ST BENEDICT'S

SUBJECT	**AP Mathematics**	**PAPER**	**Prelim Paper 1**
GRADE	**12**	**DATE**	7 September 2020
EXAMINER	MH Povall	**MARKS**	200
NAME		**MODERATOR**	Cluster moderated
TEACHER	**MH Povall**	**DURATION**	2 Hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Proof by Mathematical Induction	14	
2	Roots of equations and complex numbers	15	
3	Limits/continuity and absolute values	32	
4	Inverses, ln and e	18	
5	Differentiation	36	
6	Application of differentiation and Newton-Raphson	10	
7 & 8	Rational Functions	12 + 12	
9	Area under curve	6	
10	Integration	30	
11	Application of Integration	15	
TOTAL		200	

INSTRUCTIONS:

1. This paper consists of 11 questions and 10 pages.

2. Read the questions carefully.

3. Answer all questions.

4. Number your answers clearly and use the same numbering as in the question paper.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Round off your answers to two decimal digits where necessary.

7. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks. Equations may not be solved solely with a calculator.

8. It is in your interest to write legibly and present your work neatly.

9. A four-page information booklet is provided.

QUESTION 1 **14 MARKS**

Use Mathematical Induction to prove that:

$$\frac{1}{1\,.\,2}+\frac{1}{2\,.\,3}+\frac{1}{3\,.\,4}+\cdots\cdots\cdots\cdots+\frac{1}{n\,.\,(n+1)}=\frac{n}{n+1}$$

QUESTION 2 **15 MARKS**

a) 1) If it is given that $x = 1 - 2i$ is a zero of $g(x)$.

Show that $x^2 - 2x + 5$ is a factor of $g(x)$. (5)

2) Given that $g(x) = x^4 - 6x^3 + 18x^2 - 30x + 25$

According to the mathematician Gauss, a polynomial of degree n has n zeros.

Therefore $g(x)$ has 4 zeros.

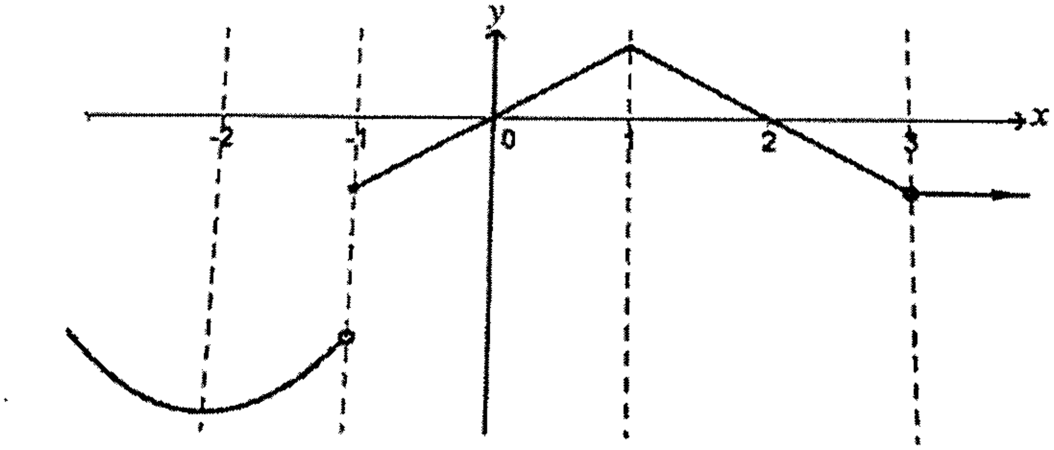
Determine the other two zeros of $g(x)$ if $x^2 - 2x + 5$ is a factor of $g(x)$. (5)

b) If it is given that $x = a + bi$ satisfies the equation $(2 + i)(x + 3i) = 8i + 6$

Find the values of a and b. (5)

QUESTION 3 **32 MARKS**

a) The function f is represented by the graph below:



Give all the values of x (no reasons required) for which:

1) the limit exists, but the function is not defined. (2)

2) the left-hand and right-hand limits exist, but they are not equal. (2)

3) f is continuous but not differentiable. (2)

4) $f'(x) = 0$ (3)

5) $f''(x) > 0$ (3)

b) $g(x) = \begin{cases} \frac{(x+3)(x+1)}{(x+3)} & if\ x < -1 \\ x^2 + 1 & if\ x \geq -1 \end{cases}$

Determine, **with algebraic motivation**, whether g is continuous at the following points and state the type of discontinuity where applicable.

1) $x = 3$

2) $x = -1$

3) $x = -3$ (10)

c) 1) Sketch the graph of $f(x) = |2x + 3| + x + 5$ (8)

2) Given that $g(x) = kx$. For which values of k will the graphs of f and g have no points of intersection. (2)

QUESTION 4 18 MARKS

a) Consider the function $f(x) = e^{x+2} - 1$

1) Find the equation of $f^{-1}(x)$. (3)

2) Sketch the graphs of f and f^{-1} on the same set of axes. (8)

b) Given that $x = \ln a$ is a solution to the equation $10e^{2x} - 7e^{x} = 26$, find, without using a calculator, the value(s) of a. (7)

QUESTION 5 **36 MARKS**

a) Find all the values of a for which the $\lim_{x \to a} \frac{x^3+1}{x^3-x}$ does not exist. (3)

b) 1) Find $f'(x)$ by first principles if $f(x) = \sqrt{1-2x}$. (6)

2) Give the domain of f'. (2)

c) The functions f, g and h are defined as follows:

$$f(x) = e^{3x+\sqrt[4]{x}}$$

$$g(x) = 2x.sec3x$$

$$h(x) = \sin^2(\tan(2x))$$

Determine $f'(x)$, $g'(x)$ and $h'(x)$. (10)

d) 1) Find an expression for $\frac{dy}{dx}$ in terms of x and y if $x^2 - 4xy + 4y + 8 = 0$. (9)

2) Hence, find the *x*-coordinates of the stationary points on this curve. (6)

QUESTION 6 **10 MARKS**

Two heat sources H_1 and H_2 are 10m apart and a point P lies on the line joining them, at a distance x metres from H_1. The temperature T°C at P is given by $T = \frac{8}{x^2} + \frac{1}{(10-x)^2}$

a) Set up an equation so that the temperature at P will be a minimum. (4)

b) Solve the equation found in a) above using the Newton-Raphson method.
Show all the steps that you have followed. (5)

c) Explain where point P is in relation to H_1. (1)

QUESTION 7 **12 MARKS**

Match the following rational functions to the appropriate graphs A – F below and give a reason for your selection:

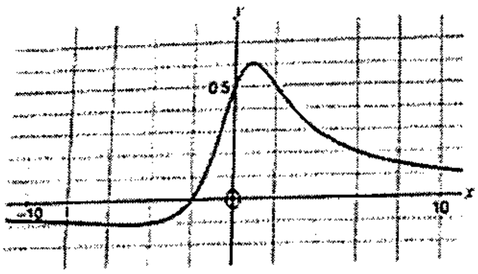
1. $f(x) = \frac{x^2-4}{x+2}$

2. $f(x) = \frac{x-2}{x^2-x-6}$

3. $f(x) = \frac{x^2+x-6}{x^2-6x+9}$

4. $f(x) = \frac{x^2-4x+4}{x-3}$

A



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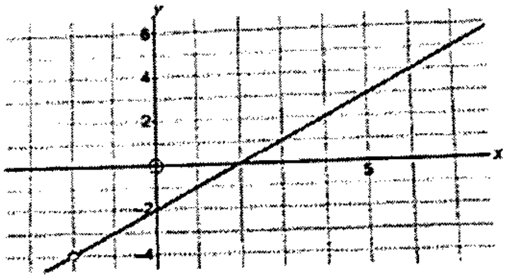

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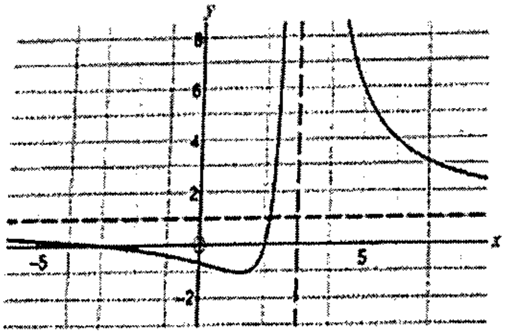

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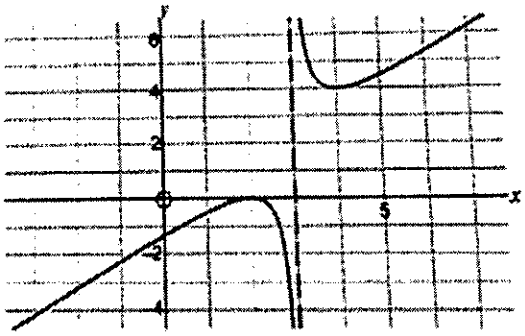

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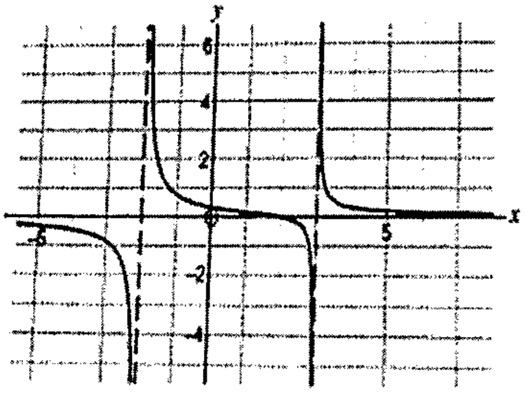

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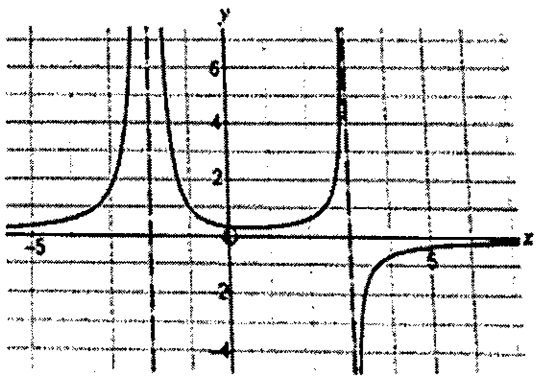


QUESTION 8 **12 MARKS**

Consider $f(x) = \frac{6x^2-x-1}{px-2}$

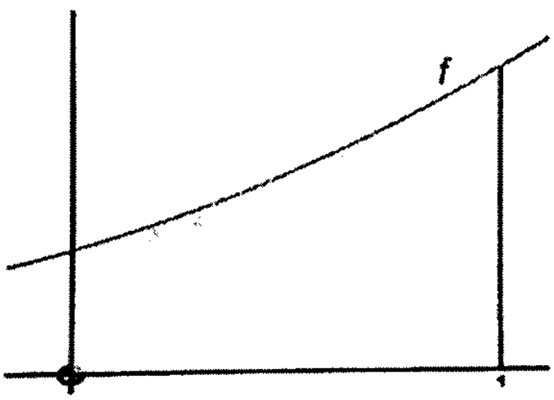
a) For which value(s) of p will $y = 2x + 1$ be an asymptote of the graph of f. (3)

b) Sketch the graph of f if $p = 4$. (4)

c) Determine f' when $p = 3$ and show that f has two stationary points. (5)

QUESTION 9 **6 MARKS**

Serema is attempting a Riemann sum to determine the area bounded by the curve f , the $x-$axis, the $y-$axis and the line $x = 1$ as shown below:



He has correctly determined that, if he uses n rectangles, then the area will be:

$$Area = \frac{10}{3} + \frac{3}{2n} + \frac{1}{6n^2}$$

a) Determine the area when 4 rectangles are used. (2)

b) Will this be an under-approximation or an over-approximation?
Explain your answer. (2)

c) Determine the exact area. (2)

QUESTION 10 **30 MARKS**

a) Determine the following integrals:

1) $\int (2x^3 - sec^2\left(\frac{x}{2}\right) - \frac{1}{\sqrt[3]{x}})\, dx.$ (3)

2) $\int \frac{e^x}{1+e^x}\, dx$ (3)

3) $\int cos^2 3x.\, sin3x\, dx$ (4)

4) $\int cos4\theta.\, sin5\theta\, d\theta$ (5)

5) $\int y\sqrt{y+3}\, dy$ (5)

b) 1) Decompose $\frac{3x+5}{(x+1)(x+2)(x+3)}$ into partial fractions. (6)

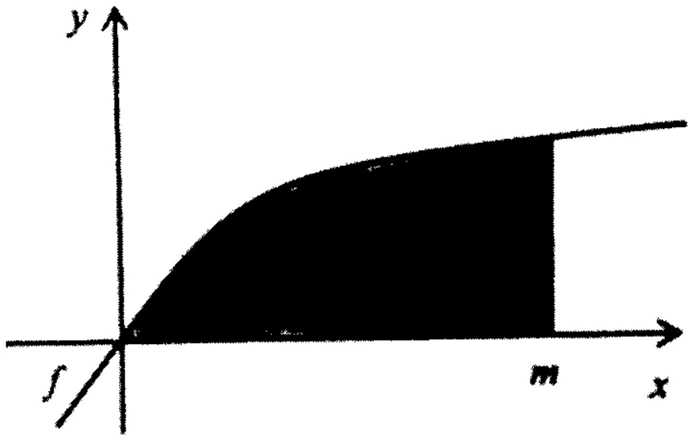
2) Hence, without a calculator, evaluate:

$$\int_1^2 \frac{3x+5}{(x+1)(x+2)(x+3)}\, dx$$

expressing your answer in the form $\ln\frac{a}{b}$ where a and b are integers. (4)

QUESTION 11 **15 MARKS**

The sketch shows the graph of $f(x) = \frac{x}{\sqrt{2x^2+1}}$ which cuts the axes at the origin.



The shaded region is the area between the graph, the $x-$axis and the line $x = m$.

a) Determine the shaded area in terms of m. (9)

b) Calculate the area if $m = 2$. (2)

c) Set up, but do not integrate or calculate, an expression in terms of m that represents the volume of the solid that will be formed if the shaded region is rotated around the $x-$axis. (4)