QUESTION 1

1.1 Solve for $x \in R$ without the use of a calculator and showing all necessary working details:

(a) $|x^2 - 7| = 2x + 1$ (8)

(b) $e^x - 6e^{-x} - 5 = 0$ (6)

1.2 Given: $f(x) = e^{x+3} - 2$

(a) Determine the equation of $f^{-1}(x)$ and state its domain and range. (7)

(b) Hence sketch the graph of $f^{-1}(x)$ on the Answer Sheet, clearly showing all intercepts with the axes and any asymptotes. If necessary round any answers off to one decimal place. (6)

1.3 If it is given that $x = 2 + i$ is a root of the equation $x^3 + px + q = 0$, where p and q are real numbers, determine the values of p and q. (6)

[33]

QUESTION 2

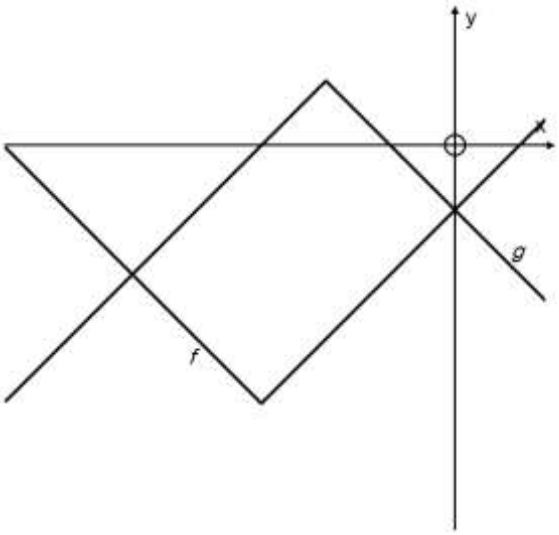
Use mathematical induction to prove that

$$\sum_{p=1}^{n} p \ln x = \frac{n}{2} \ln x^{n+1}$$

for all $n \in N$. **[13]**

QUESTION 3

Consider the functions $f(x) = |x + p| + q$ and $g(x) = -|x + 2| + 1$. $f(x) = g(x)$ for $x \in \{-5; 0\}$



3.1 Determine the values of p and q. (8)

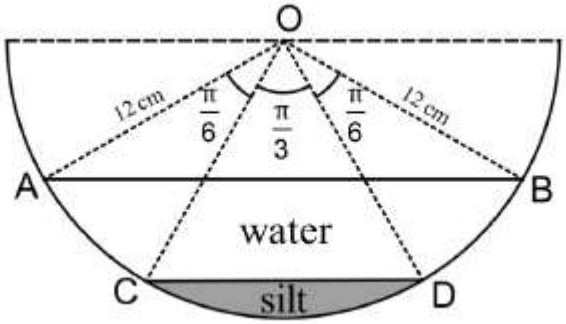
3.2 Hence, solve for x in $|x + 2| + |x + 3| \leq 5$ (6)

[14]

QUESTION 4

Consider the diagram below. It represents the cross-section of a semi-circular gutter with O the centre of the semi-circle. There is silt at the bottom of the gutter. The surface area of the silt, CD, is parallel to the surface of the water, AB. Important angles are shown in the diagram. If the radius of the gutter is 12 cm and the gutter is 2 m long, then calculate the volume of water I the gutter, to the nearest litre.

Remember: $1\ cm^3 = 1\ ml\ \ and\ 1\ litre = 1000\ ml$



[10]

QUESTION 5

Consider the function $f(x) = \begin{cases} (x+2)^2 & if \quad x \leq -1 \\ 3e^x + 1 & if \quad -1 < x < 1 \\ 3e + 1 & if \quad x \geq 1 \end{cases}$

5.1 Sketch the graph of f on the Answer Sheet. (8)

5.2 Is $f(x)$ continuous at $x = -1$? Justify your answer algebraically.

If discontinuous, state the type of discontinuity. (6)

5.3 Is $f(x)$ differentiable at $x = 1$? Justify your answer algebraically. (6)

[20]

QUESTION 6

6.1 Determine $f'(x)$ from first principles if $f(x) = \frac{1}{\sqrt{2-x}}$ (10)

6.2 Given: $f(x) = \sin(\tan(2x))$

$$g(x) = x^{\frac{2}{3}}\left(x + \sqrt[4]{x}\right)$$

$$h(x) = \frac{2x}{\cos x}$$

(a) Determine $f'(x),\ g'(x)$ and $h'(x)$. (You need not simplify your answers) (12)

(b) Arrange $f'(\pi), g'(1) and\ h'(\pi)$ in descending order. (4)

6.3 It is given that the curves with equations $y = 6\ln x$ and $y = -x^2 + 8x - 3$ intersect at a single point.

(a) Show that the point of intersection lies between $x = 5$ and $x = 6$. (4)

(b) Use Newton's method to determine the $x-$ coordinate of this point of intersection to 7 decimal places. (7)

[37]

QUESTION 7

Given: $f(x) = \frac{x^3+ax^2+bx+c}{x^2-1}$

7.1 If the oblique asymptote is: $y = x + 2$, write down the equations of any other asymptotes. (2)

7.2 If the oblique asymptote cuts the curve at $x = -\frac{1}{2}$, determine the values of a, b and c where $a; b; c \in Z$. (7)

7.3 Determine the $x-$ coordinate of one of the stationary points that lies close to $x = -2$. (You need NOT use Newton's method to solve the equation) Give your answer correct to two decimal places. (8)

7.4 Determine the nature of this stationary point. (4)

[21]

QUESTION 8

8.1 Given: $y = \frac{3x+5}{x^3+5x^2+7x+3}$

(a) Decompose into partial fractions. (13)

(b) Hence, determine $\int \frac{3x+5}{x^3+5x^2+7x+3}\,dx$ (6)

8.2 Determine the following integrals:

(a) $\int 4x(x^2+2)^5 dx$ (6)

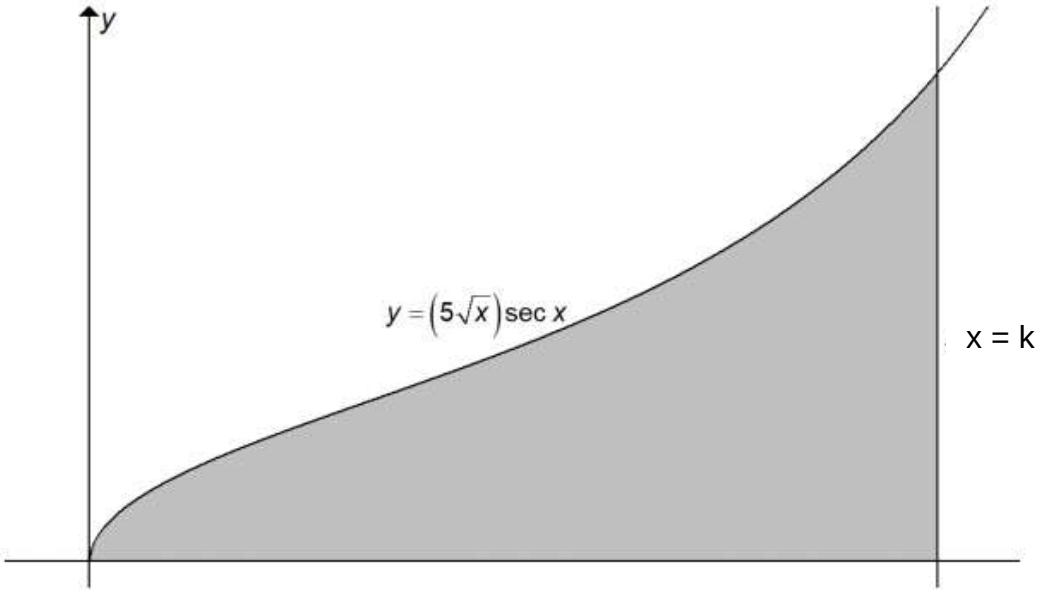
(b) $\int \frac{1}{1+\cot^2 x} dx$ (8)

[33]

QUESTION 9

9.1 Use integration by parts to determine $\int x\ sec^2 x\ dx$. (10)

9.2 The region bounded by the curve $y = 5\sqrt{x}\sec x$, the $x-$ axis and the lines $x = 0$ and $x = k$ is shown in the diagram below.



The volume of the solid generated by rotating the area about the $x-$ axis through $360°$ is 73,97 $units^2$. Determine the value of k. Give your answer rounded to the nearest whole number. (9)

[19]

Total: 200 marks