SAHETI SCHOOL

PRELIMINARY EXAMINATION 2020

GRADE 12 - ADVANCED PROGRAMME MATHEMATICS

Time: 2 hours **Total: 200**

Examiner: P R Mhuka

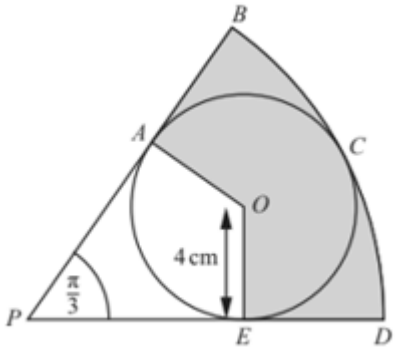
Moderators: N Elefetheriades
E Zachariou
M Torrani

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 6 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. **Answers must be rounded off to two decimal places in SECTION A**
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1:

The diagram shows a circle, centre O, radius $4\ cm$, enclosed within a sector $PBCDP$ of a circle, centre P. The circle centre O touches the sector at points A, C and E. Angle BPD is $\frac{\pi}{3}$ radians.



a) Calculate the exact value of PA and PB (4)

b) Calculate the area of this shaded region. (5)

c) The perimeter of the shaded region. (4)

[13]

QUESTION 2:

Prove that for all positive integers of *n* that:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=\frac{2^n-1}{2^n}$$ **[12]**

QUESTION 3:

a) Given that $\frac{3w+7}{5}=\frac{p-4i}{3-i}$ where p is a real constant.

1) Express w in the form $a+bi$, where a and b are real constants. Give your answer in its simplest form in terms of p (8)

2) Given that $arg\ \ w=-\frac{\pi}{2}$, calculate the value of p. (2)

b) Find the root of the equation: $2+ln\sqrt{1+x}+3ln\sqrt{1-x}=ln\sqrt{1-x^2}$ (8)

[18]

QUESTION 4:

a) Given $\lim_{x\to 2} f(x) = 5$ and $\lim_{x\to 2} g(x) = 2$, use limits laws to compute the following limits.

1) $\lim_{x\to 2}\big(f(x)g(2)\big)$ (3)

2) $\lim_{x\to 2}\big(2f(x) - g(x)\big)$ (3)

b) Given $g(x) = \begin{cases} -x^2 - x + 3, & x \le 0 \\ |x-3|, & x > 0 \end{cases}$

Prove that g is differentiable at $x = 0$ (9)

[15]

QUESTION 5:

The function has equation $f(x) = 1 + 2ln(4 - x)$

a) Sketch $f(x)$, labelling the equations of any asymptotes and the points where the graph crosses the axes. (5)

b) Sketch $f(|x|)$ (3)

c) Calculate the equation of $f^{-\prime}(x)$ (4)

d) Sketch the inverse function $f^{-\prime}(x)$. (4)

[16]

QUESTION 6:

a) Find $\frac{dy}{dx}$ for $y = ln\big(x\sqrt{1-x^2}.cosx\big)$ (8)

b) The curve has equation $y = ln\left(\frac{x}{3-x}\right)$, for $0 < x < 3$.

1) Show that the values of x for which the gradient of the curve above is equal to the constant k satisfy the equation $kx^2 - 3kx + 3 = 0$. (6)

2) Find the exact range of values of k for which the equation $kx^2 - 3kx + 3 = 0$ has no real roots. (3)

c) Given, the functions $f(x) = e^{2x}$ and $g(x) = x^2 + 2x - 5$.
Calculate the x-coordinate of the point of intersection between the two graphs for $x \leq -2$ using Newton's method to 5 decimal places. (8)

d) Calculate $\frac{dy}{dx}$ if $x = y.sec\left(\frac{5}{y}\right)$ (9)

[34]

QUESTION 7:

Given the graph of $t(x) = \frac{2x^2+3x-1}{2x+1}$

a) Find equations of any asymptotes (6)

b) Find the coordinates of the stationary points and intercepts with the axis (8)

c) Sketch the graph of $f(x)$ (8)

[22]

QUESTION 8:

a) Evaluate the following integrals without the use of a calculator:

1) $\int_1^{ln3} \frac{e^x - e^{3x}}{1+e^x}\, dx$ (7)

2) $\int \frac{cosx+sinx}{sin2x}\, dx$ (7)

3) $\int x^3 ln2x\, dx$ (7)

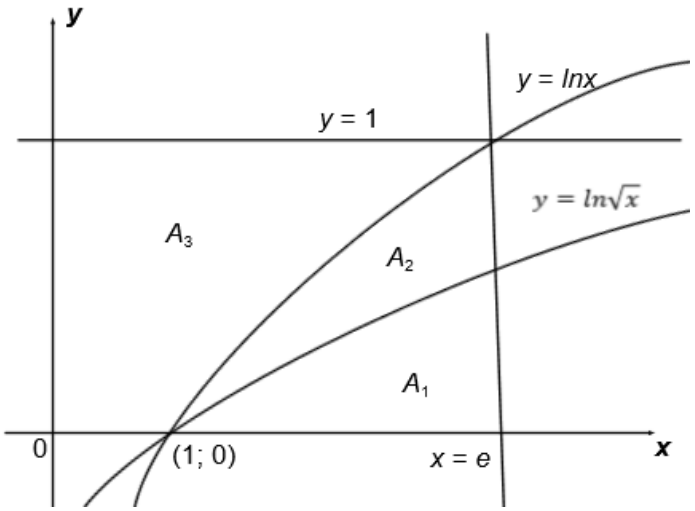
b) Find the value of $\int_1^2 \frac{6x+1}{6x^2-7x+2}.dx$, expressing your answer in the form

$mlnp + nlnq$ where m, p, n and q are integers. (11)

[32]

QUESTION 9:

The region bounded by the curve $y = lnx$, x-axis, the lines $x = 1$ and $x = e$ is divided by the curve $y = ln\sqrt{x}$ into two regions with areas A_1 and A_2. The region bounded by the curve $y = lnx$, the axes, the lines $y = 1$ and $x = e$ has area A_3.



a) Express each of A_1 and A_2 as integral. Without performing any intergration, show that $A_1 = A_2$. (6)

b) Find the exact value of A_3 (8)

c) Using your results in parts (a) and (b), find the value of A_1 (4)

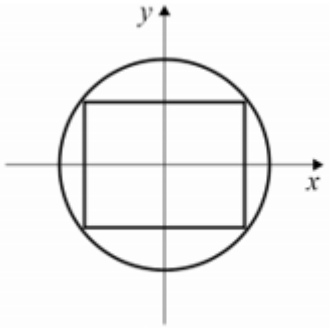
[18]

QUESTION 10:

a) The equations $y = \frac{x}{12-x^3}$, $x = 0$, $x = 2$ and $y = 0$ define the bounds of a plane region. Find the volume of the solid obtained by rotating the region about the x-axis. (8)

b) A company is designing a logo. The logo is a circle of radius 4 cm with an inscribed rectangle. The rectangle must be as large as possible.

The company models the logo on an x-y plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle. Fully justify your answer. (12)

[20]