Question 1

a)

| X | -1 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X=x) | k | k | 4k | 9k |

$P(X=x) = kx^2$ ✓

✓

$15k = 1$ ✓

$\therefore k = \frac{1}{15}$ ✓

(4)

b) $E(X) = \sum x\,P(X=x)$

$= -1 \times \frac{1}{15} + 1 \times \frac{1}{15} + 2 \times \frac{4}{15} + 3 \times \frac{9}{15}$ ✓

$= \frac{35}{15}$

$= \frac{7}{3}$

$= 2\frac{1}{3}$ ✓

(2)

Mean

$Var(x) = \sum p(x) \times (x-\mu)^2$

$= \frac{1}{15} \times (-1-2\frac{1}{3})^2 + \frac{1}{15}(1-2\frac{1}{3})^2 + \frac{4}{15}(2-2\frac{1}{3})^2 + \frac{9}{15}(3-2\frac{1}{3})^2$ ✓

$= \frac{52}{45}$ ✓ OR 1,1556

(2)

OR

$Var(x) = E(x^2) - [E(x)]^2$

$= 1^2 \times \frac{1}{15} + 1^2 \times \frac{1}{15} + 2^2 \times \frac{4}{15} + 3^2 \times \frac{9}{15} - \left(\frac{7}{3}\right)^2$ ✓

$= \frac{52}{45}$ OR ✓

(2)

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## Question 2

a) $\dfrac{\binom{12}{1}\binom{19}{6}}{\binom{31}{7}}$ ✓ ✓ ✓ ✓ (4)

$= 0,1238$

b) B B B ; B′B′B ; B′B B ; BB′B

$\frac{12}{31} \times \frac{11}{30} \times \frac{10}{29} + \frac{19}{31} \times \frac{18}{30} \times \frac{12}{29} + \frac{19}{31} \times \frac{12}{30} \times \frac{11}{29} + \frac{12}{31} \times \frac{19}{30} \times \frac{11}{29}$ ✓ ✓ ✓ ✓

$= \dfrac{1320 + 4104 + 2508 + 2508}{26970}$ (6)

$= \dfrac{10440}{26970}$

$= \frac{12}{31}$ ✓✓ or $0,3871$

(10)

## Question 3

a) i) $\binom{5}{2}(0.1)^2(0.9)^3 = 0,0729$ ✓ ✓ ✓ ✓ (4)

ii) $P(X = x) = \begin{cases} \binom{5}{x}(0.1)^x(0.9)^{5-x} & x \leq 5;\ x \in \mathbb{N}_0 \\ 0 & \text{otherwise} \end{cases}$ ✓✓ ✓ ✓

(4)

b) Check if binomial approximation can be used:

$np = 160 \times 0,1$ ✓
$= 16 > 5$ ✓

$n(1-p)$
$= 160(1-0,1)$ ✓
$= 144 > 5$ ✓

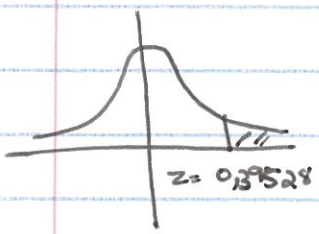
$\therefore$ binomial approximation can be used.

$\mu = np$
$= 160 \times 0,1$
$= 16$ ✓

$\sigma^2 = np(1-p)$
$= 16(0,9)$
$= \frac{75}{5}$
$= 14.4$ ✓
$\therefore \sigma = \sqrt{14.4}$

$P(X > 17) \longrightarrow P(X > 17.5)$ ✓✓

$z = \frac{x - \mu}{\sigma}$
$= \frac{17.5 - 16}{\sqrt{14.4}}$
$= 0,39528$




$= P(Z > 0,39528)$ ✓
$= 0,5 - 0,1554$ ✓
$= 0,3446$ ✓

(10)

18.

## Question 4

a) $\int_{30}^{60} a(x-30)^2 \, dx = 1$ ✓ ✓ ✓

$a \int_{30}^{60} (x-30)^2 \, dx = 1$

$\left[\frac{1}{3}(x-30)^3\right]_{30}^{60} = \frac{1}{a}$ ✓ ✓

$\frac{1}{3}(30)^3 - \frac{1}{3}(0)^3 = \frac{1}{a}$ ✓ (7)

$\frac{1}{a} = 9000$

$a = \frac{1}{9000}$ ✓

b) $\int_{30}^{m} \frac{1}{9000}(x-30)^2 \, dx = 0,5$ ✓ ✓ ✓

$\frac{1}{9000} \int_{30}^{m} (x-30)^2 \, dx = 0,5$

$\left[(x-30)^3\right]_{30}^{m} = 13500$ ✓ ✓

$(m-30)^3 - 0 = 13500$

$(m-30)^3 = 13500$ ✓ (7)

$m - 30 = 23,8110...$

$m = 53,8110$ ✓

(14)

## Question 5

a) $\frac{52}{160} = \frac{13}{40} = 0,325$ (2)

b) $p(\text{student is a boy}) = \frac{96}{160}$

$p(\text{student studies music}) = \frac{52}{160}$

$p(\text{student is a boy}) \times p(\text{student studies music}) = \frac{96}{160} \times \frac{52}{160}$

$\approx 0,195$

$p(\text{student is a boy and studies music}) = \frac{40}{160} = 0,25$

$\therefore$ Not equal

$\therefore$ Not independent. (5)

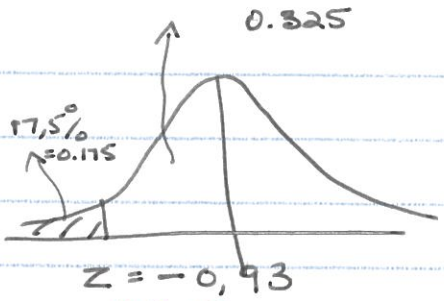
c) $p(D'/G)$

$= \frac{27}{64} = 0,4219$ (3)

10

## Question 6

a) $P(x < 7,7) = 0.175$ ✓




$z = \frac{x - \mu}{\sigma}$ ∴

$-0,93 = \frac{7,7 - \mu}{0.684}$ ✓✓

$-0,63612 = 7,7 - \mu$ (6)

$\mu = 8,34$ ✓

b) $P(x < 8,2)$ ✓✓

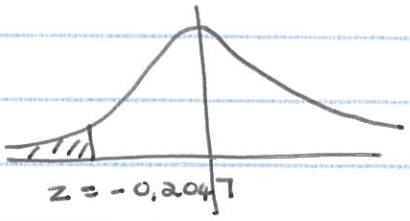
$= P(z < -0,2047)$

$= 0,5 - 0,0987$

$= 0,4013$ ✓

$z = \frac{8,2 - 8,34}{0,684}$

$= -0,2047$




∴ $\binom{5}{3}(0.4013)^3(0.5987)^2$ ✓✓

$= 0.2316$ ✓ (6)

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## Question 7

a) $\bar{x} = \frac{59+63}{2} = 61.$ ✓✓ (2)

b)

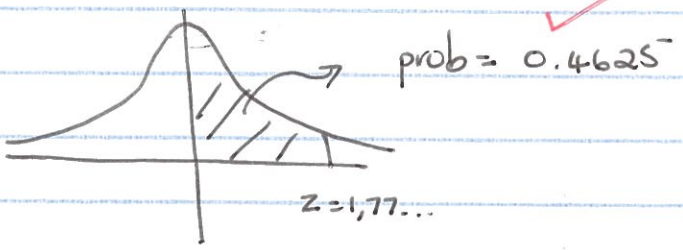
$$\mu = \bar{x} \pm z\frac{\sigma}{\sqrt{n}}.$$

$$\mu - \bar{x} = z\frac{\sigma}{\sqrt{n}}$$

$$63 - 61 = z \cdot \frac{9}{\sqrt{64}}$$ ✓✓

$$\frac{2 \times 8}{9} = z$$ ✓

$$z = 1,77\ldots$$ ✓



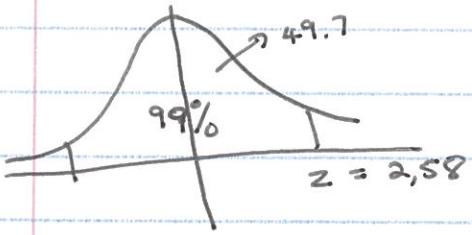

∴ level of confidence = $2 \times 46,25$ ✓✓

$= 92.5\%$ ✓ (8)

10

## Question 8

a) $\hat{p} = \frac{22}{600} = \frac{22}{600} = \frac{11}{300}$ ✓




$$p = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$ ✓✓

$$= \frac{11}{300} \pm 2.58\sqrt{\frac{\frac{11}{300} \times \frac{289}{300}}{600}}$$

$$0.0169 < p < 0.0565$$ ✓ ✓

OR $(0,0169 ; 0,0565)$

(5)

b) $$z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq 0.04.$$

$$2.58\sqrt{\frac{\frac{11}{300} \times \frac{289}{300}}{n}} < 0.04$$ ✓✓

$$\sqrt{\frac{\frac{3179}{9000}}{n}} < \frac{2}{129}$$ ✓

$$\frac{3179}{9000} < \frac{4}{16641} \times n$$ ✓

(5)

$$146,94 < n$$ ✓

∴ need to throw die 147 times

(10)

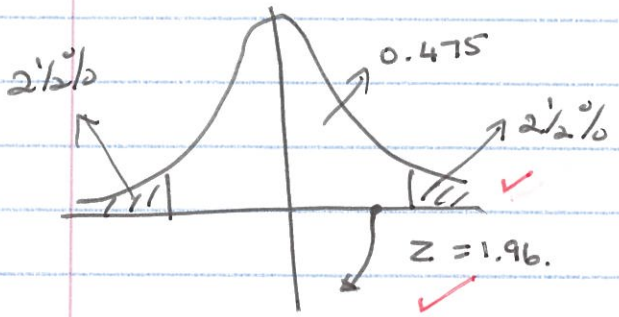
## Question 9

a) $H_0$: $\mu_x = \mu_y$ ✓

$H_1$: $\mu_x \neq \mu_y$ ✓

$\mu_x$ – everbright pop mean

$\mu_y$ – everstrong pop mean.




$$Z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$= \frac{58.5 - 53.5}{\sqrt{\frac{20^2}{100} + \frac{19^2}{100}}}$$ ✓

$$= 1.8124\ldots$$

∴ insufficient evidence to support claim at a 5% significance level that there is a difference bet two types of batteries. ✓✓ (7)

b) $Z = 1.8124$

prob $= 0.5 - 0.4649$ ✓

$= 0.0351$ ✓

∴ significance level will need to be $> 0.0351 \times 2 \times 100$

$\geq 7.02\%$ ✓

(3)

10