} \cdot -2x$

$= 4\sqrt{16-x^2} - \frac{4x^2}{\sqrt{16-x^2}} = 0$

$4(16-x^2) - 4x^2 = 0$

12

$64 = 8x^2$

$x = 2\sqrt{2}$

$\therefore$ Max Area $= 32$

AP PRELIM STATS

2020 – MARKING GUIDELINE

QUESTION 1

C . D . S

$^{26}C_2 \cdot 9 \cdot 5 \times 4!$

$26 \cdot {}^{9}C_2 \cdot 5 \times 4!$

$26 \cdot 9 \cdot {}^{5}C_2 \times 4!$ (6)

TOTAL = 519 480

QUESTION 2:

a) 1. $E(X) = 11$ $Var(X) = 4{,}95$

$Var(X) = E(X^2) - (E(X))^2$

$4{,}95 = E(X^2) - (11)^2$ (2)

$E(X^2) = 125{,}95$

2. $np = 11$

$np(1-p) = 4{,}95$

$1 - p = \frac{4{,}95}{11}$ (5)

$\frac{11}{20} = p$

$\therefore n = 20$

(6)

3. $P(12 \mid 13;14) = \frac{P(X=12)}{P(X=12)+P(X=13)}$

$= \frac{{}^{20}C_{12}\left(\frac{11}{20}\right)^{12}\cdot\left(\frac{9}{20}\right)^{8}}{{}^{20}C_{12}\left(\frac{11}{20}\right)^{12}\left(\frac{9}{20}\right)^{8} + {}^{20}C_{13}\left(\frac{11}{20}\right)^{13}\left(\frac{9}{20}\right)^{7}}$

$= \frac{117}{205}$ $(0{,}571)$

b) $n = 100$ $p = 0{,}3$

$E(X) = 30$ $Var(X) = 21$

$P(X < 35) = P\left(Z < \frac{34{,}5 - 30}{\sqrt{21}}\right)$

$= P(Z < 0{,}9819)$ (7)

$= 0{,}5 + 0{,}3365$

$= 0{,}8365$

QUESTION 3:

a) $P = 0{,}78 \pm 2{,}33\sqrt{\frac{0{,}78 \cdot 0{,}22}{n}}$

$2{,}33\sqrt{\frac{0{,}78 \times 0{,}22}{n}} = 0{,}05$ (6)

$n = 372{,}6$

$\therefore n = 373$

b) 1. $\mu = 105{,}84 \pm 2{,}58 \cdot \frac{14{,}27}{\sqrt{31}}$

$= [99{,}23 ; 112{,}45]$ (6)

$\therefore$ margin of error $= 6.61$

2. We have a voluntary response sample which might be biased

- we need a simple random sample, the data must be collected correctly. (2)

QUESTION 4:

a 1. $P\left(z > \frac{3,6-\mu}{\sigma}\right) = 0,5$

$\frac{3,6-\mu}{\sigma} = 0$ ✓

$\mu = 3,6$ ✓

$P\left(z > \frac{2,8-3,6}{\sigma}\right) = 0,6554$ (5)

$\frac{2,8-3,6}{\sigma} = -0,4$ ✓✓

$\sigma = 2$ ✓

2. $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$ ✓

$= {}^4C_2(0,6554)^2(0,3446)^2 + {}^4C_3(0,6554)^3(0,3446)^1$

$+ {}^4C_4(0,6554)^4(0,3446)^0$ ✓ (5)

$= 0,8786$ ✓

b. $\frac{{}^7C_3\,{}^8C_5 + {}^7C_5\,{}^8C_5}{{}^{17}C_{10}}$ ✓ (6)

$= \frac{392}{2431}$ $(0,1613)$ ✓

QUESTION 5

a) $K\int_{-a}^{a}(a^2-x^2)\,dx = 1$ ✓

$K\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a} = 1$ ✓

$K\left(a^3 - \frac{a^3}{3} - \left(-a^3 + \frac{a^3}{3}\right)\right) = 1$

$K\left(2a^3 - \frac{2a^3}{3}\right) = 1$ (5)

$K = \frac{3}{4a^3}$ ✓

b) $E(X) = 0$ ✓✓ (2)

c) $y = 4X + 5$

$E(y) = E(4X+5)$

$= 4E(X) + 5$ ✓✓ (3)

$= 4 \cdot 0 + 5$

$= 5$ ✓

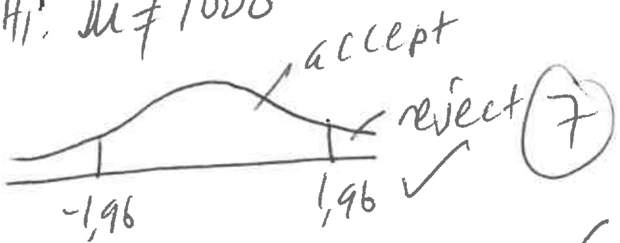
QUESTION 6:

a) Unbiased estimate:

$\bar{x} = \frac{38100}{40} = 952,50$ (2)

b. $H_0: \mu = 1000$

$H_1: \mu \neq 1000$



(7)

$$z = \frac{952,50 - 1000}{\frac{\sqrt{18764,10}}{\sqrt{40}}} = -2,1931$$

There is sufficient evidence at 5% level of significance that the mean amount of loans borrowed by its clients differs from 1000.

c) The meaning of at the 5% significance level is that there is a probability of 0,05 of rejecting the claim that the mean amount of loans borrowed is 1000 given that it is true. (2)

Or. there is a probability of 0,05 that it was wrongly concluded that the mean amount of the loans differs from 1000.

d) $z = \frac{k - 1000}{250/\sqrt{40}}$

$k = 1000 \pm 1,96 \frac{250}{\sqrt{40}}$

$(922,524 \,;\, 1077,4758)$

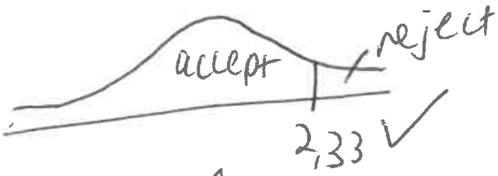
$(922,53 \,;\, 1077,47)$ (5)

QUESTION 7

$L = x \quad U = y$ (10)

$H_0: \mu_x = \mu_y$

$H_1: \mu_x > \mu_y$



$$z = \frac{99,0 - 80,5}{\sqrt{\frac{25^2}{15} + \frac{10^2}{10}}} = 2,57$$

No enough evidence to accept H_0, concluding that the true mean of alkalinity of water in the lower reaches of the river is greater than that in the upper reaches.

QUESTION 8:

a) $P(A/B) = p \qquad P(B) = p$

$$\frac{P(A \cap B)}{P(B)} = p.$$

$$P(A \cap B) = p^2$$

5

$$0{,}8 = 3p - 1 + p - p^2$$

$$p^2 - 4p + 1{,}8 = 0$$

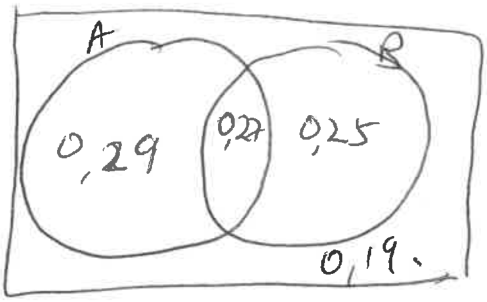
$$\underline{p = 0{,}52} \qquad p \neq 3{,}48$$

b) $P(A' \cup B') = 1 - P(A \cap B)$

$$= 1 - 0{,}27$$

$$= \underline{0{,}73}$$

3



OR: $P(A' \cup B') = P(A)' + P(B') - P(A' \cap B')$

$$= 0{,}44 + 0{,}48 - 0{,}19$$

$$= \underline{0{,}73}$$