Memo

MODULE 2 STATISTICS

QUESTION 1

1.1

(a) $P(X=x)=\begin{cases}\binom{10}{x}(0{,}4)^{x}(0{,}6)^{10-x} & ;\quad x\in\{0;1;2;\ldots\ldots\ldots 10\} \\ 0 & elsewhere\end{cases}$ ✓ ✓ ✓ ✓ ✓ (5)

(b) $P(X>3)=1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)]$

$=1-\binom{10}{0}(0{,}4)^{0}(0{,}6)^{10}-\binom{10}{1}(0{,}4)^{1}(0{,}6)^{9}-\binom{10}{2}(0{,}4)^{2}(0{,}6)^{8}-\binom{10}{3}(0{,}4)^{3}(0{,}6)^{7}$ ✓ ✓ ✓ ✓ ✓

$=0{,}618$ ✓ (10)

1.2

(a) $np=50\times 0{,}4=20>5$ ✓

$nq=50\times 0{,}6=30>5$ ✓ (2)

(b) $\mu=20$

$\sigma=\sqrt{npq}=\sqrt{50.0{,}4.0{,}6}=2\sqrt{3}$

$P(X>15)$

$\therefore with\ error\ of\ approximation$

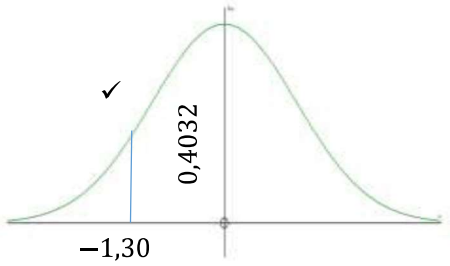
$P(X>15{,}5)$ ✓

$=P\left(Z>\frac{15{,}5-20}{2\sqrt{3}}\right)$ ✓ ✓ ✓

$=P(Z>-1{,}30)$ ✓

$=0{,}5+0{,}4032$ ✓

$=0{,}9032$ ✓



(8)

1.3 $P(X \geq 1) = 0,9$ ✓

$1 - P(X = 0) = 0,9$ ✓

$\therefore -P(X = 0) = -0,1$

$\therefore P(X = 0) = 0,1$ ✓

$\binom{n}{0}(0,4)^0(0,6)^n = 0,1$ ✓ ✓ ✓

$(0,6)^n = 0,1$ ✓

$n = \frac{log0,1}{log0,6}$ or use calculator

$n = 4,5075 \ldots.$ ✓

$\therefore n = 5$ ✓ (9)

[34]

QUESTION 2

$E[R] = 0,2$

$\therefore (-2 \times 0,3) + (0 \times b) + (a \times c) + (4 \times 0,1) = 0,2$ ✓✓

$-0,6 + ac + 0,4 = 0,2$

$\therefore ac = 0,4 \ldots\ldots\ldots\ldots eq\ 1$ ✓

$Var[R] = E[R^2] - (E[R])^2 = 3,56$

$\therefore (4 \times 0,3) + (0 \times b) + (a^2 \times c) + (16 \times 0,1) - (0,2)^2 = 3,56$ ✓✓

$1,2 + a^2c + 1,6 - 0,04 = 3,56$

$\therefore a^2c = 0,8 \ldots\ldots\ldots eq\ 2$ ✓

$eq\ 2 \div eq\ 1$: $a = 2$ ✓

$\therefore c = 0,2$ ✓

$0,3 + b + c + 0,1 = 1$ ✓

$0,3 + b + 0,2 + 0,1 = 1$ ✓

$\therefore b = 0,4$ ✓ (11)

[11]

QUESTION 3

3.1 $\bar{x} = \frac{29,74+31,86}{2}$

$= 30,8$ ✓

$\therefore 30,8 + 1,96\left(\frac{\sigma}{\sqrt{n}}\right) = 31,86$ ✓✓

$1,96\left(\frac{\sigma}{\sqrt{n}}\right) = 1,06$

$\frac{\sigma}{\sqrt{n}} = 0,54$ ✓ (4)

3.2 $\mu \in (30,8 - 1,65 \times 0,54 \; ; \; 30,8 + 1,65 \times 0,54)$ ✓✓✓✓

$\therefore \mu \in (29,909 \; ; \; 31,691)$ ✓✓ (6)

[10]

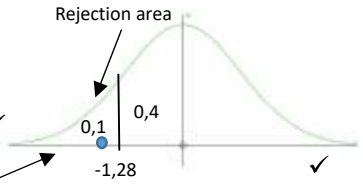
QUESTION 4

$H_0: \mu = 66,5$ ✓✓

$H_1: \mu < 66,5$ ✓✓

$z = \frac{65,5-66,5}{\frac{21,2}{\sqrt{750}}}$ ✓✓✓✓

$= -1,29$ ✓



✓

There is sufficient evidence at a 10% level of significance to reject the null hypothesis. The advertising campaign has reduced the consumption of chocolate. ✓

(11)

[11]

QUESTION 5

5.1

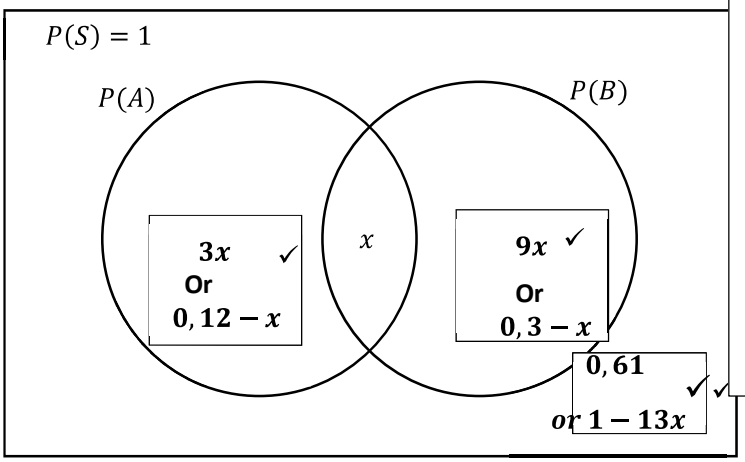
$P(B|A) = \frac{1}{4}$ $\quad$ $P(A|B) = \frac{1}{10}$

$\therefore P(B|A) = \frac{P(B \cap A)}{P(A)}$ ✓ $\quad$ $\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$ ✓

$\therefore \frac{1}{4} = \frac{P(B \cap A)}{P(A)}$ ✓ $\quad$ $\therefore \frac{1}{10} = \frac{P(A \cap B)}{P(B)}$ ✓

$\therefore P(A) = 4x$ ✓ $\quad$ $\therefore P(B) = 10x$ ✓

$\therefore P(A \cap B') = 3x$ ✓ $\quad$ $\therefore P(B \cap A') = 9x$ ✓



Or:

$P(A \cap B) = \frac{1}{4}P(A) = \frac{1}{10}P(B)$

$P(A \cup B) = P(A) + P(B) - P(AB)$

$\frac{39}{100} = \frac{4}{10}P(B) + P(B) - \frac{1}{10}P(B)$

$\therefore P(B) = 0{,}3$

$\therefore P(B \cap A') = 0{,}3 - x$

Simililarly:

$\therefore P(A) = 0{,}12$

$\therefore P(A \cap B') = 0{,}12 - x$

(12)

5.2 $3x + x + 9x + 0{,}61 = 1$ ✓✓

$13x = 0{,}39$ ✓

$x = 0{,}03 \quad or \quad \frac{3}{100}$ ✓ CA marking (4)

[16]

QUESTION 6

6.1 (a) $7! = 5040$ ✓✓ (2)

(b) $n(no\ black\ jersey\ at\ top\ and\ bottom) = 5 \times 5! \times 4 = 2400$ ✓✓

$n(no\ black\ jersey\ at\ top) = 5 \times 5! \times 2 = 1200$ ✓✓

$n(no\ black\ jersey\ at\ bottom) = 5 \times 5! \times 2 = 1200$ ✓✓

$\therefore total\ n = 4800$ ✓✓ (8)

OR: $n(Total) - n(B\ on\ top\ and\ bottom)$

$= 7! - 2.5!$

$= 4800$

6.2 $P = P(0\ marsh\ eggs) or\ P(1\ marsh\ egg) or\ P(2\ marsh\ eggs)$

$$= \frac{\overset{\checkmark\checkmark}{\binom{20}{0}\binom{27}{3}} + \overset{\checkmark\checkmark}{\binom{20}{1}\binom{27}{2}} + \overset{\checkmark\checkmark}{\binom{20}{2}\binom{27}{1}}}{\binom{47}{3}_{\checkmark\checkmark}}$$

$$= \frac{1005}{1081}$$

$$= 0{,}93 \ \checkmark$$ (8)

[18]

OR: $\boldsymbol{P = 1 - P(all\ marshmellows)}$

$$= \mathbf{1} - \frac{\binom{20}{3}\binom{27}{0}}{\binom{47}{3}}$$

$$= \mathbf{0{,}93}$$

OR: $\boldsymbol{P = 1 - \frac{20}{47} \times \frac{19}{46} \times \frac{18}{45}}$ **(tree diagram)**

$$= \mathbf{0{,}93}$$

GRAND TOTAL: 100 marks