Brescia House School

SERVIAM

AUGUST 2021

ADVANCED PROGRAMME MATHEMATICS: PAPER I

MODULE 1: CALCULUS AND ALGEBRA

Time: 2 hours 200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages and an Information Booklet of 2 pages (i–ii). Please check that your question paper is complete.
2. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
3. All necessary calculations must be clearly shown, and writing must be legible.
4. Diagrams have not been drawn to scale.
5. Round off your answers to 2 decimal digits, unless otherwise indicated.

EXAMINATION NUMBER:

Question Number	1	2	3	4	5	6	7	8	9	Total
Mark Achievable	43	24	7	11	28	10	10	28	39	200
Mark Attained										

Examiner : Mrs K Raeburn

Moderator : Mr B Dannatt **External Moderator : Mrs C Kennedy**

QUESTION ONE

1.1 Solve for $x \epsilon \mathbb{R}$ in each of the following. Leave answers in terms of ln or e if necessary.

(a) $50 = e^{50x} - 1$ (4)

(b) $2e^{x} - 1 = e^{-x}$ (6)

(c) $x(3|x| - 1) = -10$ (8)

1.2 Given: $f(x) = e^{2x} - 9$ and $g(x) = \ln(x - 1)$ $for\ x > 1$

Solve for x if $f\left(g(x)\right) = 0$ (9)

1.3 The temperature T (in ℃) of a cooling cup of tea, after a time t (in minutes), can be modelled by the equation:

$T = 20 + Ae^{-kt}$, where A and k are constants.

(a) Write down the room temperature. (2)

(b) Given that the initial temperature is 85 ℃ and that the temperature is decreasing at the rate of 5 ℃ per minute, initially, determine the value of k. (9)

(c) Determine the length of time, to the nearest minute it takes for the tea to cool to 50 ℃. (5)

[43]

QUESTION TWO

2.1 It is given that $f\left(g(x)\right) = \frac{1}{x-1} + x^2 - 2x + 1$ and $g\left(g(x)\right) = x - 2$.

Determine $g(f(2))$. (6)

2.2 The following function is given:

$$p(x) = \begin{cases} 2x + 1 & if \quad x \leq q \\ x^2 - 4x + 10 & if \quad x > q \end{cases}$$

(a) For what value(s) of q is $p(x)$ continuous at $x = q$? (6)

(b) Is p differentiable at all points? Motivate your answer. (6)

2.3 Given : $f(x) = \frac{\sqrt{2x-1} - \sqrt{x}}{x-1}$

Determine : $\lim_{x \to 1} f(x)$ (6)

[24]

QUESTION THREE

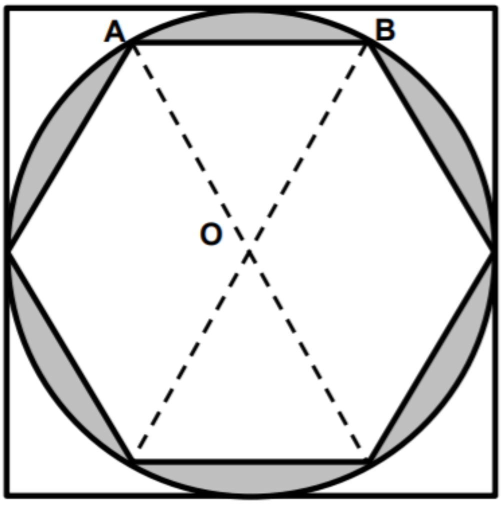
In using the induction method to prove the accuracy of a statement, you assume the statement is true for $n = k$.

Assuming $2^k + 2^{k+1} + 2^{k+2}$ is divisible by 7, prove that it will be true for the next natural number $n = k + 1$.

[7]

QUESTION FOUR

In the diagram a square tile is shown, in which a regular hexagon is inscribed in a circle with centre O. The circle fits exactly inside the square. The area of the shaded region is 54 cm^2.



4.1 Write down the size of $A\hat{O}B$ in radians, leaving the answer in terms of π if necessary. (2)

4.2 Determine the area of the square tile, correct to 2 decimal places. (9)

[11]

QUESTION FIVE

5.1 Determine the derivatives of the following

(a) $y = (3x^4 - 10x)^{15}.\sqrt{4x^4 + 64}$ (No need to simplify) (5)

(b) $f(x) = \sin^2(3x - 4)$ (simplify to one trigonometric ratio) (5)

(c) $y = \tan(3x - 4)^2$ (5)

5.2 Determine the gradient of the curve $x^2 - 3x^2y + y^3 = -1$ at (2;3) (8)

5.3 State the equation of the tangent to $f(x) = e^{2x} - 3x$ at the y intercept. (5)

[28]

QUESTION SIX

Given : $y = x + 1$ and $y = 2tanx$

6.1 Create a function $f(x)$ that can be used to determine the x value of the point of intersection of the two graphs. (2)

6.2 Show that $f(x) = 0$ has a solution in the interval [0 ; 1]. (3)

6.3 Calculate this x value using Newton's interpolation method. Round off your answer to 4 decimal places. (5)

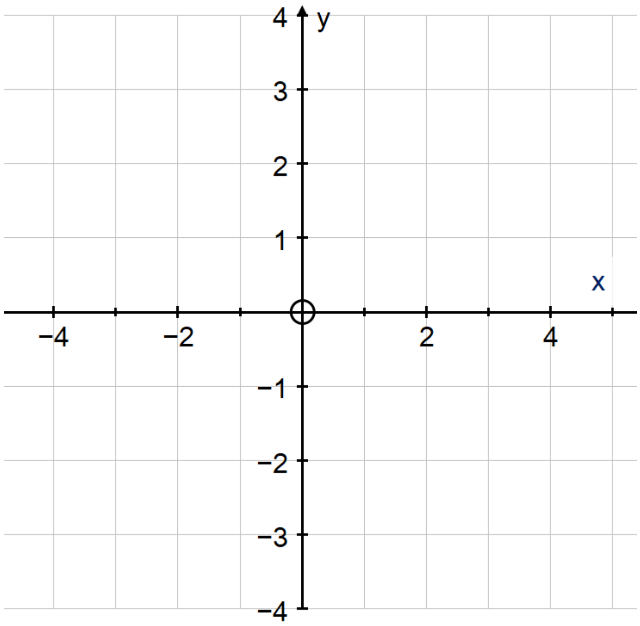
[10]

QUESTION SEVEN

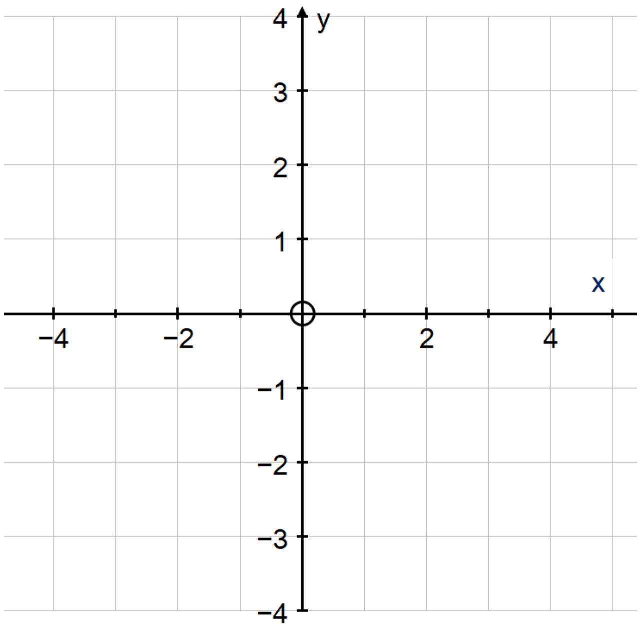
Given: $f(x) = 2^x - 2$

Sketch the following curves on the axes provided. Label your intercepts and asymptotes.

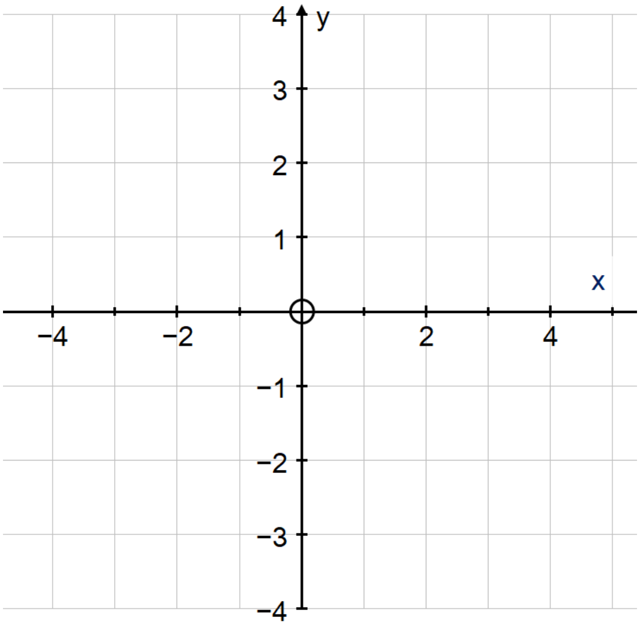
7.1 $y = f(x)$ (2)



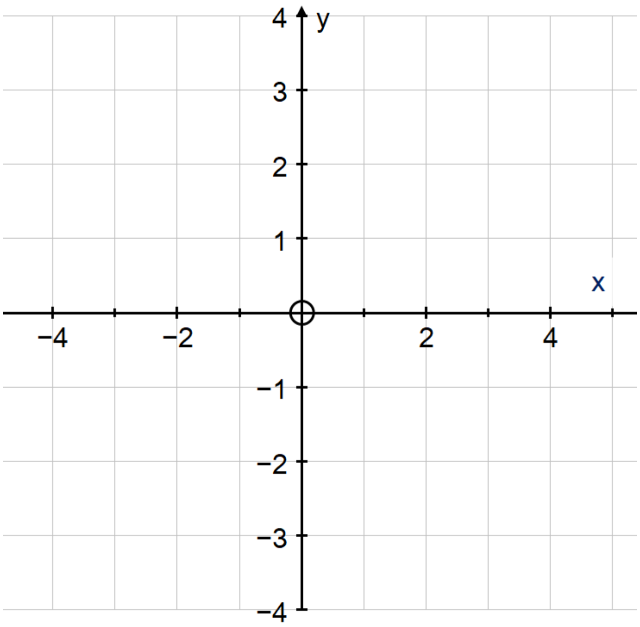
7.2 $y = |f(x)|$ (2)



7.3 $y = f(|x|)$ (3)



7.4 $y = f^{-1}(x)$ (3)



[10]

QUESTION EIGHT

Determine the following integrals:

8.1 $\int \frac{1}{x^2\left(1+\frac{1}{x}\right)^3}\ dx$ (8)

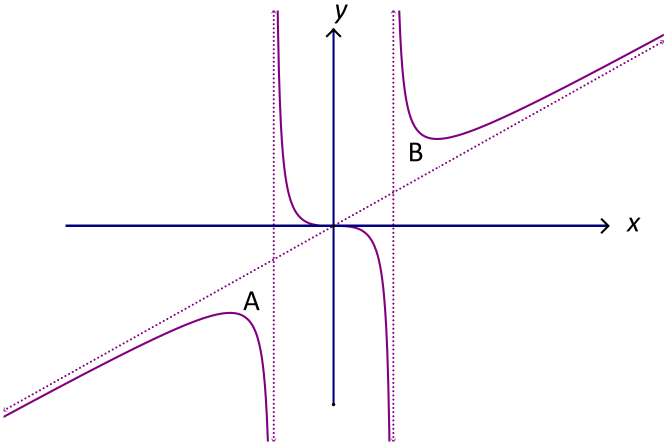
8.2 $\int \ln x\, dx$ (8)

8.3 $\int \frac{1}{a^2 - x^2}\, dx$, with a a constant ,using partial fractions. Your final answer must be as simplified as much as possible. (12)

[28]

QUESTION NINE

9.1 The sketch given below represents $h(x) = \dfrac{2x^3}{x^2 - 4}$

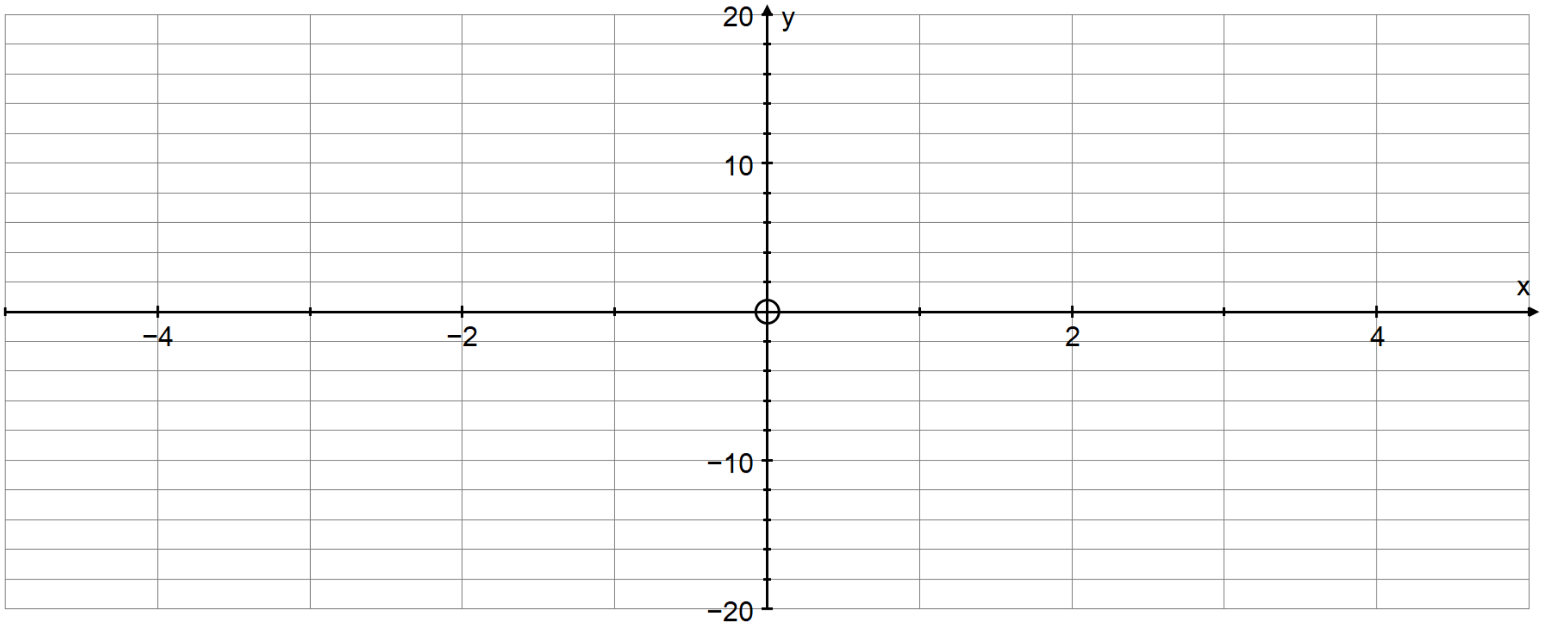


(a) Calculate the coordinates of A and B, the local maximum and minimum of $h(x)$. (8)

(b) Determine the equation of the three asymptotes. (7)

(c) From the graph, calculate the value(s) of x if $\sqrt{\dfrac{2x^3}{x^2-4}}$ is real. (3)

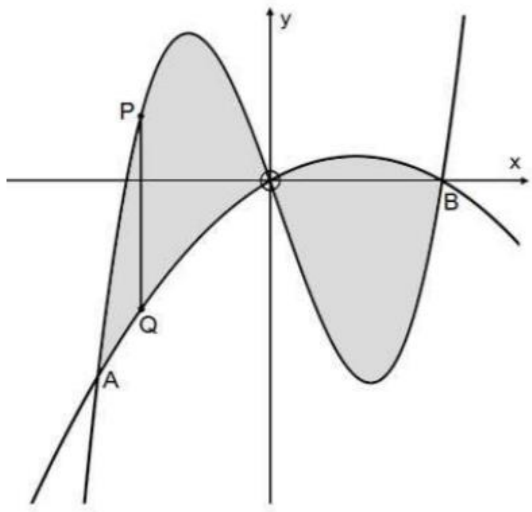
(d) Sketch $h(-x)$, on the diagram provided. Clearly show the asymptotes.



(7)

9.2 Refer to the figure below showing the graphs of:

$f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$



(a) Calculate the area of the shaded region. (8)

(b) PQ is a vertical line with P on f, Q on g between A and O. Determine the maximum length of PQ. (6)

[39]

Total : 200 marks