



PRELIMINARY EXAMINATION 2020

GRADE 12 - ADVANCED PROGRAMME MATHEMATICS

Time: 1 hour

Total: 100

Examiner: P R Mhuka

Moderators: N Elefetheriades
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PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 4 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. **Answers must be rounded off to four decimal places.**
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1:

A password for Katy's computer consists of 4 characters in a particular order. The characters are chosen from the following.

- The 26 capital letters A to Z
- The 9 digits 1 to 9
- The 5 symbols #, ~, *, ?, !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Katy can make. **[6]**

QUESTION 2:

a) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4,95.

1) Calculate $E(X^2)$. (2)

2) Find the number of plants per box. (5)

3) If $n = 20$, find the probability that a box contains 12 plants which produce yellow flowers given that between 11 and 14 plants produce yellow flowers. (6)

b) Another garden shop sells polyanthus plants in boxes of 100. The shop's advertisement states that the probability of any polyanthus plant producing a pink flower is 0,3. Use a suitable approximation to find the probability that a box contains fewer than 35 plants which produce pink flowers. (7)

[20]

QUESTION 3:

a) Suppose you wish to determine the proportion of varsity students at a University in Johannesburg who were tested for covid-19 virus. You want to be 98% confident of your results and have a maximum error of 5%. Calculate the minimum sample size needed to meet these requirements given that the national health office at the institution estimates the percentage to be 78%. (6)

b) We have IQ test scores of 31 grade 12 girls in Gauteng school district. We have calculated that sample mean is 105,84 and the standard deviation is 14,27.

1) Give a 99% confidence interval for the average score in the population. What is the margin of error? (6)

2) In fact, these are the scores of 31 girls who volunteered to share their results with the researchers. Explain carefully why we cannot trust the confidence interval above. (2)

[14]

QUESTION 4:

a) In a normally distribution with mean μ and standard deviation σ . Given that $P(X > 3,6) = 0,5$ and $P(X > 2,8) = 0,6554$.

1) Find the value of μ and σ . (5)

2) If four observations are taken at random from this distribution, find the probability that at least two observations are greater than 2,8 (5)

b) A team of 5 men and 5 women is to be picked from 8 men and 9 women such that two of the 9 women, Yolanda and Sofia, must both be selected or not at all. Calculate the probability in which this can be done? (6)

[16]

QUESTION 5:

The continuous random variable X has probability density function defined by:

$$f(x) = \begin{cases} k(a^2 - x^2) & -a \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

a) Find k in terms of a . (5)

b) State the value of $E(X)$. (2)

c) Variable Y is related to X by the equation $Y = 4X + 5$, find $E(Y)$ (3)

[10]

QUESTION 6:

An accountant wishes to investigate the figures provided by credit card companies for the amount of loans borrowed by each client, Rx . He carries out an online survey from clients in a certain company. The responses from a random sample of 40 clients are summarized by: $\sum x = 38100$, unbiased estimate of standard deviation, $s^2 = 18764.10$

a) Calculate unbiased estimate of the population mean. (2)

b) The company claims its clients will borrow R1 000 on average. Test, at the 5% level of significance, whether the mean amount of loans borrowed by the clients differs from R1 000. (7)

- c) Explain, in the context of the question, the meaning of 'at the 5% level of significance' (2)
- d) The company revised their loan policy and the new population standard deviation is known to be 250. A new random sample of 40 clients is taken and the mean amount of loans borrowed for this sample is Rk . A test at the 5% significance level indicates that the null hypothesis would not be rejected for this revised loan policy. Find the range of values of k . (5)

[16]

QUESTION 7:

The alkalinity, in milligrams per litre, of water in the upper reaches of rivers in a particular region is known to be normally distributed with standard deviation of 10mg/l. Alkalinity readings in the lower reaches of rivers in the same region are also known to be normally distributed, but with a standard deviation of 25mg/l. Ten alkalinity readings made in the upper reaches of a river in the region and fifteen in the lower reaches of the same river with the following results.

Upper reaches	91	75	91	88	94	63	86	77	71	69				
Lower reaches	86	95	135	121	68	64	113	108	79	62	143	108	121	85
	97													

Investigate, at the 1% level of significance, the claim that the true mean alkalinity of water in the lower reaches of this river is greater than that in the upper reaches. [10]

QUESTION 8:

Events A and B are such that $P(A) = 3p - 1$ and $P(B) = P(A/B) = p$.

- a) Given that $P(A \cup B) = 0,8$, find a quadratic equation satisfied by p and find the value(s) of p , correct to 2 decimal places. (5)
- b) Find $P(A' \cup B')$. (3)

[8]